Systems and Control
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Introduction to computer control systems

Master program in embedded systems, period 2, 2011

Problem solving session IV (Ex4) - Solutions

1. The system is unobservable but controllable.

2. (a) The system is controllable.

   (b) Let $B_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ and check the controllability of the pair (A, B_1). The system controllable from u_1.

   (c) Calculate

   $$Bu = \begin{bmatrix} 6u_2 \\ 2u_2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

   Set

   $$B_1 = \begin{bmatrix} 0 & 6 \\ 0 & 2 \end{bmatrix}$$

   and check the controllability of the pair (A, B_1).
   With such coupling between inputs the system is not controllable.

3. Ths given system is controllable.

$$x(k+1) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} u'(k) = Ax(k) + B_1 u'(k)$$

   Check the controllability os the pair (A, B_1).

   The system is not controllable from $u'(k)$.

4. (a) $x'(t)$ is achieved in one sampling interval with $u(0) = 3.4$.

   (b) The state $x'(t)$ cannot be achieved in two sampling intervals. This is so because the system is not controllable, i.e. there is no imput u that in finite times drives the system state vector from $x(0)$ to $x'(t)$.

$$S(A, B) = \begin{bmatrix} 1 & 0.4 \\ 2 & 0.8 \end{bmatrix}$$

$rank(S(A,B)) = 1$, being the controllable subspace spanned by $\begin{bmatrix} v & 2v \end{bmatrix}^T$,$\forall v \neq 0$.
In (a), both $x(0)$ and $x'(t)$ lay in the controllable subspace so it is possible to drive the system from one of this states to the other in a finite time. This does not happen in (b), which makes it not possible to find $u(t)$ that in finite time drives the system from $x(0)$ to $x'(t)$.