Systems and Control
Department of Information Technology
UPPSALA UNIVERSITY
www.it.uu.se/research/syscon

Introduction to computer control systems

Master program in embedded systems, period 2, 2011

Solutions for Problem solving session VIII (Ex8)

1. (a) $\omega_n = \sqrt{2}$; $\xi = 3/(2\sqrt{2})$; $k = 1/2$ (system gain).

   (b)

$$\begin{aligned} W(s) &= \frac{G(s)H(s)}{1+G(s)H(s)} \\ W(s) &= \frac{k_D s^2 + k_P s + k_I}{s^3 + (3+k_D)s^2 + (2+k_P)s + k_I} \end{aligned}$$

   (c) coefficients of the different powers of s in the desired closed-loop characteristic polynomial $Q_c(s) = (s-p_1)(s-p_2)(s-p_3)$ (with $p_i, \{i = 1,2,3\}$ as given) must be equal to coefficients of corresponding powers of s in $s^3 + (3+k_D)s^2 + (2+k_P)s + k_I$ (denominator of the transfer function obtained in (a));

   $k_P = 127$; $k_I = 290$; $k_D = 17$

2. (a) closed-loop transfer function

$$W(s) = \frac{20k_P s + 20k_I}{s^2 + (2+20k_P)s + 20k_I} \tag{1}$$

   denominator of the transfer function should be made equal to $s^2 + 2\xi\omega_n s + \omega_n^2$ with ξ and χ as given;

   $k_I = 0.8$; $k_P = 0.1828$

   (b) The static gain of the system is one (due to the integral action that regulates away the static error).

3. (a) System given by $eig(A-BK)eig(A-LC)$:
   $eig = \{-4.0; -3.0; -2.5; -18.7 \pm 1.9i; -22.7\}$; first three eigenvalues are given by the state feedback (closed-loop poles) (from A-BK) while the three last ones are given by the oberver dynamics (from A-LC). Note that the observer poles are faster than the closed-loop ones, as desirable.