Introduction to computer control systems: 
Selected exercises for the problem solving sessions 
Master program in embedded systems, period 2, 2011

Problem solving session VI (Ex6)

1. (Exercise 3.20 from [1])
   Given the system
   \[(q^2 + 0.4q)y(k) = u(k),\]
   for which values of \(K\) in the proportional controller
   \[u(k) = K(u_c(k) - y(k))\]
   is the closed-loop system stable?

2. Consider the system defined by
   \[
   \begin{align*}
   x_1(k+1) &= x_1(k) + 0.2x_2(k) + 0.4 \\
   x_2(k+1) &= 0.5x_1(k) - 0.5
   \end{align*}
   \]
   (a) Find the equilibrium point.
   (b) Obtain the state space form.
   (c) Is the model stable?

3. (Based on Exercise 3.22 from [2])
   A dynamic system is given by a scalar differential equation with an algebraic expression given by
   \[
   \begin{align*}
   \frac{d}{dt}\xi &= -\xi + u\eta^3 \\
   0 &= -\eta + u^2e^\eta
   \end{align*}
   \]
   (a) A control system should be designed to keep the system at a given stationary point \(\xi_0\). Determine the full operating point \((\xi_0, u_0, \eta_0)\) when \(\eta_0 = 1.1843\).
(b) The system input is $u$ and its output is $y = \eta \xi$. Determine a linear state model, valid near the operating point determined in (a).

(c) How is the stability of the stationary operating point ($\xi_0$)?

4. (Based on Exercise 3.26 from [2])

In an autonomous biological process there are two organisms ($A$ and $B$). The two organisms interact so that they grow in proportion to both concentration, $c_A$ and $c_B$. The organisms are dying off at a speed proportional to their number. The process is described by the following bilinear equations:

\[
\begin{align*}
\frac{dc_A}{dt} &= -c_A + \alpha c_A c_B \\
\frac{dc_B}{dt} &= -c_B + \beta c_A c_B
\end{align*}
\]

The output of the system is the arithmetic mean $c_M = 0.5(c_A + c_B)$.

(a) Determine the two possible steady states and find the linearized state models around these working points.

(b) Are the two models stable for all combinations of process parameters $\alpha$ and $\beta$?

References
