Introduction to computer control systems
Master program in embedded systems, period 2, 2011

Solutions for Problem solving session VIII (Ex8)

1. (a) $\omega_n = \sqrt{2}; \xi = 3/(2\sqrt{2}); k = 1/2$ (system gain).
   (b) 
   \[
   W(s) = \frac{G(s)H(s)}{1 + G(s)H(s)} \\
   W(s) = \frac{kD s^2 + kp s + ki}{s^3 + (3 + kD)s^2 + (2 + kp)s + ki}
   \]
   (c) coefficients of the different powers of $s$ in the desired closed-loop characteristic polynomial $Q_c(s) = (s - p_1)(s - p_2)(s - p_3)$ (with $p_i, \{i = 1, 2, 3\}$ as given) must be equal to coefficients of corresponding powers of $s$ in $s^3 + (3 + kD)s^2 + (2 + kp)s + ki$ (denominator of the transfer function obtained in (a));
   
   $k_P = 127; k_I = 290; k_D = 17$

2. (a) closed-loop transfer function
   \[
   W(s) = \frac{20kp s + 20ki}{s^2 + (2 + 20kp)s + 20ki}
   \]
   denominator of the transfer function should be made equal to $s^2 + 2\xi\omega_n s + \omega_n^2$ with $\xi$ and $\chi$ as given;
   
   $k_I = 0.8; k_P = 0.1828$
   (b) The static gain of the system is one (due to the integral action that regulates away the static error).

3. (a) System given by $eig(A - BK)eig(A - LC)$:
   $eig = \{-4.0; -3.0; -2.5; -18.7 \pm 1.9i; -22.7\}$; first three eigenvalues are given by the state feedback (closed-loop poles) (from A-BK) while the three last ones are given by the observer dynamics (from A-LC). Note that the observer poles are faster than the closed-loop ones, as desirable.