Systems and Control
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Introduction to computer control systems: Selected exercises for the problem solving sessions

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Problem solving session VI (Ex6)

1. (Exercise 3.20 from [1])

   Given the system

   $$(q^2 + 0.4q)y(k) \;=\; u(k),$$

   for which values of K in the proportional controller

   $$u(k) \;=\; K(u_c(k) - y(k))$$

   is the closed-loop system stable?

2. Consider the system defined by

   $$\begin{aligned} x_1(k+1) &= x_1(k) + 0.2x_2(k) + 0.4 \\ x_2(k+1) &= 0.5x_1(k) - 0.5 \end{aligned}$$

   (a) Find the equilibrium point.

   (b) Obtain the state space form.

   (c) Is the model stable?

3. (Based on Exercise 3.22 from [2])

   A dynamic system is given by a scalar differential equation with an algebraic expression given by

   $$\begin{aligned} \frac{d}{dt}\xi &= -\xi + u\eta^3 \\ 0 &= -\eta + u^2 e^{\eta} \end{aligned}$$

   (a) A control system should be designed to keep the system at a given stationary point ξ_0. Determine the full operating point $(\xi_0,\, u_0,\, \eta_0)$ when $\eta_0 = 1.1843$.

(b) The system input is u and its output is $y = \eta\xi$. Determine a linear state model, valid near the operating point determined in (a).

(c) How is the stability of the stationary operating point (ξ_0)?

4. (Based on Exercise 3.26 from [2])

In an autonomous biological process there are two organisms (A and B). The two organisms interact so that they grow in proportion to both concentration, c_A and c_B. The organisms are dying off at a speed proportional to their number. The process is described by the following bilinear equations:

$$\begin{aligned} \frac{d}{dt} c_A &= -c_A + \alpha c_A c_B \\ \frac{d}{dt} c_B &= -c_B + \beta c_A c_B \end{aligned}$$

The output of the system is the arithmetic mean $c_M = 0.5(c_A + c_B)$.

(a) Determine the two possible steady states and find the linearized state models around these working points.

(b) Are the two models stable for all combinations of process parameters α and β?

References

[1] Karl J. Åström and Björn Wittenmark. *Computer-Controlled Systems.* Prentice Hall, 1997.

[2] Mikael Johansson and Torsten Söderström. *Exercises Control Theory.* Uppsala University and Royal Institute of Technology, 2010.