Automatic control II (Reglerteknik II, 1RT495)

Homework assignments, 2010

Deadlines
Assignment 1  September 23, 24.00
Assignment 2  October 07, 24.00

Assignment 1 gives up to 6 hw points, and assignment 2 up to 9 hw points.

The points achieved from the homework assignments are then transformed into
bonus points exchangeable for the last problem in the final exam, according to

\[ bp = \min(6, \floor{hwp/2}) \]

expressed in Matlab code, and where \( bp \) and \( hwp \) should be read as bonus points (on
the exam) and homework points, respectively. (Hence, full 6 bp can be obtained
if \( 12 \leq hwp \leq 15 \).)
1 Homework assignment 1 – Sampling

Contents:
- Sampling a continuous-time state-space system
- Sampling a continuous-time state-space system with time delay

1.1 Problem 1

a) Consider the system with two inputs and one output:

\[ G(s) = \begin{bmatrix} K_1/sT_1 + 1 & K_2/sT_2 + 1 \end{bmatrix} \]

The system is of first order for each input. Write the system in state-space form.

*Hint*: It will be handy to use a diagonal form.

b) Sample the continuous-time state-space system obtained from a) and calculate the discrete state-space matrices:

\[ F = e^{Ah} \]

\[ G = \int_0^h e^{At} B dt \]

where \( h \) is the sampling period.

(1 p)

c) Show that the discrete transfer operator for the system is given by:

\[ H(q) = \begin{bmatrix} b_1/q - a_1 & b_2/q - a_2 \end{bmatrix} \]

and determine \( a_1, a_2, b_1, b_2 \) as functions of \( K_1, K_2, T_1 \) and \( T_2 \) and \( h \).

(1 p)

1.2 Problem 2

Consider a time-delayed continuous system described by

\[ \frac{dx(t)}{dt} = Ax(t) + Bu(t - \tau) \]

Assume that the time delay \( \tau \) is less than or equal to the sampling period \( h \).

Show that the sampled system is given by

\[ x((k+1)h) = Fx(kh) + G_0u(kh) + G_1u((k-1)h) \]

where

\[ F = e^{Ah} \]

\[ G_0 = \int_0^{h-\tau} e^{At} B dt \]
\[ G_1 = e^{A(h-\tau)} \int_0^\tau e^{At} B dt \] 

*Hint:* Split the integral

\[ \int_{kh}^{kh+h} e^{A(kh+s)} Bu(s - \tau) \, ds \]

into two parts.
2 Homework assignment 2 – LQG design

Consider the DC servo model

\[ G(s) = \frac{1}{s(s + 1)} \]

which we represent in state space form as

\[
\begin{align*}
\dot{x} &= \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\
y &= \begin{pmatrix} 1 & 0 \end{pmatrix} x
\end{align*}
\] (2.1)

Design linear controllers based on LQ(G) theory for this system. The regulator has the structure

\[ u = -Lx + mr \]

where \( r \) is the reference signal, in case all state variables can be measured; and

\[ u = -L\dot{x} + mr \]

in case an observer must be included.

Design constraints

- It is required that the step response from \( r(t) \) to \( y(t) \) does not have any stationary error.
- During the design the input for a step response must at all times be constrained so that

\[ |u(t)| \leq 4 \forall t \]

Evaluate the closed loop system with the designed controller, and simulate it with the reference signal

\[ r(t) = \begin{cases} 0, & 0 \leq t \leq 4 \\ 1, & 4 < t \leq 8 \end{cases} \]

Problems to solve

(a) Assume that both state variables are assessible. Use LQ theory in the design. What constraints have to be applied to the penalty matrices \( Q_1 \) and \( Q_2 \) in order to meet the design constraint on \( u(t) \)? Define the risetime of the closed loop system as the time it takes for the step response to go from 10% to 90% of its final value. How small can the risetime be made? (6 p)

(b) Next assume that only \( y(t) = x_1(t) \) is available for measurements. Design a regulator using LQG theory. Simulate the closed loop system as above. Use the initial values

\[ x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \dot{x}(0) = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} \]

How did you choose the matrices \( Q_1 \), \( Q_2 \), \( R_1 \), \( R_2 \) ? (3 p)
For both cases, include with your solution the program files you have used, and plots of the step responses (plot $y(t)$ and $u(t)$). It is open what state variable(s) to penalize in the LQ criterion.

Hints:
Below is part of the Matlab code you can use to find the solution. You will have to input and test various matrices $Q_1$, $Q_2$, $R_1$, $R_2$. The template m-file can be downloaded, see the webpage with course material. The risetime can be determined using the Matlab command `stepinfo`.

Your solution will be graded using the following guidelines.

Part a).
- A correct answer is obtained, but only with experimentation. max 3 p
- A motivation is also added, and relevant combinations of the penalty matrices are evaluated. max 4 p
- A theoretical analysis is also carried out. max 6 p

Hint. For a theoretical analysis, first find out, by experimentation or otherwise, at what time $\max |u(t)|$ occurs. Then use this to find a condition on $m$ and $L$, and convert this to a condition on $Q_1$ and $Q_2$.

Part b).
For this part, a theoretical analysis is not feasible.
- A correct answer, but without appropriate motivation. max 2 p
- A correct answer, with appropriate motivation. max 3 p

```matlab
% input matrices A,B,C,D; define open loop system
sysservo=ss(A,B,C,D)
% define reference signal
t1=0:0.01:8;t1=t1(:);
r = ones(size(t1));
r(1:400) = zeros(400,1);

% LQ design
% input Q1 and Q2

[L,S] = lqr(A,B,Q1,Q2)
% Compute m
m =
% Define closed loop system with y and u as outputs
sysservo2=ss(A-B*L,B*m,[C;-L],[D;m]);
% simulate the closed loop system, plot y and u
```
\[
[z2, t2] = lsim(sysservo2, r, t1);
\]

\[
\text{subplot}(2,1,1)
\]

\[
\text{plot}(t2, z2(:,1))
\]

\[
ylabel(\text{\textquoteleft output\textquoteleft}), xlabel(\text{\textquoteleft time\textquoteleft})
\]

\[
\text{subplot}(2,1,2)
\]

\[
\text{plot}(t2, z2(:,2))
\]

\[
ylabel(\text{\textquoteleft input\textquoteleft}), xlabel(\text{\textquoteleft time\textquoteleft})
\]

\[
% \text{compute max value of } u
umax = \text{max}(\text{abs}(z2(:,2)))
\]

% LQG case
% Input Q1, Q2, R1, R2
%
% Compute L and S
[L,S] = lqr(A,B,Q1,Q2)
% Determine m
m =
% Compute K and P
[K,P]=lqe(A,eye(2),C,R1,R2)
% Define closed loop system with y and u as outputs
sysservo3 = ss([A,-B*L;K*C,A-B*L-K*C],[B*m;B*m],...
[C,zeros(1,2);zeros(1,2),-L],[0;m]);
% Define initial values for the closed loop system
xinitial = [0; 0; 0; 0.5];
% Simulate closed loop system
[z3, t3] = lsim(sysservo3, r, t1, xinitial);
\[
\text{subplot}(2,1,1)
\]

\[
\text{plot}(t3, z3(:,1))
\]

\[
ylabel(\text{\textquoteleft output\textquoteleft}), xlabel(\text{\textquoteleft time\textquoteleft})
\]

\[
\text{subplot}(2,1,2)
\]

\[
\text{plot}(t3, z3(:,2))
\]

\[
ylabel(\text{\textquoteleft input\textquoteleft}), xlabel(\text{\textquoteleft time\textquoteleft})
\]

\[
6
\]