Systems and Control
Department of Information Technology
UPPSALA UNIVERSITY
www.it.uu.se/research/syscon

Introduction to computer control systems

Master program in embedded systems, period 2, 2011

Problem solving session V (Ex5) - Solutions

1. (a) $h_{1,0} = 0.907$, $h_{2,0} = 1.148$, $h_{3,0} = 1.440$

$$A = \begin{bmatrix} -1.3781 & 0 & 0 \\ 1.1025 & -0.8711 & 0 \\ 0 & 0.1667 & -1.0417 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.8333 & -0.8333 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1.0417 \end{bmatrix}$$

(b)

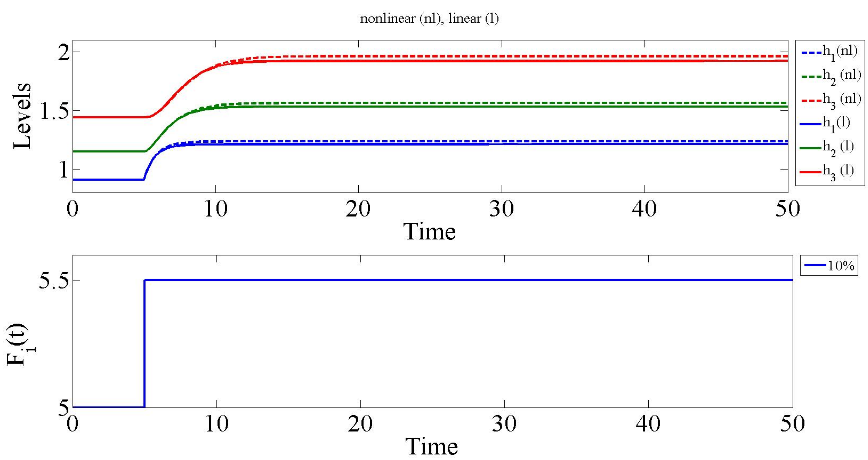

Figure 1: Step change of +10% in the input.

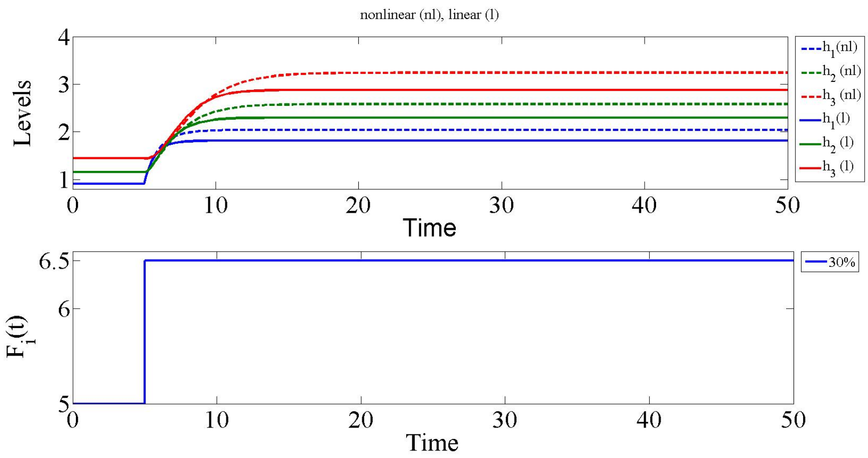

Figure 2: Step change of +30% in the input.

(c) $G(s)|_{s=0} = \begin{bmatrix} 0.1269 & -0.1269 \end{bmatrix}$

2. (a) Let $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$

   $A = \begin{bmatrix} 0 & 1 \\ -9.8 & -2/3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1/3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

   (b) $G(s) = \frac{1/3}{s^2+\frac{2}{3}s+9.8}$

   (c) SISO system: zeros of $G(s)$ are the values of s such that $G(s) = 0$ $\Rightarrow$ the system has no zeros;

   Poles are the roots of $det(sI - A)$ i.e. eigenvalues of $A \Rightarrow$ the system has poles located in $s = -0.33 \pm 3.11i$

   (d) $y(t) = A.M(\omega_0)\sin(\omega_0 + \phi(\omega_0))$, where

   $M(\omega) = \sqrt{Im^2(G(j\omega)) + Re^2(G(j\omega))}$ and $\phi(\omega) = \arctan\frac{Im(G(j\omega))}{Re(G(j\omega))}$, with $Re(\cdot)$ as the real part of its argument and $Im(\cdot)$ as the imaginary part of its argument.

   Using $G(s)$ from (c), and substituting s by $j\omega$,

$$G(j\omega) = \frac{1/3(9.8 - \omega^2)}{(9.8 - \omega^2)^2 + 4/9\,\omega^2} + \frac{1/3(-2/3\,\omega)}{(9.8 - \omega^2)^2 + 4/9\,\omega^2}j$$

   Substitute A and ω by the given values to obtain the response of the sinusoidal system for each case.