Problem solving session V (Ex5) - Solutions

1. (a) \( h_{1,0} = 0.907, h_{2,0} = 1.148, h_{3,0} = 1.440 \)

\[
A = \begin{bmatrix}
-1.3781 & 0 & 0 \\
1.1025 & -0.8711 & 0 \\
0 & 0.1667 & -1.0417
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.8333 & -0.8333 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0 & 0 & 1.0417
\end{bmatrix}
\]

(b) Figure 1: Step change of +10% in the input.
(c) \( G(s)|_{s=0} = \begin{bmatrix} 0.1269 & -0.1269 \end{bmatrix} \)

2. (a) Let \( x_1(t) = \theta(t) \) and \( x_2(t) = \dot{\theta}(t) \)
\[
A = \begin{bmatrix} 0 & 1 \\ -9.8 & -2/3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/3 \end{bmatrix}, \quad C = [1 \quad 0]
\]

(b) \( G(s) = \frac{\frac{1}{3}}{s^2 + \frac{1}{3}s + 9.8} \)

(c) SISO system: zeros of \( G(s) \) are the values of \( s \) such that \( G(s) = 0 \)
\( \Rightarrow \) the system has no zeros;
Poles are the roots of \( \text{det}(sI - A) \) i.e. eigenvalues of \( A \) \( \Rightarrow \) the system has poles located in \( s = -0.33 \pm 3.11i \)

(d) \( y(t) = A.M(\omega_0)\sin(\omega_0 + \phi(\omega_0)), \) where
\[
M(\omega) = \sqrt{\text{Im}^2(G(j\omega)) + \text{Re}^2(G(j\omega))} \quad \text{and} \quad \phi(\omega) = \arctan\frac{\text{Im}(G(j\omega))}{\text{Re}(G(j\omega))},
\]
with \( \text{Re}() \) as the real part of its argument and \( \text{Im}() \) as the imaginary part of its argument.
Using \( G(s) \) from (c), and substituting \( s \) by \( j\omega \),
\[
G(j\omega) = \frac{1/3(9.8 - \omega^2)}{(9.8 - \omega^2)^2 + 4/9\omega^2} + \frac{1/3(-2/3 \omega)}{(9.8 - \omega^2)^2 + 4/9\omega^2}j
\]
Substitute \( A \) and \( \omega \) by the given values to obtain the response of the sinusoidal system for each case.