A periodic task $\tau_i$ generates an unbounded sequence of identical jobs. It is specified as $\tau_i = (C_i, D_i, T_i) \in \mathbb{N}^3$, where

- $C_i$ is the worst-case execution time (WCET),
- $D_i$ is the relative deadline, and
- $T_i$ is the period.
Recap

A periodic task $\tau_i$ generates an unbounded sequence of identical jobs. It is specified as $\tau_i = (C_i, D_i, T_i) \in \mathbb{N}^3$, where

- $C_i$ is the worst-case execution time (WCET),
- $D_i$ is the relative deadline, and
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Synchronous periodic tasks

We scheduled task sets $\mathcal{T}$ of synchronous periodic tasks

- with a (static) cyclic executive, and
- on the fly with EDF.

Last time
We will look into the following today.

- The *Fixed-Priority* (FP) scheduling algorithm
  - Including schedulability analysis
- Some variations and extensions of the task model:
  - Sporadic tasks
  - Asynchronous periodic tasks
  - Jitter
  - Resource sharing (which you have already seen)
  - Preemption overhead
Another scheduling algorithm

Fixed-Priority (FP) scheduling

Preprocessing: Assign a static priority to every task $\tau_i$.

Scheduling rule: Choose among the ready jobs to execute the job from the task with the highest priority (ties can be broken in various ways).
Another scheduling algorithm

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Preprocessing: Assign a *static priority* to every task $\tau_i$.
Scheduling rule: *Choose among the ready jobs to execute the job from the task with the highest priority* (ties can be broken in various ways).

We write $\tau_i \prec \tau_j$ to mean that $\tau_i$ has higher priority than $\tau_j$. 
Another scheduling algorithm

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Preprocessing: Assign a static priority to every task $\tau_i$.

Scheduling rule: Choose among the ready jobs to execute the job from the task with the highest priority (ties can be broken in various ways).

We write $\tau_i \prec \tau_j$ to mean that $\tau_i$ has higher priority than $\tau_j$.

Warning: Some authors use higher numbers for higher priorities, and others do it the other way around.
Let’s try it...

Schedule these tasks

\[ \mathcal{J} = \{ \tau_1, \tau_2, \tau_3 \} = \{(1,4,4), (2,3,5), (3,9,10)\} \]
Let’s try it...

\[ T = \{ \tau_1, \tau_2, \tau_3 \} = \{(1, 4, 4), (2, 3, 5), (3, 9, 10)\} \]
Let’s try it...

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Let’s try it…

\[ \mathcal{T} = \{ \tau_1, \tau_2, \tau_3 \} = \{(1, 4, 4), (2, 3, 5), (3, 9, 10)\} \]
Questions

Are some priority orderings better than others?
Are some priority orderings better than others?  YES!
Questions

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YES!

For any feasible task set $\mathcal{T}$, does there always exist some priority ordering with which $\mathcal{T}$ is FP-schedulable?

NO! Fixed-priority scheduling is suboptimal.
Questions

Are some priority orderings better than others?  YES!

For any feasible task set $\mathcal{T}$, does there always exist some priority ordering with which $\mathcal{T}$ is FP-schedulable?  NO!

Fixed-priority scheduling is suboptimal.
Questions

Are some priority orderings better than others? **YES!**

For any feasible task set $\mathcal{T}$, does there always exist some priority ordering with which $\mathcal{T}$ is FP-schedulable? **NO!**

Fixed-priority scheduling is *suboptimal*. 
Determining FP-schedulability

The worst case response time (WCRT)

The worst case response time $R_i$ of task $\tau_i$ is the longest possible duration between the release time and finishing time of any job from $\tau_i$ under a given scheduling policy.
**Determining FP-schedulability**

**The worst case response time (WCRT)**

The worst case response time $R_i$ of task $\tau_i$ is the longest possible duration between the release time and finishing time of any job from $\tau_i$ under a given scheduling policy.

**Observation**

A task set $\mathcal{T}$ is schedulable under the given scheduling policy iff $R_i \leq D_i$ for all $\tau_i \in \mathcal{T}$. 

---

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A task set $\mathcal{T}$ is schedulable under the given scheduling policy iff $R_i \leq D_i$ for all $\tau_i \in \mathcal{T}$.

Response times are fairly complicated to compute under EDF, but it is more straightforward under FP.
Calculating response times

$T = r_1; r_2; r_3$,
$g = f(1; 4; 4)\; (3; 6; 6)\; (3; 20; 20)$
with $r_1 \prec r_2 \prec r_3$.

An example task set

$\mathcal{J} = \{r_1, r_2, r_3\} = \{(1, 4, 4), (3, 6, 6), (3, 20, 20)\}$,
with $r_1 \prec r_2 \prec r_3$. 

Calculating response times

Let \( T = f_1; 2; 3 \) and \( g = f(1; 4; 4); (3; 6; 6); (3; 20; 20) \), with \( 1 \prec 2 \prec 3 \).

An example task set

\[ T = \{ \tau_1, \tau_2, \tau_3 \} = \{ (1, 4, 4), (3, 6, 6), (3, 20, 20) \}, \]

with \( \tau_1 \prec \tau_2 \prec \tau_3 \).

Challenge

Calculate the response time of *the first job* from each task.
Iteratively refining our estimate

1. Make an initial underestimation $R^1_i$ of the response time (for example, $R^1_i = C_i$).

2. Refine the estimate $R^k_i$ to $R^{k+1}_i$ by including the interference from higher priority tasks we can see in that interval of length $R^k_i$.

3. If $R^k_i = R^{k+1}_i$, then the estimate is the response time. Otherwise, repeat step (2).
The response time $R_i$ of the *first job* from $\tau_i$ is the *smallest positive fixed point* to the recurrence relation

$$
R_i = C_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j,
$$

where $\text{hp}(\tau_i)$ denotes the set of tasks with higher or equal priority than $\tau_i$ (excluding $\tau_i$ itself).
Some insights!

A recurrence relation

The response time $R_i$ of the first job from $\tau_i$ is the smallest positive fixed point to the recurrence relation

$$R_i = C_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j,$$

where $\text{hp}(\tau_i)$ denotes the set of tasks with higher or equal priority than $\tau_i$ (excluding $\tau_i$ itself).

For a task set with constrained deadlines, the response time of the first job of $\tau_i$ is the worst-case response time of $\tau_i$. 
Response Time Analysis (RTA)

Theorem (Joseph & Pandya, 1986)

For a task set with constrained deadlines under preemptive FP-scheduling on a single processor, the worst case response time $R_i$ of $\tau_i$ is the smallest positive fixed point to the recurrence relation

\[ R_i = C_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j. \]
Response Time Analysis (RTA)

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Using this method we can test FP-schedulability in \textit{pseudo-polynomial time} because we only care if $R_i \leq D_i$. 
Response Time Analysis (RTA)

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Using this method we can test FP-schedulability in pseudo-polynomial time because we only care if $R_i \leq D_i$.

This FP-schedulability problem is NP-complete (RTSS 2017).
How to pick a priority ordering?

Under the Rate-Monotonic (RM) priority ordering, tasks with shorter periods get higher priorities.

Under the Deadline-Monotonic (DM) priority ordering, tasks with shorter relative deadlines get higher priorities.
How to pick a priority ordering?

Rate-Monotonic ordering

Under the \textit{Rate-Monotonic (RM)} priority ordering, tasks with shorter periods get higher priorities.
How to pick a priority ordering?

Rate-Monotonic ordering

Under the *Rate-Monotonic (RM)* priority ordering, tasks with shorter periods get higher priorities.

Deadline-Monotonic ordering

Under the *Deadline-Monotonic (DM)* priority ordering, tasks with shorter relative deadlines get higher priorities.
RM/DM optimality

Theorem (Liu & Layland, 1973)

*RM* is an *optimal priority ordering* for synchronous periodic task sets with *implicit deadlines* executed on a single preemptive processor.
**RM/DM optimality**

**Theorem (Liu & Layland, 1973)**

*RM* is an **optimal priority ordering** for synchronous periodic task sets with *implicit deadlines* executed on a single preemptive processor.

**Theorem (Leung & Whitehead, 1982)**

*DM* is an **optimal priority ordering** for synchronous periodic task sets with *constrained deadlines* executed on a single preemptive processor.
RM is an optimal priority ordering for synchronous periodic task sets with implicit deadlines executed on a single preemptive processor.

DM is an optimal priority ordering for synchronous periodic task sets with constrained deadlines executed on a single preemptive processor.

(The latter subsumes the former.)
A task set $T$ of $n$ tasks with *implicit deadlines* is FP-schedulable with *RM priority ordering* on a single preemptive processor if

$$U(T) \leq n(2^{\frac{1}{n}} - 1).$$
A task set $\mathcal{T}$ of $n$ tasks with *implicit deadlines* is FP-schedulable with *RM priority ordering* on a single preemptive processor if

$$U(\mathcal{T}) \leq n(2^{\frac{1}{n}} - 1).$$

*Sufficient*, but not necessary!
A simpler (but famous) test!

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$$\lim_{n \to \infty} n(2^{\frac{1}{n}} - 1) = \ln(2) \approx 0.693$$
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*Theorem (Liu & Layland, 1973)*

*Sufficient*, but not necessary!

$$\lim_{n \to \infty} n(2^{\frac{1}{n}} - 1) = \ln(2) \approx 0.693$$

Compare to $U(\mathcal{T}) \leq 1$ for EDF.
Another common task model

The sporadic task model

A *sporadic* task \( \tau_i \) is also given by a triple \((C_i, D_i, T_i) \in \mathbb{N}^3\), where

- \( C_i \) is the worst-case execution time (WCET),
- \( D_i \) is the relative deadline, and
- \( T_i \) is the *minimum inter-arrival time* (but often just called “the period”).
Another common task model

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- \( C_i \) is the worst-case execution time (WCET),
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- \( T_i \) is the minimum inter-arrival time (but often just called “the period”).

Each sporadic task can generate an infinite number of job sequences!
A job sequence to end them all

The synchronous arrival sequence (SAS)

Out of all the job sequences that a set of sporadic task can generate, the hardest to schedule (both for EDF and FP) on a single preemptive processor is the one where all tasks

1. release their first jobs at the same time, and
2. then release jobs as early as possible.
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This is the sequence generated by a set of synchronous periodic tasks!
A job sequence to end them all

The synchronous arrival sequence (SAS)

Out of all the job sequences that a set of sporadic task can generate, the hardest to schedule (both for EDF and FP) on a single preemptive processor is the one where all tasks

1. release their first jobs at the same time, and
2. then release jobs as early as possible.

This is the sequence generated by a set of synchronous periodic tasks!

The tests for synchronous periodic tasks are also valid for sporadic tasks! (But you can’t do the static schedule.)
Yet another task model...

The asynchronous periodic task model

An *asynchronous periodic* task $\tau_i$ is given by a quadruple $(O_i, C_i, D_i, T_i) \in \mathbb{N}^4$, where

- $O_i$ is the *offset* (the arrival time of the first job),
- $C_i$ is the worst-case execution time (WCET),
- $D_i$ is the relative deadline, and
- $T_i$ is the period.

Sometimes asynchronous periodic tasks are just called “periodic tasks”.
Analyzing asynchronous tasks

Because the synchronous arrival sequence (SAS) is always the hardest to schedule, an asynchronous periodic task set can never be harder to schedule for EDF or FP than the corresponding synchronous periodic task set.
Analyzing asynchronous tasks

Observation

Because the synchronous arrival sequence (SAS) is always the hardest to schedule, an asynchronous periodic task set *can never be harder* to schedule for EDF or FP than the corresponding synchronous periodic task set.

The tests for synchronous periodic (or sporadic) tasks are still *sufficient* for asynchronous periodic tasks!
Analyzing asynchronous tasks

Observation

Because the synchronous arrival sequence (SAS) is always the hardest to schedule, an asynchronous periodic task set can never be harder to schedule for EDF or FP than the corresponding synchronous periodic task set.

The tests for synchronous periodic (or sporadic) tasks are still sufficient for asynchronous periodic tasks!

Exact tests for asynchronous periodic tasks are generally more involved and computationally harder.
An extension to the model: jitter

A task is said to experience \textit{jitter} if there is some variation in the time it takes for its jobs to become available for execution after their arrival.
An extension to the model: jitter

A task is said to experience *jitter* if there is some variation in the time it takes for its jobs to become available for execution after their arrival.

\[ \tau_i = (C_i, D_i, T_i, J_i) = (2, 6, 10, 3) \]
An extension to the model: jitter

A task is said to experience *jitter* if there is some variation in the time it takes for its jobs to become available for execution after their arrival.

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\]
An extension to the model: jitter

A task is said to experience jitter if there is some variation in the time it takes for its jobs to become available for execution after their arrival.

\[ D_i \leq J_i = (C_i, D_i, T_i, J_i) = (2, 6, 10, 3) \]
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An extension to the model: Jitter

A task is said to experience \textit{jitter} if there is some variation in the time it takes for its jobs to become available for execution after their arrival.

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A task is said to experience *jitter* if there is some variation in the time it takes for its jobs to become available for execution after their arrival.

\[
\tau_i = (C_i, D_i, T_i, J_i) = (2, 6, 10, 3)
\]

\[D_i \leq J_i\]
For a task set with constrained deadlines and jitter under preemptive FP-scheduling on a single processor, the worst case response time $R_i$ of $\tau_i$ is at most

$$R_i = j_i + w_i,$$

where $w_i$ is the smallest positive fixed point to

$$w_i = c_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lceil \frac{w_i + j_j}{t_j} \right\rceil \cdot c_j.$$
Extending RTA with jitter

For a task set with constrained deadlines and jitter under preemptive FP-scheduling on a single processor, the worst case response time $R_i$ of $\tau_i$ is at most

$$R_i = J_i + w_i,$$

where $w_i$ is the smallest positive fixed point to

$$w_i = C_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lfloor \frac{w_i + J_j}{T_j} \right\rfloor \cdot C_j.$$

Note 1: Exact for sporadic tasks, but only sufficient for periodic tasks.
For a task set with *constrained deadlines and jitter* under preemptive FP-scheduling on a single processor, the worst case response time $R_i$ of $\tau_i$ is *at most*

$$R_i = J_i + w_i,$$

where $w_i$ is the smallest positive fixed point to

$$w_i = C_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lceil \frac{w_i + J_j}{T_j} \right\rceil \cdot C_j.$$

Note 1: Exact for sporadic tasks, but only sufficient for periodic tasks.

Note 2: DM might not be the optimal priority ordering here.
Recap: With a given resource sharing protocol (PIP, PCP, etc.), task $\tau_i$ may experience up to $B_i$ time units of \textit{blocking due to priority inversion}. (See Wang’s previous slides and Buttazzo’s book for details on how to compute $B_i$.)
Recap: With a given resource sharing protocol (PIP, PCP, etc.), task $\tau_i$ may experience up to $B_i$ time units of \textit{blocking due to priority inversion}. (See Wang’s previous slides and Buttazzo’s book for details on how to compute $B_i$.)

For a task set with \textit{constrained deadlines and blocking} under preemptive FP-scheduling on a single processor, the worst case response time $R_i$ of $\tau_i$ is the smallest positive fixed point to

$$R_i = C_i + B_i + \sum_{\tau_j \in \text{hp}(\tau_i)} \left\lceil \frac{R_i}{T_j} \right\rceil \cdot C_j.$$
So far we have assumed that preemptions are *free*, but in reality they are not. Preemptions may inflate execution times due to

- Context-switch overheads
  - Saving current execution state
  - Loading new execution state
  - ...
- Tainting cache states
- Other microarchitectural features
  - ...

*Preemption overhead*
Counting preemptions

A nice property

For both EDF and FP, there can be \textit{at most one} new pre-emption per job release.
Counting preemptions

A nice property

For both EDF and FP, there can be at most one new pre-emption per job release.

If we can estimate an upper bound on the total cost of doing one preemption, then a safe (but likely pessimistic) approach is to simply inflate every WCET by this cost.
Counting preemptions

A nice property

For both EDF and FP, there can be \textit{at most one} new preemption per job release.

If we can estimate an upper bound on the total cost of doing one preemption, then a safe (but likely pessimistic) approach is to simply inflate every WCET by this cost.

Reducing and managing preemptions is an active area of research, see Buttazzo’s book (chapter 8) for some more details.
FP scheduling is more commonly found than EDF in RTOSs.
FP is suboptimal: It can not schedule all feasible task sets.
The priority ordering matters greatly.
  • Rate-monotonic (RM) is optimal for implicit deadlines.
  • Deadline-monotonic (DM) is optimal for constrained deadlines.
With response time analysis (RTA) we can find WCRTs of tasks
  • We get a schedulability test by checking if $R_i \leq D_i$ for all $\tau_i$.
  • Test runs in pseudo-polynomial time with constrained deadlines.
    • Which is quite fast in practice.
    • More complicated for arbitrary deadlines (we didn’t cover this).
    • Test can be extended to handle things like jitter.
Liu & Layland’s utilization bound offers a simpler sufficient test.
  • Only for RM and implicit deadlines.
Sporadic tasks are often more flexible than periodic tasks.
  • Because the SAS is the worst-case job sequence, (most) schedulability tests are the same as for synchronous periodic tasks.
Asynchronous periodic tasks have offsets for the first job.
  • Tests for sporadic tasks are still sufficient.