Systems and Control
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# Introduction to computer control systems

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## Problem solving session I (Ex1) - Solutions

1.

$$\begin{aligned}
\begin{bmatrix} \dot{h}_1(t) \\ \dot{h}_2(t) \end{bmatrix} &= \begin{bmatrix} -\beta/A_1 & \beta/A_1 \\ \beta/A_2 & -(\alpha+\beta)/A_2 \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} 1/A_1 & 0 \\ 0 & 1/A_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \\
y(t) &= \begin{bmatrix} 0 & \alpha \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix}
\end{aligned}$$

Substitute $A_1 = 1$ and $A_2 = 1$ and use $G(s) = C(sI - A)^{-1}B + D$, with $D = 0$ and A, B, and C as in the state-space description above to obtain

$$G(s) = \begin{bmatrix} \frac{\alpha\beta}{s^2+(2\beta+\alpha)s+\alpha\beta} & \frac{\alpha(s+\beta)}{s^2+(2\beta+\alpha)s+\alpha\beta} \end{bmatrix} \tag{1}$$

2. Let

$$\begin{aligned}
\dot{x}(t) &= Fx(t) + Gu(t) \\
y(t) &= Hx(t).
\end{aligned}$$

Note that the first column of G only affects $u_1(t)$, and, similarly, the second column of G only affects $u_2(t)$. Hence

$$\begin{aligned}
\dot{x}(t) &= Fx(t) + \begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \\
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= F \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}
\end{aligned}$$

If the two rows in the previous state-space representation are summed up,

$$\dot{x_1}(t) + \dot{x_2}(t) = F(x_1(t) + x_2(t)) + \begin{bmatrix} G_1 & G_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}.$$

Let $x(t) = x_1(t) + x_2(t)$. Hence

$$y(t) = Hx(t) = H(x_1(t) + x_2(t)) = Hx_1(t) + Hx_2(t) = y_1(t) + y_2(t),$$

being $y_1(t)$ the output for subsystem 1 and $y_2(t)$ the output of subsystem 2 (superposition principle).

If the two subsystems are written in the observable canonical form (pg. 36 course book), $A(p)y_1(t) = B(p)u_1(t)$ and $A(p)y_2(t) = C(p)u_2(t)$ are obtained, respectively. Using $y(t) = y_1(t) + y_2(t)$,

$A(p)y(t) = B(p)u_1(t) + C(p)u_2(t)$ is obtained with

$$\begin{aligned} A(p) &= p^n + a_1 p^{n-1} + \ldots + a_n \\ B(p) &= b_1 p^{n-1} + \ldots + b_n \\ C(p) &= c_1 p^{n-1} + \ldots + c_n \end{aligned}$$

3.

$$\begin{aligned} G(s) &= \begin{bmatrix} \frac{s^2+s+1}{(s+1)(s+2)(s^2+s+1)} & \frac{(s+3)(s+2)}{(s+1)(s+2)(s^2+s+1)} \end{bmatrix} \\ Y(s) &= \frac{s^2+s+1}{s^4+4s^3+6s^2+5s+2} U_1(s) + \frac{s^2+5s+6}{s^4+4s^3+6s^2+5s+2} U_2(s) \end{aligned}$$

Using the observable canonical form,

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -4 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ -5 & 0 & 0 & 1 \\ -2 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(t) \end{aligned}$$

4. In Ex2.

5. First obtain $x_2(1) = 0$, $x_1(1) = 1$ to get

$$x(3) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$