Systems and Control
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www.it.uu.se/research/syscon

Introduction to computer control systems

Master program in embedded systems, period 2, 2010

Problem solving session X (Ex10) - Solutions

1. (a)
$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k/m & -b/m\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 1/m\end{bmatrix} r(t)$$
$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix} x(t)$$

   (b)
$$G(s) = \frac{1}{ms^2 + bs + k}$$
   $1/k$ is the static gain of the system

   (c)
$$y(t) = C\Phi(t)x(0) + C\int_0^t \Phi(t-\tau)u(\tau)Bd\tau$$
$$y(t) = 1/3(e^{-2.366t} - e^{-0.634t}) + 2/3$$

2. —

3. (a)
$$A_d = \begin{bmatrix}1 & 0.0952\\ 0 & 0.9048\end{bmatrix}$$
$$B_d = \begin{bmatrix}0.048\\ 0.952\end{bmatrix}$$
$$C_d = C = \begin{bmatrix}1 & 0\end{bmatrix}$$

   (b) The discrete-time system is observable and controllable.

   (c) The poles $p_{1,2} = -3 \pm 2i$ are in continuous-time. Note the use of the letter p and, if they were in discrete-time, they will give rise to an unstable system, which is not desirable for a closed-loop system.

The corresponding poles of the discretized system are given by $z = e^{Tp}$, where T is the sampling time.

Hence $z_{1,2} = 0.726 \pm 0.147i$.

Equating the coefficients in z of the characteristic polynomials:

$p(z) = \det(zI - (A_d - B_d K))$ and

$\alpha_c(z) = (z - z_1)(z - z_2)$ it follows that

$K = \begin{bmatrix} 1.01 & 0.424 \end{bmatrix}$.