Systems and Control
Department of Information Technology
UPPSALA UNIVERSITY
www.it.uu.se/research/syscon

Introduction to computer control systems: Selected exercises for the problem solving sessions

Master program in embedded systems, period 2, 2011

Problem solving session VII (Ex7)

1. (Based on Exercise 5.9 from [1])

   Given the system (sampled double integrator)

$$\begin{aligned} x(t+T) &= \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} x(t) + \begin{pmatrix} T^2/2 \\ T \end{pmatrix} u(t) \\ y(t) &= (1 \quad 0)x(t), \end{aligned}$$

   determine

$$K = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

   in the observer

$$\hat{x}(t+T) = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \hat{x}(t) + \begin{pmatrix} T^2/2 \\ T \end{pmatrix} u(t) + K(y(t) - (1 \quad 0)\hat{x}(t))$$

   so that estimation error becomes zero at two sampling intervals. (Dead-beat estimation)

2. (Based on Exercise 5.12 from [1])

   Given the system

$$\begin{aligned} x(t+1) &= \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} x(t) + \begin{pmatrix} 2 \\ 3 \end{pmatrix} u(t) \\ y(t) &= (1 \quad 0)x(t), \end{aligned}$$

   and the observer for the state x_2

$$\begin{aligned} \hat{x}_2(t+1) &= 0.9\hat{x}_2(t) + 0.1y(t) + 3u(t) + K(y(t+1) - 0.8y(t) \\ &\quad -0.2\hat{x}_2(t) - 2u(t)), \end{aligned}$$

   select the number K (in this reduced observer) so that the estimation error becomes zero after a finite number of steps.

3. (Based on Exercise 6.7 from [1])

   Consider the process described by the following state model:

$$\begin{aligned}\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\end{aligned}$$

   The state feedback here refers to the matrix $L = (l_1 \quad l_2)$ such that a control law of the form $u = Kr - Lx$ is obtained.

   (a) Determine the matrix L so that the closed-loop system poles are $p = -1.6 \pm j1.2$.

   (b) Determine the transfer function from the $R(s)$ to $Y(s)$ for the closed loop system.

   (c) Determine the gain parameter K so that the closed-loop static gain is one.

4. Consider the three-tank system analysed in Exercise 1 of problem solving session V (Ex5). Let $f_i(t)$ to be the system input and $f_3(t)$ the system output. The process has two flowmeters to measure $f_i(t)$ and $f_3(t)$. The tank levels are not measured. The state space representation obtained at the equilibrium point $f_{i,0} = 5$ m^3/h and $f_{0,0} = 2$ m^3/h is given by

$$\begin{aligned}\dot{x} &= \begin{pmatrix} -1.6537 & 0 & 0 \\ 1.6537 & -1.3067 & 0 \\ 0 & 1.3067 & -1.0417 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & 0 & 1.0417 \end{pmatrix} x\end{aligned}$$

   (a) Design a control law $u = -K\hat{x}$ ($\hat{x}$ are the estimated states), so that the closed loop system poles are in $p_1 = -2.5$, $p_2 = -3$ and $p_3 = -4$.

   (b) Draw a block diagram of the closed-loop system.

   (c) Validate the designed observer and state feedback controller by simulations. Analyse the results.

References

[1] Mikael Johansson and Torsten Söderström. *Exercises Control Theory.* Uppsala University and Royal Institute of Technology, 2010.