Systems and Control
Department of Information Technology
UPPSALA UNIVERSITY
www.it.uu.se/research/syscon

Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2011

Problem solving session IV (Ex4)

1. (Exercise 3.6 from [1])

   Is the following system (a) observable, (b) controllable?

$$\begin{aligned} x(k+1) &= \begin{pmatrix} 0.5 & -0.5 \\ 0 & 0.25 \end{pmatrix} x(k) + \begin{pmatrix} 6 \\ 4 \end{pmatrix} u(k) \\ y(k) &= \begin{pmatrix} 2 & -4 \end{pmatrix} x(k) \end{aligned}$$

2. (Based on [2])

   Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix} x + \begin{pmatrix} 1 & 8 \\ 2 & 6 \end{pmatrix} u \\ y &= \begin{pmatrix} 3 & 7 \end{pmatrix} x \end{aligned}$$

   (a) Is the system controllable?

   (b) Suppose that we have only access to the input u_1. Is the system controllable in this case?

   (c) Suppose that the two control signals are coupled so that $u_1 + 2u_2 = 0$ always holds. Is the system controllable in this case?

3. (Exercise 3.7 from [1])

   Is the following system controllable?

$$x(k+1) = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} x(k) + \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} u(k)$$

   Assume that a scalar input $u'(k)$ such that

$$u(k) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} u'(k)$$

   is introduced. Is the system controllable from $u'(k)$?

4. (Exercise 3.4 from [3])

   Consider the discrete-time system

$$x(t+1) = \begin{pmatrix} 0.2 & 0.1 \\ 0.2 & 0.3 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u(t)$$

   Determine, in case it is possible, the input $u(t)$ so that the state vector change from x(0) to $x'(t)$ in at most two sampling intervals, when

   (a)

$$x(0) = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, x'(t) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

   (b)

$$x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, x'(t) = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

   In both cases, it thus holds $t' = 1$ or $t' = 2$. Explain the achieved results.

References

[1] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems*. Prentice Hall, 1997.

[2] Automatic Control Group (Linköpings University) and Systems and Control Group (Uppsala University). *Exercise Manual for Automatic Control*. Uppsala University, 2001.

[3] Mikael Johansson and Torsten Söderström. *Exercises Control Theory*. Uppsala University and Royal Institute of Technology, 2010.