Systems and Control
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Introduction to computer control systems

Master program in embedded systems, period 2, 2011

Problem solving session II (Ex2) - Solutions

1. (a) Let the discrete-time state-space of the system be given by

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) \\ y(k) &= Cx(k) \end{aligned}$$

with A, B, C being the continuous-time state-space description matrices and

$$\begin{aligned} \Phi &= e^{Ah} \\ \Gamma &= \int_{\tau=0}^{h} e^{A\tau} d\tau B. \end{aligned}$$

$$Ah = \begin{bmatrix} -0.0197h & 0 \\ 0.0178h & -0.0129h \end{bmatrix} \tag{1}$$

For a general h ($h \neq 0$), Ah has distinct eigenvalues ($eig(Ah) = \{-0.0197h; -0.0129h\}$) $\Rightarrow$ Ah distinct eigenvectors $\Rightarrow$ Ah diagonalizable by $Ah = P\Lambda P^{-1}$, where the columns of P are the eigenvectors of $eig(Ah)$, and $\Lambda = \Lambda(h)$ is a diagonal matrix with $eig(Ah)$ as diagonal components.

$$\Phi = e^{Ah} = Pe^{\Lambda}P^{-1} = \begin{bmatrix} 0.7895 & 0 \\ 0.1757 & 0.8566 \end{bmatrix}$$

$$\Gamma = \int_{\tau=0}^{h} e^{A\tau} d\tau B = P \int_{\tau=0}^{h} e^{A\tau} d\tau B = \begin{bmatrix} 0.2810 \\ 0.0296 \end{bmatrix}$$

(b) Substitute C, Φ, and Γ in

$$H_0(q) = C(qI - \Phi)^{-1}\Gamma$$

2. The impulse response is the system output when the input signal is a impulse (Dirac) and initial conditions are zero.

$h(t) = C\Phi(t)B = Ce^{At}B$

3. (a) $h(t) = 0.0374(-e^{-0.0197t} + e^{-0.0129t})$ is the impulse response of the continuous-time system.

   $h(k) = C\Phi^{k-1}\Gamma$ $(k > 0)$ is the impulse response for the discrete-time system.

   (b) $x(t) = -A^{-1}Bu(t) = -A^{-1}B \times 3.8 = \begin{bmatrix} 5.07 \\ 7.00 \end{bmatrix}$ are the steady-state values for the tank levels.

   $y(t) = 0.5429 \times 7.00 = 3.8L/min$ is the steady-state output flow.

   (c) $x_0 = x(t_0 = 0) = \begin{bmatrix} 5.07 \\ 7.00 \end{bmatrix}$, and $u(t) = 5$ for $t \geq 0$

   continuous-time:

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

$$x_0 = x(k_0 = 0) = \begin{bmatrix} 5.07 \\ 7.00 \end{bmatrix}$$

   discrete-time:

$$x(k) = \Phi^k x_0 + \sum_{i=0}^{k-1} \Phi^{k-i-1}\Gamma u(i)$$

   Use the given A, B, Φ and Γ to calculate the time responses $x(t)$ and $x(k)$.