Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2011

Assignment: Solve the exercises listed below individually for next Friday (2011/12/09).

Problem solving session X (Ex10)

1. The differential equation describing a mass-spring-damper system is

$$m\,\frac{d^2y(t)}{dt^2} + b\,\frac{dy(t)}{dt} + k\,y(t) = r(t)$$

Consider the input variable as the external force $r(t)$, and the output as the position $y(t)$ of the mass.

(a) Find the state space representation of the system.

(b) Obtain the transfer function of the system and calculate its static gain.

(c) Determine the step response ($y(t)$ for $t > 0$) of the system for $m = 1$, $b = 3$ and $k = 3/2$ and initial conditions $x(0) = [0 \quad 1]^T$.

2. Consider the system

$$\begin{aligned} \dot{x}_1 &= x_1(u - \beta x_2) \\ \dot{x}_2 &= x_2(-\alpha + \beta x_1) \end{aligned}$$

where u is the system input and α and β positive constants.

(a) Is the system linear, nonlinear? Time varying, time invariant? Justify.

(b) Determine the system equilibrium points (x_0, u_0) for $u_0 = 2$.

(c) Near the positive equilibrium point from (b), find a linearized state-space model.

(d) Is the linearized model stable?

3. Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

(a) Discretize the system for $T = 0.1$s.

(b) Is the discrete system controllable? Observable?

(c) Given the state feedback $u(k) = -K\, x(k)$, determine K such that the closed-loop system poles are in $p_{1,2} = -3 \pm 2i$.