Introduction to computer control systems  
Master program in embedded systems, period 2, 2011

Problem solving session IV (Ex4) - Solutions

1. The system is unobservable but controllable.

2. (a) The system is controllable.
    (b) Let $B_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ and check the controllability of the pair $(A, B_1)$.
        The system controllable from $u_1$.
    (c) Calculate 
        $$Bu = \begin{bmatrix} 6u_2 \\ 2u_2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$ 
        Set 
        $$B_1 = \begin{bmatrix} 0 & 6 \\ 0 & 2 \end{bmatrix}$$ 
        and check the controllability of the pair $(A, B_1)$.
        With such coupling between inputs the system is not controllable.

3. The given system is controllable.

$$x(k + 1) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} u'(k) = Ax(k) + B_1 u'(k)$$

Check the controllability of the pair $(A, B_1)$.

The system is not controllable from $u'(k)$.

4. (a) $x'(t)$ is achieved in one sampling interval with $u(0) = 3.4$.
    (b) The state $x'(t)$ cannot be achieved in two sampling intervals. This is so because the system is not controllable, i.e. there is no input $u$ that in finite times drives the system state vector from $x(0)$ to $x'(t)$.

$$S(A, B) = \begin{bmatrix} 1 & 0.4 \\ 2 & 0.8 \end{bmatrix}$$
$\text{rank}(S(A, B)) = 1$, being the controllable subspace spanned by $[v \quad 2v]^T, \forall v \neq 0$.

In (a), both $x(0)$ and $x'(t)$ lay in the controllable subspace so it is possible to drive the system from one of these states to the other in a finite time. This does not happen in (b), which makes it not possible to find $u(t)$ that in finite time drives the system from $x(0)$ to $x'(t)$. 