Control structures, frequency descriptions

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F4: Quiz!

1) PI-control
   a. May suppress constant disturbances ↑
   b. Guarantees zero stationary control errors ↑
   c. Guarantees zero stable closed-loop system ↓
F4: Quiz!

1) PI-control
   a. May suppress constant disturbances ↑
   b. Guarantees zero stationary control errors ↑
   c. Guarantees zero stable closed-loop system ↓

2) Routh’s algorithm is a method that
   a. Checks for oscillations in systems ↑
   b. Aids design of stable closed-loop systems ↑
   c. Guarantees stable systems ↓
Simple vs. general linear feedback
Simple linear feedback control

Exemple: PID-controller

Simple feedback using control error:

\[ U(s) = F(s) (R(s) - Y(s)) = F(s)R(s) - F(s)Y(s). \]

Closed-loop system:

\[ Y(s) = G(s) F(s) \]

\[ 1 + G(s) F(s) \]

\[ R(s) \]

\[ y \]

\[ e \]

\[ u \]

\[ F \]

\[ G \]

\[ r \]

\[ u \]

\[ y \]

\[ e \]

\[ u \]

\[ y \]
Simple linear feedback control

Exemple: PID-controller

Simple feedback using control error:

\[ U(s) = F(s)(R(s) - Y(s)) = F(s)R(s) - F(s)Y(s). \]

Closed-loop system:

\[ Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)}R(s) \]
General linear feedback control

General feedback with *all measured* signals:

\[ U(s) = F_r(s)R(s) - F_y(s)Y(s). \]
General linear feedback control

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![Block diagram showing the feedback control system with signals](#)
General linear feedback control

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Closed-loop system:

\[ Y(s) = \frac{G(s)F_r(s)}{1 + G(s)F_y(s)} R(s) \]

- \( F_y(s) = F_r(s) = F(s) \) ⇒ simple linear feedback
- \( F_y(s) \neq F_r(s) \) ⇒ more *degrees of freedom*. 
General linear feedback control

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Closed-loop system:

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- \( F_y(s) \neq F_r(s) \) ⇒ more *degrees of freedom.*

See state-feedback controller F8-F10.
Feedback with measureable disturbances
In some systems we are also able to measure disturbances. Example:
Feedforward control

Scenarios with measurable disturbances

General feedback with measurable signals including disturbance:

\[ U(s) = F_r(s)R(s) - F_y(s)Y(s) + F_f(s)V(s) \]

control using also measured disturbance
Feedforward control

Scenarios with measurable disturbances

General feedback with measurable signals including disturbance:

\[ U(s) = F_r(s)R(s) - F_y(s)Y(s) + F_f(s)V(s). \]

[Board: derive the closed-loop system \( r, v \rightarrow y \)]
Feedforward control
Scenarios with measurable disturbances

Ideal controller:

\[ Y(s) = \frac{G_1(s)G_2(s)F_r(s)}{1 + G_1(s)G_2(s)F_y(s)} R(s) + \frac{G_1(s)(H(s) + G_2(s)F_f(s))}{1 + G_1(s)G_2(s)F_y(s)} V(s) \approx 1 \]

\[ \approx 0 \]
Feedforward control
Scenarios with measurable disturbances

Ideal disturbance suppression:

\[ H(s) + G_2(s)F_f(s) = 0 \iff F_f(s) = -\frac{H(s)}{G_2(s)} \]

Explicit disturbance compensation, but often hard to implement, due to high order in the numerator of \( F_f(s) \).
Example: tank with valve + disturbance

Example:

- **Static feedforward:**

  \[ F_f(s) = -\frac{H(0)}{G_2(0)} \]

- **Dynamic feedforward:**

  \[ F_f(s) = -\frac{H(s)}{G_2(s)} \frac{20}{s + 20} \]
Example: tank with valve + disturbance

Control using feedforward, when $r(t) \equiv 0$ and $v(t)$ is a step

$$F_f(s) = -\frac{20(s+2)}{2(s+20)}$$

PID

$K_p = 4$, $K_i = 4$, $K_d = 2$
Example: tank with valve + disturbance
Control using feedforward, when \( r(t) \equiv 0 \) and \( v(t) \) is a step

\[
F_f(s) = -\frac{20(s+2)}{2(s+20)}
\]

PID
\[
K_p = 4, \ K_i = 4, \ K_d = 2
\]

Static/dynamic feedforward vs. PID control

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Feedback with intermediate signals
Cascade control
Measurable intermediate signals

In certain systems we can measure internal or intermediate signals:
Cascade control
Measurable intermediate signals

Simple linear feedback:

\[ U(s) = F(s)R(s) - F(s)Y(s) \]
\[ = F(s)E(s) \]
Cascade control
Measurable intermediate signals

Simple linear feedback control with *measured* signals:

\[
U(s) = F_i(s)F(s)R(s) - F_i(s)F(s)Y(s) - F_i(s)Z(s) = F_i(s)(F(s)E(s) - Z(s))
\]

\[
Z_r(s)
\]
**Cascade control**

Measurable intermediate signals

Control strategy: easier to control subsystems

- Design $F_i(s)$ so that $Z(s) \approx Z_r(s)$
- Design $F(s)$ with respect to $G_1(s)$, neglect internal loop.
Combining all above
Control with respect to $R(s)$ and $Y(s)$
Control with respect to $R(s)$ and $Y(s)$ as well as measurable disturbance $V(s)$
Measurable signals and disturbances

Total controller

Control with respect to $R(s)$ and $Y(s)$ as well as measurable signal $Z(s)$ and disturbance $V(s)$

Exploits all available information!
Time-varying signals and frequency descriptions
Frequency description of systems
(Closed-loop) system response to oscillating signals

- Recall representation of cosine and sine signals:

\[ e^{i\omega t} = \cos(\omega t) + i \sin(\omega t) \]

\[ \omega = 2\pi f \quad \text{(frequency)} \]
Recall representation of cosine and sine signals:

\[ e^{i\omega t} = \cos(\omega t) + i \sin(\omega t) \]

\[ \omega = 2\pi f \quad \text{(frequency)} \]

Any signal \( x(t) \) can be decomposed into sum of cosine and sine signals:

\[ x(t) = \int_{-\infty}^{\infty} X(i\omega) e^{i\omega t} \, d\omega, \]

where the integral is over all frequencies \( \omega \) and \( X(i\omega) \) is the frequency response of the system.
Recall representation of cosine and sine signals:

\[ e^{i\omega t} = \cos(\omega t) + i \sin(\omega t) \]

\[ \omega = 2\pi f \quad \text{(frequency)} \]

Any signal \( x(t) \) can be decomposed into sum of cosine and sine signals:

\[ x(t) = \int \underbrace{X(i\omega)}_{\text{weights}} e^{i\omega t} \ d\omega, \]

\[ e^{i\omega t} \] is an eigen-function to linear time-invariant systems
Frequency description of systems
(Closed-loop) system response to oscillating signals

- Recall representation of cosine and sine signals:
  \[ e^{i\omega t} = \cos(\omega t) + i \sin(\omega t) \]
  \[ \omega = 2\pi f \] (frequency)

- Any signal \( x(t) \) can be decomposed into sum of cosine and sine signals:
  \[ x(t) = \int X(i\omega) e^{i\omega t} d\omega, \]

- \( e^{i\omega t} \) is an eigen-function to linear time-invariant systems

Fundamental property of LTI systems
Output is also a sum of the input cosine- and sine signals!
Frequency properties
Sine in/sine out

[Board: derive \( y(t) \) when \( u(t) = A \sin(\omega t) \)]
Frequency properties
Sine in/sine out

[Board: derive \( y(t) \) when \( u(t) = A \sin(\omega t) \)]

Sine in-sine out

Assume stable system \( Y(s) = G(s)U(s) \), where \( u(t) = A \sin(\omega t) \).

After all transients vanish, we obtain output:

\[
y(t) = \underbrace{|G(i\omega)| \cdot A \sin(\omega t + \arg G(i\omega))}_{\text{amplification}} \quad \underbrace{\text{phase shift}}_{\text{arg } G(i\omega)}
\]
Frequency properties
Sine in/sine out

[Board: derive \( y(t) \) when \( u(t) = A \sin(\omega t) \)]

### Sine in-sine out

Assume stable system \( Y(s) = G(s)U(s) \), where \( u(t) = A \sin(\omega t) \).

After all transients vanish, we obtain output:

\[
y(t) = |G(i\omega)| \cdot A \sin(\omega t + \arg G(i\omega))
\]

- **amplification**
- **phase shift**

- Same applies to closed-loop system

\[
Y(s) = G_c(s)R(s),
\]

with \( r(t) = A \sin(\omega t) \)

- Yields *interpretation* of complex-valued

\[
G_c(s) = |G_c(s)|e^{i\arg G_c(s)} \text{ when } s = i\omega
\]
Frequency properties of closed-loop system


**Example: inverted pendulum**

Linearized model (around $y \approx 0$):

\[
\ddot{y} - \left( \frac{3g}{2L} \right) y = \left( \frac{3}{mL^2} \right) u \iff Y(s) = \frac{3}{s^2 - \frac{3g}{L}} U(s) = \frac{1}{s^2 - 1} U(s) s
\]
Example: inverted pendulum

Response of PD-control

System $Y(s) = \frac{1}{s^2 - 1} U(s)$.

PD-control $U(s) = K(s + 3)(R(s) - Y(s))$ gives closed-loop system

$Y(s) = G_c(s)R(s)$ where $G_c(s) = \frac{K(s + 3)}{s^2 + Ks + 3K - 1}$. 

$r$ $e$ $u$ $F$ $G$ $y$
Example: inverted pendulum
Response of PD-control

System \( Y(s) = \frac{1}{s^2 - 1} U(s) \).

- PD-control \( U(s) = K(s + 3)(R(s) - Y(s)) \) give closed-loop system

\[
Y(s) = G_c(s) R(s) \quad \text{where} \quad G_c(s) = \frac{K(s + 3)}{s^2 + Ks + 3K - 1}.
\]

- With reference \( r(t) = \sin(\omega t) \) yields output

\[
y(t) = |G_c(i\omega)| \sin (\omega t + \arg(G_c(i\omega)))
\]

Affected by user parameter \( K \)!
Amplitude- and phase plot
Example: PD-control of inverted pendulum

$|G_c(i\omega)|$ and $\arg(G_c(i\omega))$ as a function of frequency $\omega$
\[ |G_c(i\omega)| \text{ and } \arg(G_c(i\omega)) \text{ as a function of frequency } \omega \]

Note amplification and phase shift at \( \omega = 1 \)
Example: PD-control of inverted pendulum

Amplification/attenuation and phase shift of \( r(t) \)

PD-control inverted pendulum when \( r(t) = \sin(1t) \).

Cf. amplitude- and phase plots for different \( K \)
Summary and recap

- General linear feedback control
  - Measured disturbances $\rightarrow$ feedforward control
  - Measured intermediate signals $\rightarrow$ cascade control

- Frequency description of system:
  - Sine in/sine out
  - System amplitude- and phase plots (Bode diagram)