Introduction to computer control systems:
Selected exercises for the problem solving sessions
Master program in embedded systems, period 2, 2011

Assignment: Solve the exercises listed below individually for next Friday (2011/12/09).

Problem solving session X (Ex10)

1. The differential equation describing a mass-spring-damper system is

\[ m \frac{d^2 y(t)}{dt^2} + b \frac{dy(t)}{dt} + ky(t) = r(t) \]

Consider the input variable as the external force \( r(t) \), and the output as the position \( y(t) \) of the mass.

(a) Find the state space representation of the system.
(b) Obtain the transfer function of the system and calculate its static gain.
(c) Determine the step response \( y(t) \) for \( t > 0 \) of the system for \( m = 1, b = 3 \) and \( k = 3/2 \) and initial conditions \( x(0) = [0 \quad 1]^T \).

2. Consider the system

\[
\begin{align*}
\dot{x}_1 &= x_1(u - \beta x_2) \\
\dot{x}_2 &= x_2(-\alpha + \beta x_1)
\end{align*}
\]

where \( u \) is the system input and \( \alpha \) and \( \beta \) positive constants.

(a) Is the system linear, nonlinear? Time varying, time invariant? Justify.
(b) Determine the system equilibrium points \( (x_0, u_0) \) for \( u_0 = 2 \).
(c) Near the positive equilibrium point from (b), find a linearized state-space model.
(d) Is the linearized model stable?
3. Consider the system

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x
\end{align*}
\]

(a) Discretize the system for \( T = 0.1 \)s.

(b) Is the discrete system controllable? Observable?

(c) Given the state feedback \( u(k) = -K x(k) \), determine \( K \) such that the closed-loop system poles are in \( p_{1,2} = -3 \pm 2i \).