Uppsala University
Department of Information Technology
Division of Systems and Control
November 5, 2013

Automatic control III

Homework assignment 1
2013

Deadline (for this assignment):
Wednesday November 20, 24.00

All homework assignments are compulsory and form an important part of the examination.

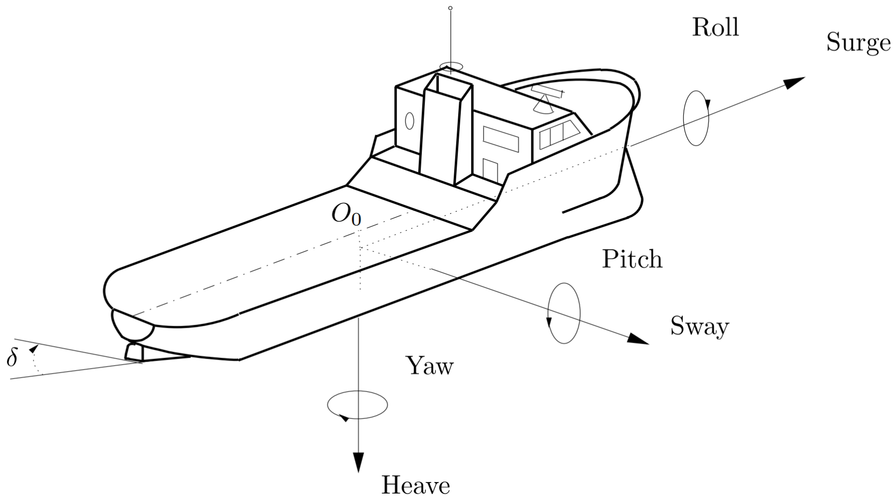
This assignment is to be solved in groups, with up to 4 students. The solution has to be handed in in written form as a pdf file. All group members are responsible for the entire report. There will also be an oral examination (on December 12–13), based on this assignment and assignment 2. Each group member will be given a few individual questions related to the assignment and the report.

Assignment 2 (not in this document) is to be solved in the same groups, and **assignment 3** (not in this document) is to be solved individually.

Instructions on how to hand in the assignments are found on the course homepage. Instructions for how to prepare the report are given at the end of this document.

Problem I Poles and zeros

The model of Son och Nomoto[1] describes the dynamics of a ship. It is used to design autopilots, which keeps the direction of the ship and stabilizes the rolling at the same time.



The motion and rotation of a ship[2]

For a given ship the model can be simplified into the following transfer function matrix

$$G(s) = \begin{bmatrix} \dfrac{2s+2}{s^3+3.2s^2+0.6s} \\ \\ \dfrac{2.5s^2-5s-20}{(s^2+3.2s+0.6)(s^2+0.4s+13)} \end{bmatrix}. \tag{1}$$

The system input is the rudder angle $u(t)$, and the outputs are the yaw angle (direction) $y_1(t)$ and the roll angle (heeling[3]) $y_2(t)$.

(a) Your colleague has transformed the system into a state space description with the states roll angle, roll rate, sway velocity, yaw rate and yaw angle. Is is assumed that the yaw angle and the roll angle are measured, and the rudder angle is known. Your colleague claims that it is possible to control the state vector to arbitrary values using this model. Is he/she correct? Motivate your answer mathematically.

(b) When going straight in a certain direction, the rudder angle is instantly set to a certain, constant value. Will the ship heel to the same side, from the beginning until the rudder angle is changed, or will the roll angle change sign during the maneuver? The impact from wind and waves is neglected. Motivate your answer mathemaically.

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[1]T.I. Fossen and T. Lauvdal, "Nonlinear Stability Analysis of Ship Autopilots in Sway, Roll and Yaw," *Proc. Int. Conf. Manoevring and Control of Marine Craft*, Southhampton, UK, 1994.

[2]T. Pèrez and M. Blanke, *Mathematical Ship Modeling for Control Applications*, Technical University of Denmark, Technical Report, 2002.

[3]Swedish: krängning

Problem II   Relative Gain Array

The following MIMO system is given:

$$G(s) = \begin{bmatrix} \dfrac{-k}{s+1} & \dfrac{ks+1}{s+1} \\[2ex] \dfrac{1}{s+2} & \dfrac{2k}{s+2} \end{bmatrix} \tag{2}$$

(a) Calculate the RGA-matrix analytically.

(b) Use MATLAB to plot the RGA matrices for $k = \{0, 0.2, 2\}$ in Bode plots. For which values of k does a decentralized controller appear to be a good approach, and for which does it not? The MATLAB command **bodemag** may be useful.

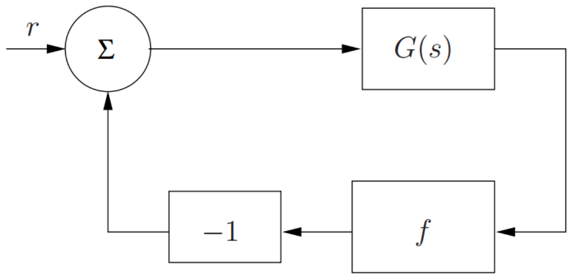
(c) For another system we have the following static RGA matrix

$$RGA(0) = \begin{bmatrix} -0.3 & 1.3 \\ 1.3 & -0.3 \end{bmatrix}. \tag{3}$$

What is preferred, to control y_1 from u_1 and y_2 from u_2, or vice versa? With the preferred choice for this case, do you expect a faster or slower control action due to the cross coupling effects?

Problem III Small gain theorem

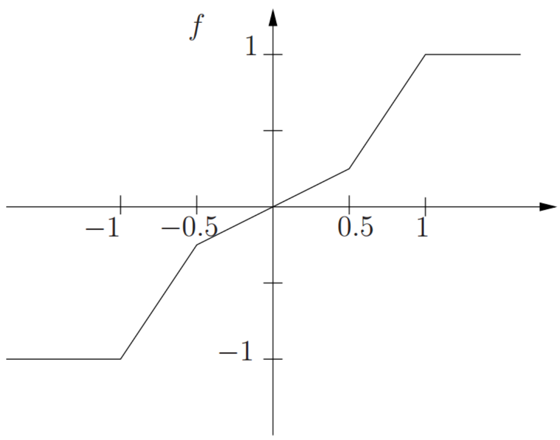
A nonlinear system is described by the block diagram



where $G(s)$ is a linear dynamical system given by

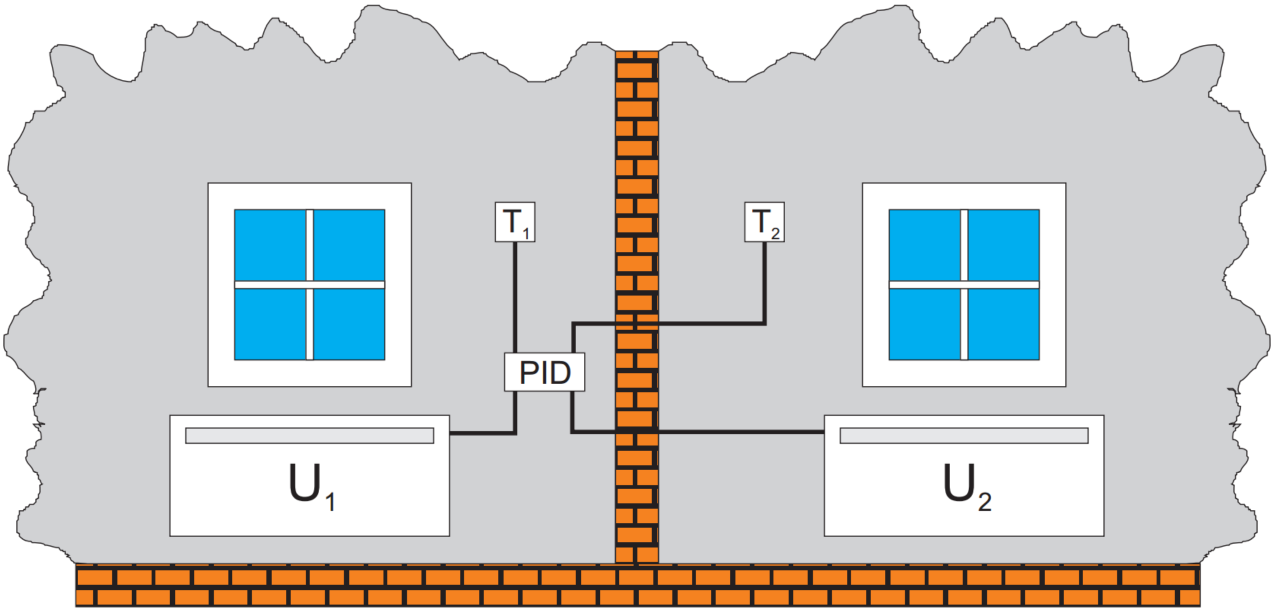
$$G(s) = \frac{1}{s(s+3)} \tag{4}$$

and f is a static nonlinearity defined as



Using the small gain theorem, can stability be guaranteed?

Problem IV RGA and IMC



You are assigned the task to design a control system for the heating of two rooms, according to the figure above. The temperatures T_1 and T_2 are measured, and the control signals are the radiator effects U_1 and U_2. U_1 and U_2 are assumed to take both negative and positive values (i.e., they can both heat and cool). After some scalings and simplifactions the transfer function can be written as

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{5s+0.025}{s^2+0.1s+0.002} & \frac{10^{-2}}{s^2+0.1s+0.002} \\ \frac{10^{-2}}{s^2+0.1s+0.002} & \frac{5s+0.025}{s^2+0.1s+0.002} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad (5)$$

(a) For complexity reasons, you are asked to use a solution based on regular SISO PID-controllers. Unfortunately, the heating in one room affects the other, so you cannot straightforwardly use these controllers. Describe a systematic way to handle this multivariable control problem using two PID controllers. You should clearly present your solution with block diagrams and all numerical values computed. In particular, the connection between the controlled system $G(s)$ and the two PIDs should be clear. However, you do not have to choose the PID parameters.

(b) Consider the transfer function (5). Design a multivariable IMC controller for the system in (5) such that the closed loop system fullfills

(a) A static gain I.

(b) A rise time of 10 ± 1 minutes for a step in each reference signal separately

(c) The order of the denominator polynomial should not be greater than 3 for any element in Q .

Simulate the closed loop system in Simulink. Enclose a figure of the Simulink diagram and plots verifying the requirements (a) and (b).

Some instructions on the report

The report should be written carefully, in order to be understood by a person without prior knowledge of the assignments. The theory that you use should also be clearly referenced. Summarize important findings in tables and illustrative plots. Make sure to describe what variables are plotted and in what units. Also try to make the figures readable, e.g., making curves with different type of lines or use colors.

Relevant MATLAB code should also be provided in electronic form preferably in an Appendix to the report. Avoid sending MATLAB code in a separate email.

Some general guidelines on how to write reports can be found in the document *Att skriva en teknisk rapport — en kort instruktion* (in Swedish) which is available as a pdf-file on the course homepage.