Systems and Control
Department of Information Technology
UPPSALA UNIVERSITY
www.it.uu.se/research/syscon

Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2011

Problem solving session III (Ex3)

1. Demonstrate that for a system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

with a diagonal $A-$matrix given by

$$A = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix}$$

The transition matrix is given by

$$e^{At} = \begin{pmatrix} e^{\lambda_1 t} & 0 & \dots & 0 \\ 0 & e^{\lambda_2 t} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & e^{\lambda_n t} \end{pmatrix}$$

2. (Based on Exercise 3.20 from [3])

Consider the electrical circuit with the input voltage u shown in Figure 1.

(a) Select the current through the inductor and the voltage across the capacitance as state variables, and the capacitance's voltage as output, and determine the state space equations

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

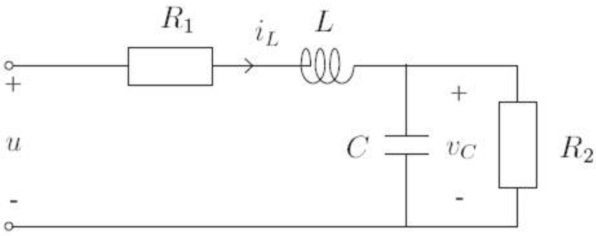

Figure 1: Electric circuit

(b) Calculate the transition matrix $\Phi(t) = e^{At}$ assuming the following values: $R_1 = R_2 = 1\ \Omega$, $L = 1\ H$, and $C = 1\ F$.

(c) Obtain the time response for the state variables if a step change is produced at the input, that is the input changes from 2 V to 0 V at t=0 s (Consider that the system is in stationary conditions for t<0 s).

(d) Repeat the previous item but considering that the input voltage changes from 2 V to 1 V.

3. (Based on Exercise 3.17 from [1])

Determine a coordinate transformation $z = Tx$ that transfers the system

$$\begin{aligned} x(k+1) &= \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} x(k) + \begin{pmatrix} 3 \\ 4 \end{pmatrix} u(k) \\ y(k) &= \begin{pmatrix} 5 & 6 \end{pmatrix} x(k) \end{aligned}$$

(a) To diagonal canonical form

(b) To controllable canonical form

(c) To observable canonical form

4. (Based on [2])

Analise the controllability and unobservability for the following continuous-time systems:

(a)

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} x \end{aligned}$$

(b)

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix} x + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} u \\
y &= \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} x
\end{aligned}
$$

References

[1] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems.* Prentice Hall, 1997.

[2] Automatic Control Group (Linköpings University) and Systems and Control Group (Uppsala University). *Exercise Manual for Automatic Control.* Uppsala University, 2001.

[3] Mikael Johansson and Torsten Söderström. *Exercises Control Theory.* Uppsala University and Royal Institute of Technology, 2010.