Summary of lecture 3 (I/II)

**Bode’s relationship** provides an upper bound on the phase, which depends on the derivative of the amplitude curve.

Hence, Bode’s relationship provides a fundamental limit by revealing a certain **coupling** between the amplitude and the phase.

Summary of lecture 3 (II/II)

For stable systems we derived **Bode’s integral theorem** stating that

\[ \int_0^\infty \log |S(i\omega)| \, d\omega = 0. \]

For unstable systems (with \( M \) poles \( \{p_i\}_{i=1}^M \) in the RHP):

\[ \int_0^\infty \log |S(i\omega)| \, d\omega = \pi \sum_{i=1}^M \text{Re}(p_i). \]

Contents – lecture 4

1. Summary of lecture 3
2. Which control design methods do we have?
3. Who should control what? Relative Gain Array (RGA)
   a) The pairing problem
   b) Decentralized control
   c) Decoupled control
4. Internal Model Control (IMC)
The most successful controller ever: PID

- Boulton och Watt 1788: speed control of steam engines, mechanical implementation.
- Hydraulic and pneumatic implementation: late 1800s
- Electronic: 1930s.
- Computer: 1950s.
- "PID-on-a-chip": 1990s.
- Applications: All.

Seam engine: http://www.youtube.com/watch?v=Hqqc6gfVrtU
PID-on-a-chip: http://www.youtube.com/watch?v=9eMWG3fwiEU

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PID

- Assumes one input signal and one output signal
- If you have several input and output signals you must pair them in twos
- Interpretation in Bode plots: lead and lag
- Tuning using intuition and experience results in not more than mediocre performance (except in very simple cases).
- Tuning using systematic analysis (poles, zeros, $S$, $T$, ...) can give controllers with very good performance.
- The first systematic approach (using poles): Maxwell 1868.


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Where PID is not enough

(Typically means that the system is multivariable and/or nonlinear)

- Internal model control (IMC)
- Minimization of quadratic criteria: LQ, LQG.
- Model predictive control (MPC)
- Systematic shaping of transfer functions: $H_2$, $H_{\infty}$.
- Nonlinear methods

The foundation of all modern control methods is to make use of models.

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MIMO – who should control what?

1. Tall system (more output than input signals):

   $G = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix}$

   All output signals cannot be controlled perfectly – prioritize.

2. Fat system (more input signals than output signals)

   $G = \begin{bmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots \\ \cdots \end{bmatrix}$

   How should the actuation be distributed among the control signals?

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If there are many input and output signals we can make the control design much easier by breaking down the system in sub-systems with *little interaction* between each other.

The *relative gain array (RGA)* is a way of measuring the level of cross coupling or interaction in a system.

### Interaction/cross coupling (I/II)

- Two-handle mixer, a system with a tough cross coupling
  - Several input signals (significantly) affects an output signal.
  - Several output signals are (significantly) affected by an input signal.

### Interaction/cross coupling (II/II)

- Mixer with one handle, a system with a “nice” cross coupling.
  - Each input signal affects (almost) only one output signal.
  - Each output is affected (almost) only by one input signal.
Interaction/cross coupling (II/II) 11(23)

- Mixer with one handle, a system with a “nice” cross coupling.
  - Each input signal affects (almost) only one output signal.
  - Each output is affected (almost) only by one input signal.

<table>
<thead>
<tr>
<th>Vinkel temperatur</th>
<th>G_{11}</th>
<th>0</th>
<th>Temperatur</th>
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<tbody>
<tr>
<td>Vinkel flöde</td>
<td>0</td>
<td>G_{22}</td>
<td>Vattenflöde</td>
</tr>
</tbody>
</table>

Decentralized control 12(23)

Idea (decentralized control): Build a controller for a MIMO system where one output signal controls one input signal.

The result is a set of “single variable loops”

\[ u_i = F_i^r r - F_i^y y_j, \]

where the individual controllers are all independent ("they do not know of each other").

- \( F_y \) is a quadratic transfer matrix. If the number of input and output signals is different some of them are simply discarded.
- The less cross couplings there are, the better this strategy works.
- We want to pair the input and output signals that have the strongest connections, the pairing problem.
- How do we determine the couplings between the various input and output signals?

Temperature control 13(23)

- Two rooms with a separating inner wall.
- The temperatures \( T_1, T_2 \) are states, which are both measured.
- Both rooms can be both heated and cooled by \( U_1 \) and \( U_2 \).

\[
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix}
= 
\begin{bmatrix}
0.005s + 0.0002438s^2 + 0.0975s - 0.05 & 9.375e-05 \\
9.375e-05 & 0.005s + 0.0002438s^2 + 0.0975s + 0.002025
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2
\end{bmatrix}
\]
Which sensor should be used in controlling which heating/cooling source (i.e. the pairing problem)?

Decentralized PI control
- $T_2$ is used for $U_1$.
- $T_1$ is used for $U_2$.

\[ F(s) = \begin{bmatrix} 0 & 1000 + \frac{500}{s} \\ 1000 + \frac{500}{s} & 0 \end{bmatrix} \]

After 10 hours 10 people enters room 1.

Is there any way in which we can predict this problem “analytically”?
Use RGA to decide which input signals to pair with which output signals.

The two main rules in designing a decentralized controller using RGA. Pair measurement signals and control signals such that the diagonal elements in
  1. \( \text{RGA}(G(i\omega_c)) \) are close to 1 in the complex plane.
  2. \( \text{RGA}(G(0)) \) are positive (if they are negative this can lead to instability)

“Pairing” implies a change of the position of rows and columns in the RGA-matrix.

RGA in Matlab: \( \text{RGA}(A) = A \cdot \text{pinv}(A.') \) (\text{pinv = pseudoinverse, handles non-square matrices}).

RGA again

Design rules from previous slide is different words:
  1. Select input-output pairs so that the diagonal elements of \( \text{RGA}(i\omega_c) \) are close to 1
  2. Avoid pairing that gives negative diagonal elements of \( \text{RGA}(G(0)) \).

In the temperature control example:

\[
\text{RGA}(i8) \approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{RGA}(G(0)) = \begin{pmatrix} 1.17 & -0.17 \\ -0.17 & 1.17 \end{pmatrix}
\]

The pairing where
  - \( T_2 \) is used for \( U_1 \) and
  - \( T_1 \) is used for \( U_2 \),

breaks both of these rules (the rows change places)!

Decoupled control

Sounds good, but how do we choose \( W_1 \) and \( W_2 \)?
  - In order to obtain a completely decoupled “virtual system” we would need \( s \)-dependent wright matrices \( W_1 \) and \( W_2 \).
  - This is in general not possible, since it would lead to a complicated and/or non-proper controller.
  - Instead we choose one frequency where the system becomes decoupled:
    - \( \omega = 0 \)
    - \( \omega = \omega_c \) (\( G^{-1}(\omega_c) \) is often approximated to get rid of complex valued elements).
  - The choice \( W_1 = G^{-1}(0) \) and \( W_2 = I \) results in decoupling in stationarity.
  - With the right choice of \( W_1 \) and \( W_2 \) we can make the two-handle mixer behave as a one-handle mixer. Easier to control!

Temperature control – decoupled

After 10 hours 10 people enters room 1.

Decoupled control using

\[
W_1 = G^{-1}(0), \quad \text{and} \quad W_2 = I.
\]
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Internal Model Control (IMC)

Feedback only using the new information $y - Gu$.

Results in (if $G_0 = G$)
- $G_c = GQ\tilde{F_r}$
- $S = I - GQ$
- $T = GQ$

How do we choose $Q$? (I/II)

The “ideal” case $Q = G^{-1}$ would result in $S = 0, G_c = I$, but it is infeasible since $\tilde{F_y} = \infty$.

Hence, we have to approximate!

Some rules of thumb:
1. If $G$ has more poles than zeros: The inverse of $G$ cannot be realized. Use
   $$Q(s) = \frac{1}{(\lambda s + 1)^n} G^{-1}(s).$$
   Choose $n$ such that $Q(s)$ is proper (# poles = # zeros).
   Choose $\lambda$ to adjust the bandwidth of the closed loop system.

How do we choose $Q$? (II/II)

2. If $G(s)$ non-minimum phase $\iff G$ has an “unstable zero” and contains a factor $(-\beta s + 1), \beta > 0$. Two alternatives
   a) Omit the factor when $Q = G^{-1}$ is formed.
   b) Replace the factor $(-\beta s + 1), \beta > 0$ with $(\beta s + 1), \beta > 0$ when $Q = G^{-1}$ is formed.

3. If $G$ contains a time delay, i.e. a factor $e^{-s\tau}$:
   a) Omit the factor when $Q = G^{-1}$ is formed.
   b) Approximate the factor using
      $$e^{-s\tau} \approx \frac{1 - s\tau/1}{1 + s\tau/2}$$
Cross couplings: The key difficulty in controlling MIMO systems is that there are cross couplings between input and output signals. If we change one input signal this affects several output signals.

Relative gain array (RGA): The relative gain array is a measure of the amount of cross couplings in a matrix ($\text{RGA}(A) = A \ast (A^\dagger)^T$).

Decentralized control: Let every input be determined by feedback from one single output.

The pairing problem: The pairing problem is to select which input-output pairs that should be used for the feedback.

Decoupled control: Decoupled control makes use of a change of variables such that suitable pairings of measurements and control signals becomes easier to see.

Internal model control (IMC): Choose $Q \approx G^{-1}$ ($y = GQr$) and let the new information in the form of $y - Gu$ be fed back to affect $u$. 