Introduction to computer control systems:
Selected exercises for the problem solving sessions
Master program in embedded systems, period 2, 2011

Problem solving session VIII (Ex8)

1. Consider the following continuous-time second order linear system

\[ G(s) = \frac{4}{4s^2 + 12s + 8} \]

and the PID controller given by

\[ H(s) = k_p + \frac{k_i}{s} + k_ds \]

(a) Obtain the resonance frequency \( \omega_n \) and the damping factor \( \xi \) for system \( G(s) \).

(b) Obtain the closed-loop transfer function.

(c) Obtain the PID parameters such that the closed-loop poles are in \( p_{1,2} = -5 \pm 2i \) and \( p_3 = -10 \).

2. Consider the continuous-time linear system given by

\[ G(s) = \frac{100}{5s + 10} \]

(a) Design a PI controller

\[ H(s) = k_p + \frac{k_i}{s} \]

so that the closed-loop poles have a resonance frequency \( \omega_n = 4 \)
and a damping factor \( \xi = 0.707 \).

(b) Compute the static gain for the closed loop system.
3. Consider the following continuous-time system

\[
\begin{align*}
\dot{x} &= \begin{pmatrix} -1.6537 & 0 & 0 \\ 1.6537 & -1.3067 & 0 \\ 0 & 1.3067 & -1.0417 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u \\
y &= \begin{pmatrix} 0 & 0 & 1.0417 \end{pmatrix} x
\end{align*}
\]

which is controlled by \( u = -K \hat{x} \) (\( \hat{x} \) are the estimated states), where \( K = [5.50 \ 6.86 \ 3.91] \). The observer is given by

\[
\dot{\hat{x}} = A \hat{x} + Bu + L(y - C \hat{x})
\]

where \( L = [2743.3 \ 756.0 \ 53.8]^T \).

(a) Compute the closed loop system poles. Compare with the poles of the open loop system.