



Intro. Computer Control Systems: F5

Control structures, frequency descriptions

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F4: Quiz!

1) PI-control

- a May suppress constant disturbances ↑
- b Ensures zero stationary control errors ↑
- c Ensures stable closed-loop system ↓

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- c Ensures stable closed-loop system ↓

2) Routh's algorithm is a method that

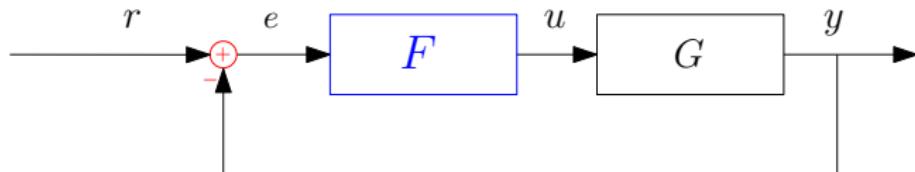
- a Checks for oscillations in systems ↑
- b Aids design of stable closed-loop systems ↑
- c Guarantees stable systems ↓

Simple linear feedback control

Exemple: PID-controller

Simple feedback using *control error*:

$$U(s) = \underbrace{F(s)(R(s) - Y(s))}_{=E(s)} = F(s)R(s) - F(s)Y(s).$$

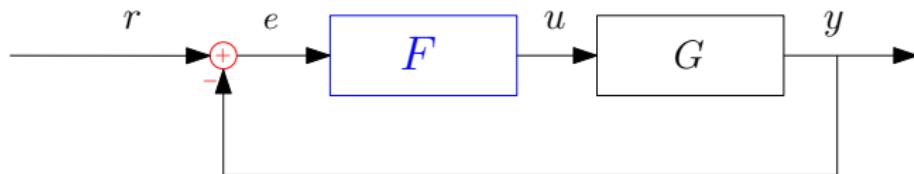


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Closed-loop system:

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s)$$

General linear feedback control

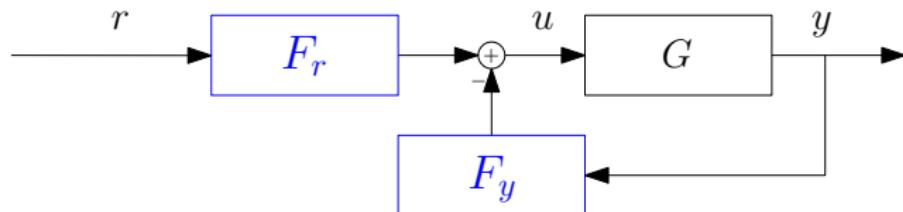
General feedback with *measured* signals:

$$U(s) = F_r(s)R(s) - F_y(s)Y(s).$$

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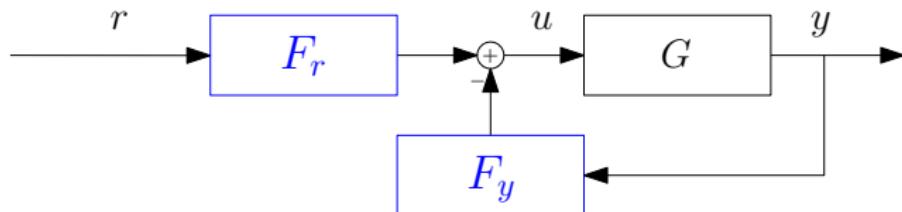
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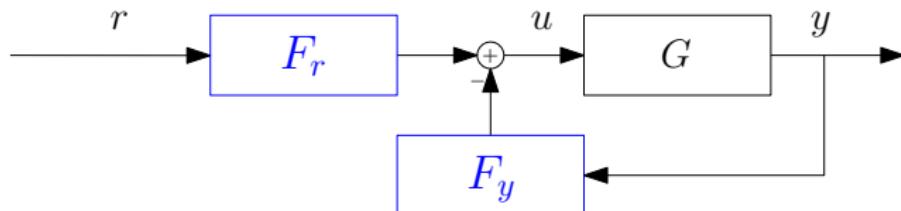
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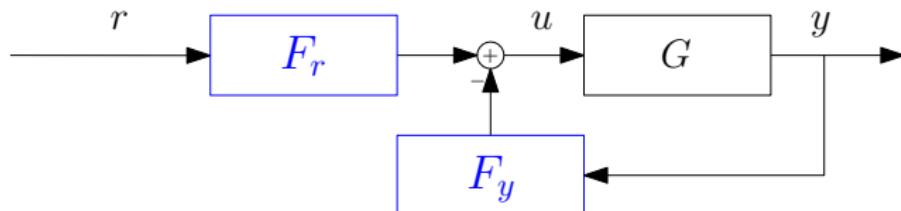
$$Y(s) = \frac{G(s)F_r(s)}{1 + G(s)F_y(s)} R(s)$$

- ▶ $F_y(s) = F_r(s) = F(s) \Rightarrow$ simple linear feedback
- ▶ $F_y(s) \neq F_r(s) \Rightarrow$ more *degrees of freedom*.

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General feedback with *measured signals*:

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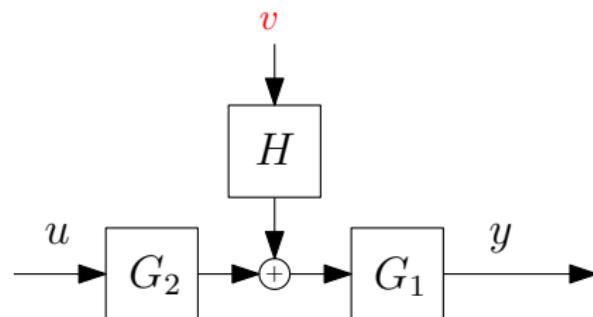
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See state-feedback controller F8-F10.

Feedforward control

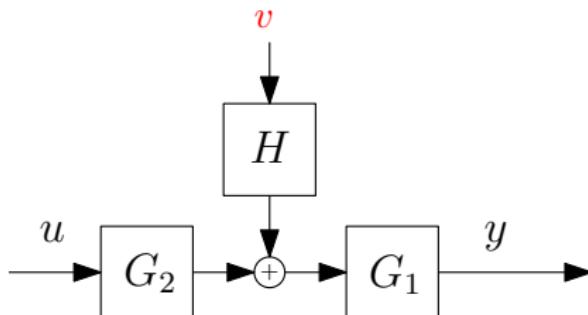
Scenarios with measurable disturbances

In some systems we are also able to measure **disturbances**.
Exempel:



Feedforward control

Scenarios with measurable disturbances



General feedback with *measurable* signals *including* **disturbance**:

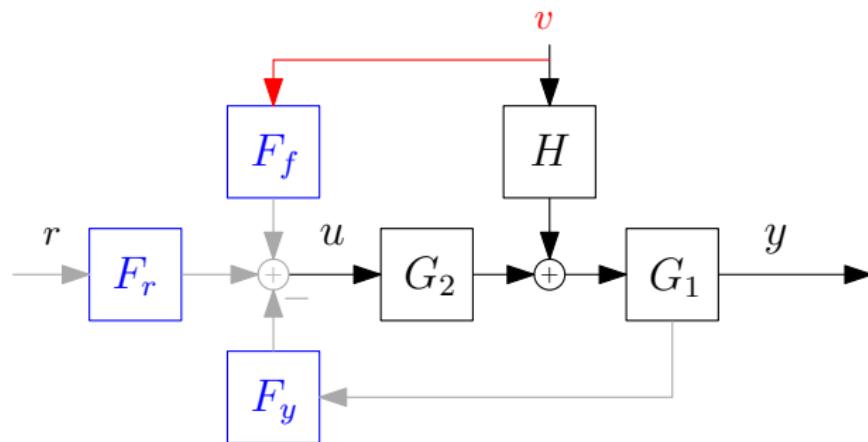
$$U(s) = F_r(s)R(s) - F_y(s)Y(s) + \underbrace{F_f(s)V(s)}_{\text{control using also measured disturbance}}$$

Feedforward control

Scenarios with measurable disturbances

General feedback with *measurable signals including disturbance*:

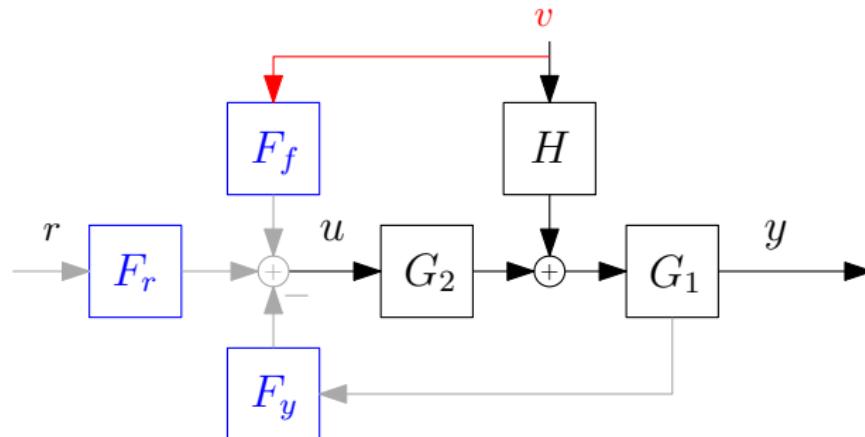
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[Board: derive the closed-loop system $r, v \rightarrow y$]

Feedforward control

Scenarios with measurable disturbances

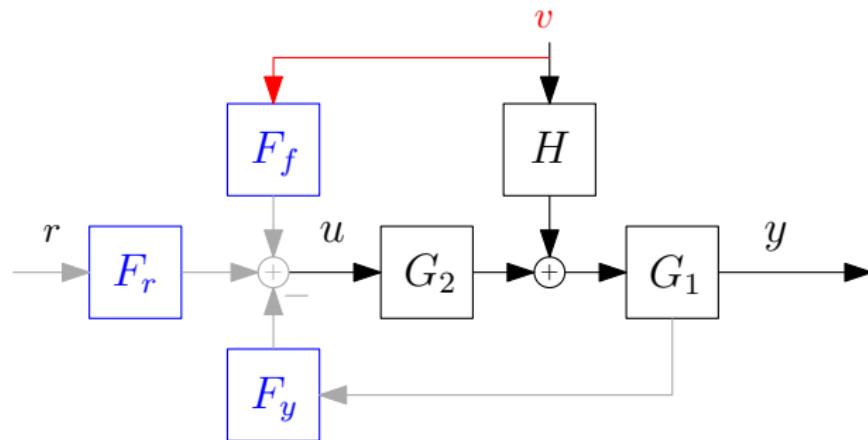


Ideal controller:

$$Y(s) = \underbrace{\frac{G_1(s)G_2(s)F_r(s)}{1 + G_1(s)G_2(s)F_y(s)}}_{\approx 1} R(s) + \underbrace{\frac{G_1(s)(H(s) + G_2(s)F_f(s))}{1 + G_1(s)G_2(s)F_y(s)}}_{\approx 0} V(s)$$

Feedforward control

Scenarios with measurable disturbances

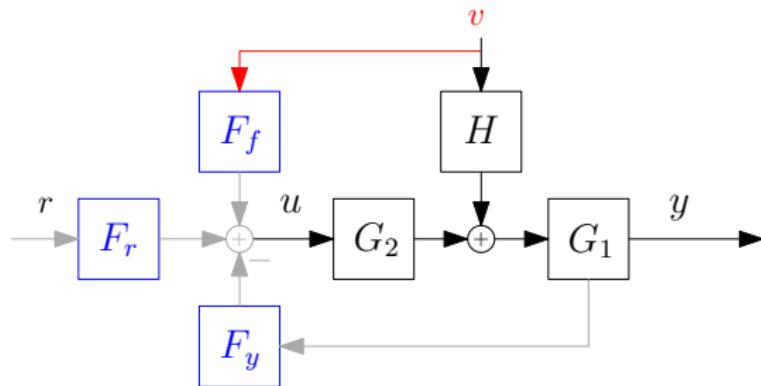


Ideal disturbance suppression:

$$H(s) + G_2(s)F_f(s) = 0 \quad \Leftrightarrow \quad F_f(s) = -\frac{H(s)}{G_2(s)}$$

Explicit disturbance compensation, but often *hard* to implement,
due to *high order in the numerator* of $F_f(s)$.

Example: tank with valve + disturbance



Example:

- ▶ Static feedforward:

$$F_f(s) = -\frac{H(0)}{G_2(0)}$$

- ▶ Dynamic feedforward:

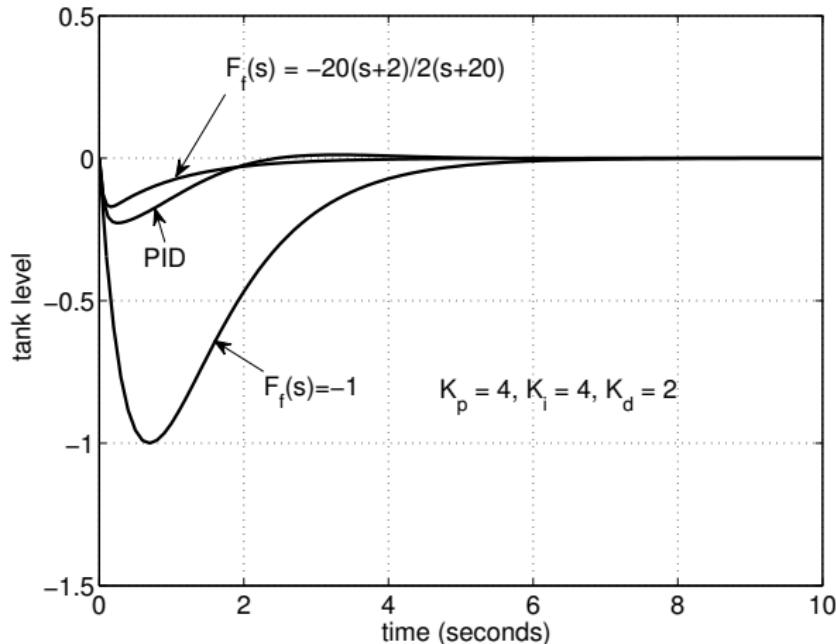
$$F_f(s) = -\frac{H(s)}{G_2(s)} \frac{20}{s + 20}$$

Example: tank with valve + disturbance

Control using feedforward, when $r(t) \equiv 0$ and $v(t)$ is a step

Example: tank with valve + disturbance

Control using feedforward, when $r(t) \equiv 0$ and $v(t)$ is a step



Static/dynamic feedforward vs. PID control

Cascade control

Measurable intermediate signals

In certain systems we can measure internal or **intermediate** signals:

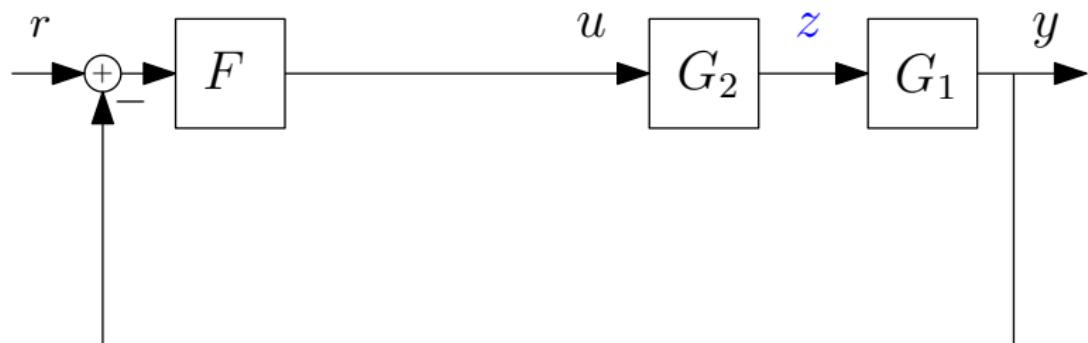


Cascade control

Measurable intermediate signals

Simple linear feedback:

$$\begin{aligned}U(s) &= F(s)R(s) - F(s)Y(s) \\&= F(s)E(s)\end{aligned}$$

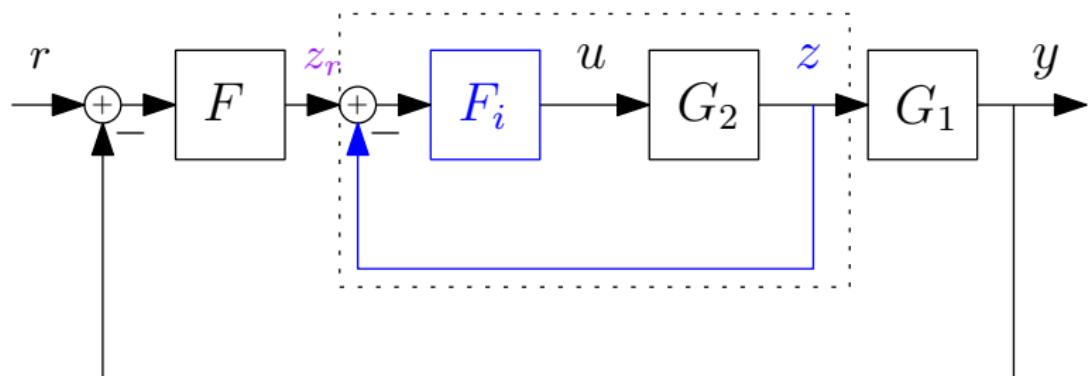


Cascade control

Measurable intermediate signals

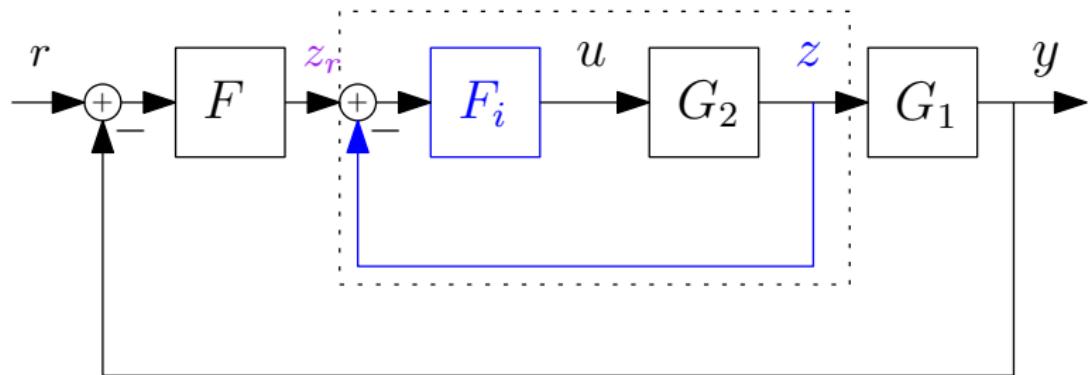
Simple linear feedback control with *measured* signals:

$$\begin{aligned} U(s) &= F_i(s)F(s)R(s) - F_i(s)F(s)Y(s) - F_i(s)Z(s) \\ &= F_i(s)\left(\underbrace{F(s)E(s)}_{Z_r(s)} - Z(s)\right) \end{aligned}$$



Cascade control

Measurable intermediate signals



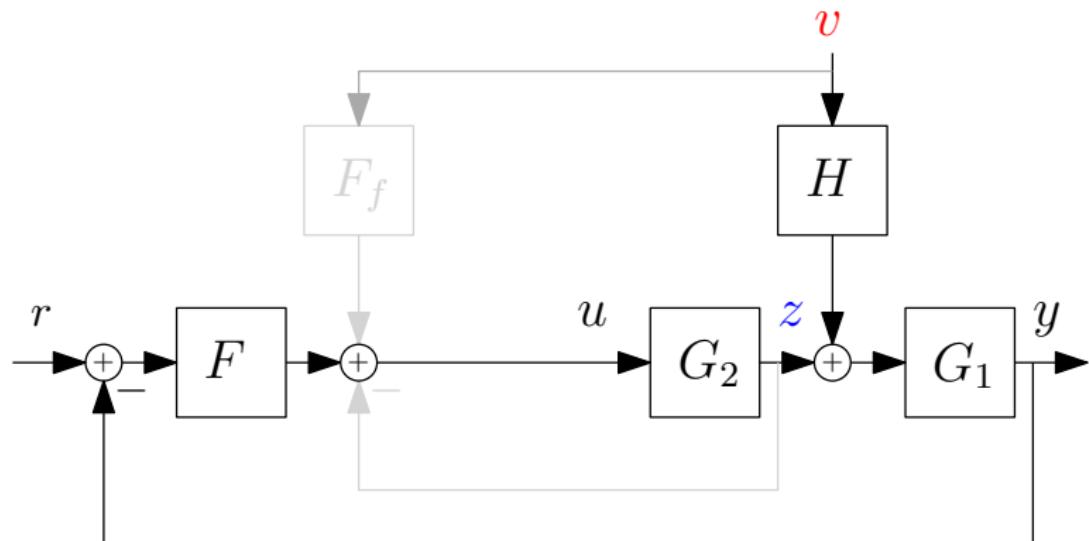
Control strategy: easier to control subsystems

- ▶ Design $F_i(s)$ so that $Z(s) \approx Z_r(s)$
- ▶ Design $F(s)$ with respect to $G_1(s)$, neglect internal loop.

Measurable signals and disturbances

Total controller

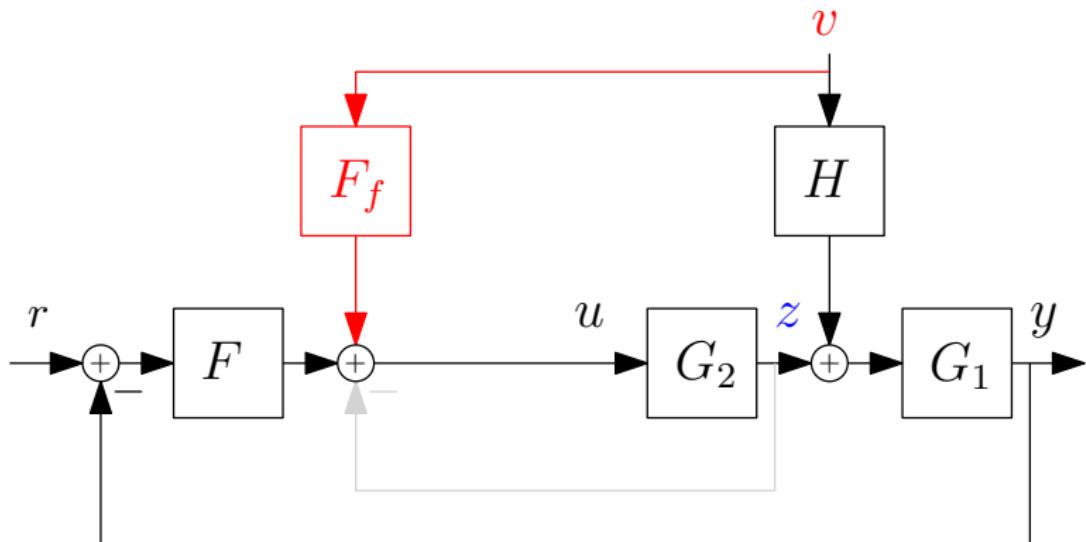
Control with respect to $R(s)$ and $Y(s)$



Measurable signals and disturbances

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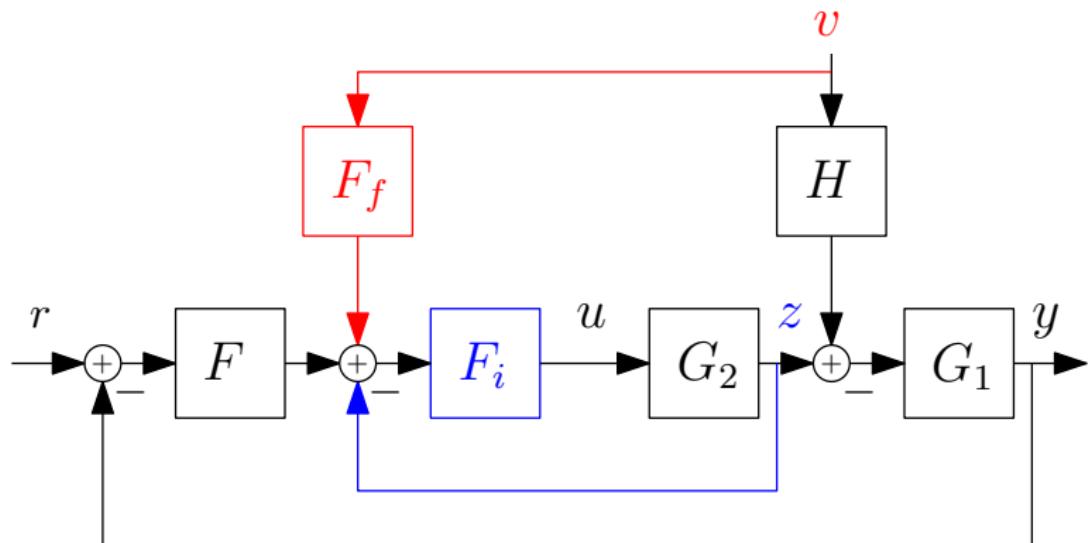
Control with respect to $R(s)$ and $Y(s)$ as well as measurable disturbance $V(s)$



Measurable signals and disturbances

Total controller

Control with respect to $R(s)$ and $Y(s)$ as well as measurable signal $Z(s)$ and disturbance $V(s)$



Exploits all available information!

Frequency description of systems

(Closed-loop) system response to oscillating signals

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- ▶ Any signal $x(t)$ can be decomposed into sum of **cosine- and sine signals**:

$$x(t) = \int \underbrace{X(i\omega)}_{\text{weights}} e^{i\omega t} d\omega,$$

where

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

$$\omega = 2\pi f \quad (\text{frequency})$$

Frequency description of systems

(Closed-loop) system response to oscillating signals

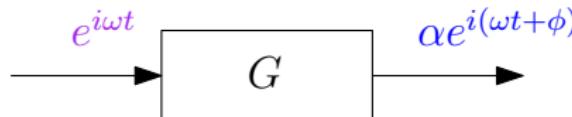
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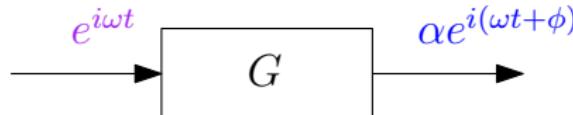
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Fundamental property of LTI systems

Output is also a sum of the *input* cosine- and sine signals!

Frequency properties

Sine in/sine out

[Board: derive $y(t)$ when $u(t) = A \sin(\omega t)$]

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Sine in-sine out

Assume stable system $Y(s) = G(s)U(s)$, where $u(t) = A \sin(\omega t)$.
After all transients vanish, we obtain output:

$$y(t) = \underbrace{|G(i\omega)|}_{\text{amplification}} \cdot A \sin(\omega t + \underbrace{\arg G(i\omega)}_{\text{phase shift}})$$

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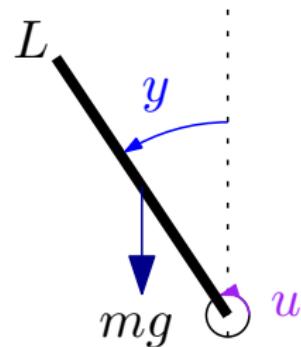
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- ▶ Same property applies $Y(s) = G_c(s)R(s)$, with $r(t) = A \sin(\omega t)$
- ▶ Yields *interpretation* of complex-valued

$$G_c(s) = |G_c(s)| e^{i \arg G_c(s)} \quad \text{when } s = i\omega$$

Example: inverted pendulum



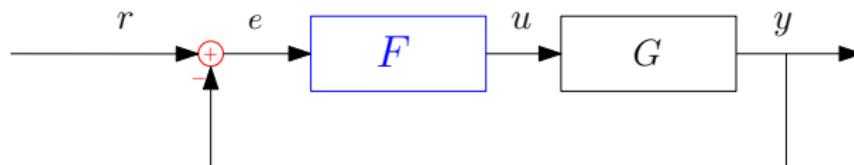
Linearized model (around $y \approx 0$):

$$\ddot{y} - \left(\frac{3g}{2L} \right) y = \left(\frac{3}{mL^2} \right) u \Leftrightarrow Y(s) = \frac{\frac{3}{mL^2}}{s^2 - \frac{3g}{L}} U(s) = \frac{1}{s^2 - 1} U(s) s$$

Example: inverted pendulum

Response of PD-control

System $Y(s) = \frac{1}{s^2 - 1} U(s)$.



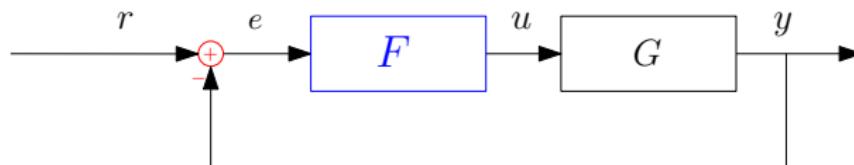
- ▶ PD-control $U(s) = K(s + 3)(R(s) - Y(s))$ give closed-loop system

$$Y(s) = G_c(s)R(s) \quad \text{where} \quad G_c(s) = \frac{K(s + 3)}{s^2 + Ks + 3K - 1}.$$

Example: inverted pendulum

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- ▶ With reference $r(t) = \sin(\omega t)$ yields output

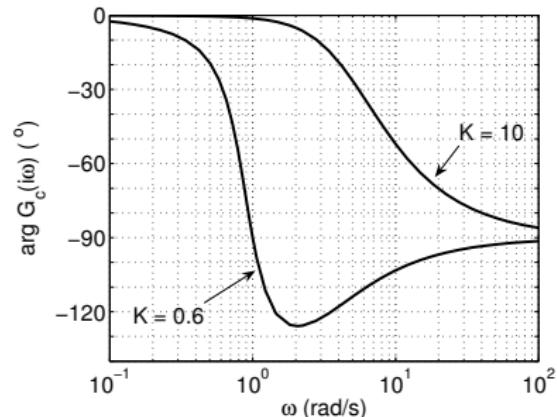
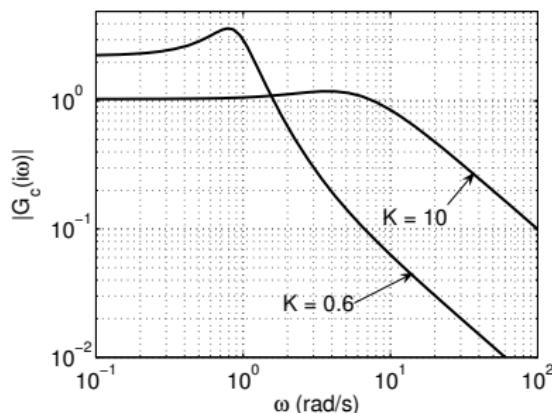
$$y(t) = |G_c(i\omega)| \sin(\omega t + \arg(G_c(i\omega)))$$

Affected by user parameter K !

Amplitude- and phase plot

Example: PD-control of inverted pendulum

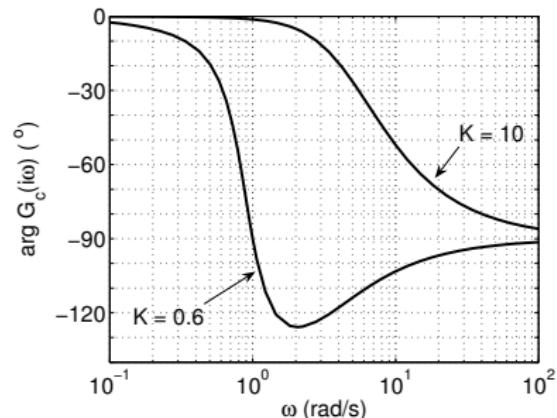
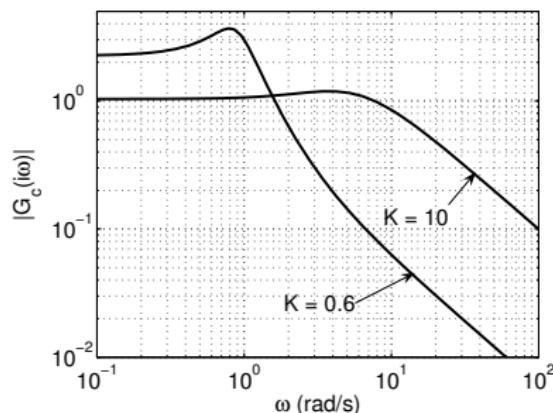
$|G_c(i\omega)|$ and $\arg(G_c(i\omega))$ as a function of frequency ω



Amplitude- and phase plot

Example: PD-control of inverted pendulum

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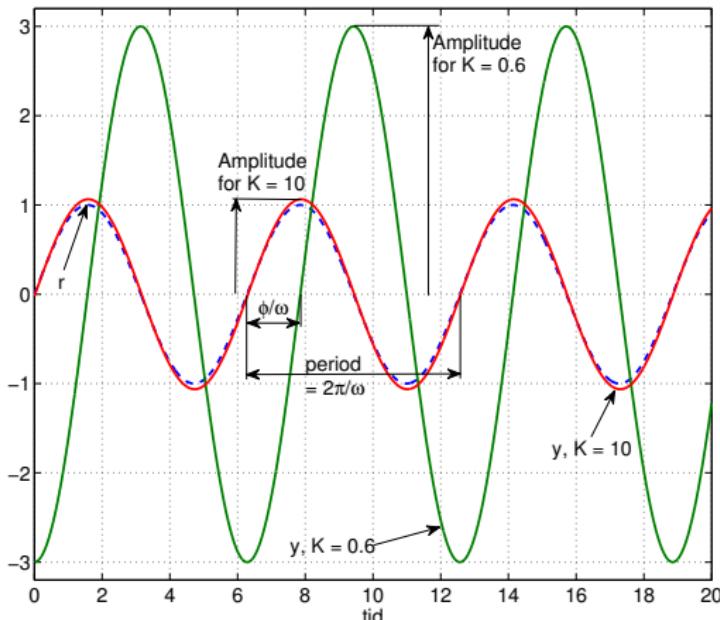


Note amplification and phase shift at $\omega = 1$

Example: PD-control of inverted pendulum

Amplification/attenuation and phase shift of $r(t)$

PD-control inverted pendulum when $r(t) = \sin(1t)$.



Cf. amplitude- and phase plots for different K

Summary and recap

- ▶ General linear feedback control
 - ▶ Measured disturbances → feedforward control
 - ▶ Measured intermediate signals → cascade control
- ▶ Frequency description of system:
 - ▶ Sine in/sine out
 - ▶ System amplitude- and phase plots (Bode diagram)