Workload Models

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(With some slides borrowed from Martin Stigge)
Recap

A sporadic task $\tau_i$ is given by a triple $(C_i, D_i, T_i) \in \mathbb{N}^3$, where

- $C_i$ is the worst-case execution time (WCET),
- $D_i$ is the relative deadline, and
- $T_i$ is the minimum inter-release separation time (or just period).

$\tau_i = (C_i, D_i, T_i) = (3, 6, 10)$
Today’s topic

1. We will look at a few *generalizations* of the sporadic task model, which allow for more complicated patterns of job releases.

2. We will outline analysis techniques for feasibility (or EDF-schedulability) for one of these task models.
Recall the feasibility test for sporadic tasks

A sporadic task set $\mathcal{T}$ is feasible on a single preemptive processor iff

$$\forall t, \text{ such that } t \geq 0 : \quad \text{dbf}(\mathcal{T}, t) \leq t.$$
Recall the feasibility test for sporadic tasks

\[ \text{Recall the feasibility test for sporadic tasks} \]

\[ (C_i, D_i, T_i) \]
Recall the feasibility test for sporadic tasks:

\[ C_i; D_i; T_i \]

\[ \text{dbf}(i; t) = \max(0; \lfloor t \cdot D_i \cdot T_i \rfloor + 1) \]

\[ \text{dbf}(T; t) = \sum_i \text{dbf}(i; t) \]

\[(C_i, D_i, T_i)\]
Recall the feasibility test for sporadic tasks

\[ C_i; D_i; T_i \]

\[ D_i T_i C_i^{dbf}(i; t) = \max(0; \lfloor t D_i T_i \rfloor + 1) \]

\[ C_i^{dbf}(T; t) = \sum_i 2 T_i^{dbf}(i; t) \]
Recall the feasibility test for sporadic tasks

$\text{Recall the feasibility test for sporadic tasks}$

$$(C_i, D_i, T_i)$$
Recall the feasibility test for sporadic tasks

\[ C_i; D_i; T_i \]

\[ t \]

\[ C_{i} \text{dbf}(T; t) = \sum_{i} C_{i} \text{dbf}(i; t) \]

\[ D_i \quad T_i \]

\[ (C_i, D_i, T_i) \]
Recall the feasibility test for sporadic tasks.

\[
dbf(\tau_i, t) = \max \left( 0, \left\lfloor \frac{t - D_i}{T_i} \right\rfloor + 1 \right) \cdot C_i
\]

\((C_i, D_i, T_i)\)
Recall the feasibility test for sporadic tasks

\begin{align*}
\text{dbf}(\tau_i, t) &= \max \left(0, \left\lfloor \frac{t-D_i}{T_i} \right\rfloor + 1 \right) \cdot C_i \\
\text{dbf}(\mathcal{T}, t) &= \sum_{\tau_i \in \mathcal{T}} \text{dbf}(\tau_i, t)
\end{align*}

\((C_i, D_i, T_i)\)
Recall the feasibility test for sporadic tasks

\[ t + t + t + t = t, \]

such that

\[ t \geq 0 : \]

\[ dbf(T; t) \leq t. \]
Recall the feasibility test for sporadic tasks

\[
dbf(T; t) \leq t
\]
Recall the feasibility test for sporadic tasks:

\[ t_1 + t_2 + t_3 + \cdots \leq t \]

such that \( t \geq 0 \):

\[ dbf(T; t) \leq t \]
Recall the feasibility test for sporadic tasks

\[ \forall t, \text{ such that } t \geq 0 : \quad \text{dbf}(\mathcal{J}, t) \leq t \]
Recall the feasibility test for sporadic tasks

\[ t^+ + t^+ + t^+ + t^+ = t^8 \]

such that \( t \geq 0 \):

\[ \text{dbf}(\mathcal{T}, t) \leq t \]

\( \forall t, \text{ such that } t \geq 0 : \text{dbf}(\mathcal{T}, t) \leq t \)
Recall the feasibility test for sporadic tasks:

$$\forall t, \text{ such that } t \geq 0 : \quad \text{dbf}(\mathcal{T}, t) \leq t$$
Recall the feasibility test for sporadic tasks:

$\text{slope} = \frac{\sum_i C_i}{\sum_i C_i}$

$\text{HP}(T) = \text{LCM of periods}$
Recall the feasibility test for sporadic tasks

\( HP(J) = \text{LCM of periods} \)
Recall the feasibility test for sporadic tasks

\[ HP(\mathcal{J}) = \text{LCM of periods} \]

\[ \text{slope} = U(\mathcal{J}) \]
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\[ \sum_i C_i \]

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Recall the feasibility test for sporadic tasks

$$\text{HP}(J) = \text{LCM of periods}$$

$$\sum_i C_i = \frac{\sum_i C_i}{1 - U(J)}$$

slope $= U(J)$
Example code structure for a periodic/sporadic task:

```plaintext
loop
  // Execute some function for, e.g.,
  // up to 11ms
  // (obtained via WCET analysis)
  delay until Previous_Period + 50ms;
end loop;
```
Generalizing sporadic tasks

• Example code structure for a periodic/sporadic task:

```
loop
    // Execute some function for, e.g.,
    // up to 11ms
    // (obtained via WCET analysis)
    delay until Previous_Period + 50ms;
end loop;
```

• What if the structure is more complicated?
A more complicated control structure

- Code is not always periodic:

```plaintext
loop

  // Execute some function
  delay until Period_Start + 50ms;

  // Execute another function
  delay until Prev_Function + 30ms;

  // Execute yet another function
  delay until Prev_Function + 70ms;

end loop;
```
A more complicated control structure

- Code is not always periodic:

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loop

// Execute some function
delay until Period_Start + 50ms;

// Execute another function
delay until Prev_Function + 30ms;

// Execute yet another function
delay until Prev_Function + 70ms;

end loop;
```

- Here a task is split in *frames*, each with own
  - Execution time
  - Inter-release separation until next frame
  - Relative deadline
The Generalized Multiframe (GMF) Task Model

- Each task *cycles* through job types
  - Vector for WCET \( (e^{(1)}, \ldots, e^{(n)}) \)
  - Vector for deadlines \( (d^{(1)}, \ldots, d^{(n)}) \)
  - Vector for minimum inter-release delays \( (p^{(1)}, \ldots, p^{(n)}) \)
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The Generalized Multiframe (GMF) Task Model

Diagram showing a cycle of tasks $j_1, j_2, j_3, j_4, j_5$ with edges connecting them:

- $j_1$ to $j_2$: labeled 15
- $j_1$ to $j_3$: labeled 12
- $j_2$ to $j_3$: labeled 5
- $j_2$ to $j_5$: labeled 30
- $j_3$ to $j_4$: labeled 10
- $j_4$ to $j_5$: labeled 5
- $j_5$ to $j_1$: labeled 4
- $j_1$ to $j_3$: labeled 4
- $j_2$ to $j_4$: labeled 30
- $j_3$ to $j_5$: labeled 27
- $j_4$ to $j_1$: labeled 10
- $j_5$ to $j_2$: labeled 8

Each task is connected to the next in the cycle.
The Generalized Multiframe (GMF) Task Model
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The Generalized Multiframe (GMF) Task Model

Diagram:

- Node $j_1$ connected to $j_2$ with weight 15
- Node $j_2$ connected to $j_3$ with weight 5
- Node $j_3$ connected to $j_4$ with weight 30
- Node $j_4$ connected to $j_5$ with weight 10
- Node $j_5$ connected to $j_1$ with weight 12

Weights:
- From $j_1$ to $j_2$: 4
- From $j_2$ to $j_3$: 2
- From $j_3$ to $j_4$: 10
- From $j_4$ to $j_5$: 3
- From $j_5$ to $j_1$: 9

Nodes:
- $j_1$, $j_2$, $j_3$, $j_4$, $j_5$
The Generalized Multiframe (GMF) Task Model

- $j_1$ connected to $j_2$ with weight 15
- $j_2$ connected to $j_3$ with weight 5
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- $j_1$ to $j_2$: 15
- $j_2$ to $j_3$: 5
- $j_3$ to $j_4$: 30
- $j_4$ to $j_5$: 10
- $j_5$ to $j_1$: 12

Task States:
- $j_1$: $\langle 4, 9 \rangle$
- $j_2$: $\langle 2, 5 \rangle$
- $j_3$: $\langle 10, 27 \rangle$
- $j_4$: $\langle 1, 10 \rangle$
- $j_5$: $\langle 3, 8 \rangle$
The Generalized Multiframe (GMF) Task Model

The diagram illustrates the relationships and timelines among the tasks, with specific event times and durations.
The Generalized Multiframe (GMF) Task Model
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  - Vector for WCET \((e^{(1)}, \ldots, e^{(n)})\)
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- Schedulability analysis for EDF?
  - Use the demand bound function, like for sporadic tasks
  - How to calculate dbf? What about the bound?
  - Exercise for the interested!
  - Read more in *Generalized Multiframe Tasks* (Baruah et al., 1999)
**Generalize differently?**

- What about *branches*?

```plaintext
loop
  // Execute some function
  delay until Period_Start + 50ms;
  if (condition) then {
    // Execute another function
    delay until Prev_Function + 30ms;
  } else {
    // Execute yet another function
    delay until Prev_Function + 70ms;
  }
end loop;
```

Each task is a tree. Vertices represent jobs. Edges represent control flow and delays. Restarted once a leaf is reached.
What about *branches*?

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```

Each task is a *tree*

- Vertices represent jobs
- Edges represent control flow and delays
- Restarted once a leaf is reached
The Recurring Branching (RB) Task Model

- Introduces \textit{branching} structures
- A \textit{tree} for each task
  - Vertices \textit{j}: job types with WCET and deadline \(\langle e(j), d(j) \rangle\)
  - Edges \((j_i, j_k)\): minimum inter-release delays \(p(j_i, j_k)\)
The Recurring Branching (RB) Task Model

- Introduces *branching* structures
- A *tree* for each task
  - Vertices $j$: job types with WCET and deadline $\langle e(j), d(j) \rangle$
  - Edges $(j_i, j_k)$: minimum inter-release delays $p(j_i, j_k)$
  - General period parameter $P$

Feasibility analysis is also dbf-based, rather involved. Read more in Feasibility analysis of recurring branching tasks (Baruah, 1998)
The Recurring Branching (RB) Task Model

- Introduces branching structures
- A tree for each task
  - Vertices \( j \): job types with WCET and deadline \( <e(j), d(j)> \)
  - Edges \( (j_i, j_k) \): minimum inter-release delays \( p(j_i, j_k) \)
  - General period parameter \( P \)

- Feasibility analysis is also dbf-based, rather involved
- Read more in Feasibility analysis of recurring branching tasks (Baruah, 1998)
Generalize further?

- Now we can model:
  - Periodic behavior
  - Multiple frames
  - Branching behavior
- Still not possible:
  - Local loops
  - Local modes
  - ...

```plaintext
loop
  f1();
  delay ...
  if (cond) then {
    while (cond2) {
      f2();
      delay ...
    }
  }
  else {
    f3();
    delay ...
  }
end loop;
```
The Digraph Real-Time (DRT) Task Model

- Generalizes sporadic, GMF, RRT (almost), ...
- **Directed graph** for each task
  - Vertices $j$: job types with WCET and deadline $\langle e(j), d(j) \rangle$
  - Edges $(j_i, j_k)$: minimum inter-release delays $p(j_i, j_k)$
DRT: Semantics

Path = (j_4)
Path = (j_4; j_2)
Path = (j_4; j_2; j_3)

\( \langle 2, 5 \rangle \)
\( \langle 1, 8 \rangle \)
\( \langle 3, 8 \rangle \)

\( \langle 2, 5 \rangle \)
\( \langle 1, 8 \rangle \)
\( \langle 5, 10 \rangle \)

\( \langle 1, 5 \rangle \)
Path $\pi = (j_4)$
Path $\pi = (j_4, j_2)$
DRT: Semantics

Path $\pi = (j_4, j_2, j_3)$
The main result about demand bound functions still holds:

A task set $\mathcal{T} = \{\tau_1, \ldots, \tau_n\}$ of DRT tasks is feasible on a single preemptive processor iff

$$\forall t \geq 0 : \sum_{\tau_i \in \mathcal{T}} \text{dbf}(\tau_i, t) \leq t.$$
The main result about demand bound functions still holds:

**Theorem**

A task set $\mathcal{J} = \{\tau_1, \ldots, \tau_n\}$ of DRT tasks is feasible on a single preemptive processor iff

$$\forall t \geq 0 : \sum_{\tau_i \in \mathcal{J}} \text{dbf}(\tau_i, t) \leq t.$$ 

Thus, do as before:

1. Compute demand bound functions $\text{dbf}(\tau_i, t)$.
   - How to do that for a given $t$?

2. Test the inequality for all $t \leq B$ for some bound $B$.
   - How to derive the bound $B$?
**Demand pairs**

- Recall demand bound function:
  - Interval length $t$, sum all demand ...
  - ... of jobs *released* and with *deadline* inside
Demand pairs

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  - Interval length $t$, sum all demand ...
  - ... of jobs released and with deadline inside

- Consider a path in a task’s graph:

![Diagram of task graph]

**Execution demand:**

$$5 + 1 + 3 = 9$$

**Interval size:**

$$20 + 15 + 8 = 43$$

**Demand pair**

$$\langle 9; 43 \rangle$$
**Demand pairs**

- Recall demand bound function:
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- Consider a path in a task’s graph:

  ![Diagram](image)

  **Execution demand:**
  \[5 + 1 + 3 = 9\]

  **Interval size:**
  \[20 + 15 + 8 = 43\]

  *Demand pair \langle 9, 43 \rangle*
From demand pair \( \langle e, d \rangle \) we learn:

- Task can create \( e \) units of exec. demand ...
- ... during interval of size \( d \)
Demand pairs (cont.)

- From demand pair $\langle e, d \rangle$ we learn:
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- Useful for the demand bound function!
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\[
\langle 9, 43 \rangle
\]

Graph showing demand pairs with \( t \) on the x-axis and demand on the y-axis.
Demand pairs (cont.)

• From demand pair $\langle e, d \rangle$ we learn:
  • Task can create $e$ units of exec. demand ...
  • ... during interval of size $d$

• Useful for the demand bound function!

Thus: Compute all demand pairs, then take "maximum"
$$dbf(i, t) = \max_{f \in \langle e, d \rangle} \text{demand pair with } d \leq t$$
Demand pairs (cont.)

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Thus: Compute *all demand pairs*, then take “maximum”

$$\text{dbf}(\tau_i, t) = \max \{ e \mid \langle e, d \rangle \text{ demand pair with } d \leq t \}$$
Demand pairs (cont. 2)

More formally

- Given path \( \pi = (\pi_1, \ldots, \pi_k) \)
- **Execution demand**: \( e(\pi) := \sum_{i=1}^{k} e(\pi_i) \)
- **Deadline**: \( d(\pi) := \sum_{i=1}^{k-1} p(\pi_i, \pi_{i+1}) + d(\pi_k) \)
- \( \langle e(\pi), d(\pi) \rangle \) is a demand pair for \( \pi \)
Demand pairs (cont. 2)

More formally

- Given path $\pi = (\pi_1, \ldots, \pi_k)$
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How to compute all demand pairs?

- Enumerate all paths?
Demand pairs (cont. 2)

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How to compute all demand pairs?

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Demand pairs (cont. 2)

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- Given path $\pi = (\pi_1, \ldots, \pi_k)$
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How to compute all demand pairs?

- Enumerate all paths? Too expensive! (Exponential)
- Better: Iteration using abstraction
- (Remark: Demand pairs are abstractions of paths)
**Demand triples**

- Idea: Start with 0-paths (one vertex), extend stepwise
Demand triples

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- We need: Abstraction which
  1. allows to *extend* paths,
  2. contains demand pair information,
  3. without visiting/storing all paths
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- Idea: *Demand triples*
  - Execution demand \( e(\pi) \)
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  - Last vertex \( \pi_k \)
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- Idea: *Demand triples*
  - Execution demand $e(\pi)$
  - Deadline $d(\pi)$
  - Last vertex $\pi_k$
- Demand triple $\langle e(\pi), d(\pi), \pi_k \rangle$ is another path abstraction!

![Diagram showing paths and demand triples](image)

Path $(j_4)$
\[ \sim \langle 5, 10, j_4 \rangle \]
Path $(j_4, j_2)$
\[ \sim \langle 6, 28, j_2 \rangle \]
Path $(j_4, j_2, j_3)$
\[ \sim \langle 9, 43, j_3 \rangle \]
Iterative procedure

- Create all demand triples up to bound $B$:
  1. Store all 0-paths, i.e., $\langle e(j), d(j), j \rangle$ for all vertices $j$
  2. Pick some stored unmarked demand triple $\langle e, d, j_i \rangle$, then mark it
  3. Create new demand triples:
     - For each successor vertex $j_k$ of $j_i$
     - $e' = e + e(j_k)$
     - $d' = d - d(j_i) + p(j_i, j_k) + d(j_k)$
     - $\langle e', d', j_k \rangle$ is a new demand triple!
  4. Store each new $\langle e', d', j_k \rangle$ if
     - it is not stored yet, and
     - $d' \leq B$
  5. Repeat from 2 until there are no more unmarked triples

More efficient procedure than enumerating all paths!

Note: Actual paths are never stored

Optimizations: Discard non-critical triples along the way

Exercise: What’s $dbf(i, 26)$ if $i$ is the task on previous slide?
Iterative procedure

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- More **efficient** procedure than enumerating all paths!
  - Note: Actual paths are never stored
  - Optimizations: Discard non-critical triples along the way
- Exercise: What’s dbf($\tau_i$, 26) if $\tau_i$ is the task on previous slide?
Which \( t \) to check?

- Recall: Want to check \( \text{dbf}(\mathcal{J}, t) \leq t \) for all \( t \)
- Find a bound for \( t \) just like for sporadic tasks!

\[ \forall t : \text{dbf}(\mathcal{J}, t) \leq t \]

- Derive linear bound for \( \text{dbf}(\mathcal{J}, t) \)
  - Intersection with \( t \) gives bound \( B \)
  - How to find the linear bound?
**Linear bound for dbf**

- Bound is based on *utilization*
  - Long-term demand “rate” (asymptotic)
  - For DRT: “most dense” cycle
  - Highest ratio execution demand (vertices) vs. duration (edges)
    - How to find this value? (Exercise!)

\[
dbf(\tau_i, t) \leq U(\tau_i) \cdot t + \sum_{k} e(j_k)
\]

- Any path can be split into vertices in cycles and not in cycles
- Leads to

- So, check dbf(\tau_i, t) for which t? (Exercise!)
DRT feasibility: Summary

- Feasibility test (or schedulability test for EDF), based on dbf
- First, compute the *utilization* for all tasks
  - Based on most dense cycles in graphs
- Derive *bound B*
- Compute $\text{dbf}(\mathcal{T}, t)$ for all $t \leq B$
  - Uses iterative procedure with *demand triples*
  - Path abstraction to reduce complexity
- If $t \leq B$ with $\text{dbf}(\mathcal{T}, t) > t$ is found, then $\mathcal{T}$ is infeasible
- Otherwise $\mathcal{T}$ is feasible

Read more in *The Digraph Real-Time Task Model* (Stigge et al., 2011) … or in Martin Stigge’s PhD thesis. Play with a Python implementation: libdrt
DRT feasibility: Summary

- Feasibility test (or schedulability test for EDF), based on dbf
- First, compute the utilization for all tasks
  - Based on most dense cycles in graphs
- Derive bound $B$
- Compute $\text{dbf}(\mathcal{T}, t)$ for all $t \leq B$
  - Uses iterative procedure with demand triples
  - Path abstraction to reduce complexity
- If $t \leq B$ with $\text{dbf}(\mathcal{T}, t) > t$ is found, then $\mathcal{T}$ is infeasible
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- Read more in *The Digraph Real-Time Task Model* (Stigge et al., 2011)
- ...or in Martin Stigge’s PhD thesis
- Play with a Python implementation: libdrt
**Generalize further? Timed automata!**

**Idea**

Annotate locations on timed automata with job parameters (WCET, relative deadline) let a job be created every time such a location is visited.

Analysis is generally costly, but sometimes fast enough.