Some exercises on basic limitations

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October 28, 2013

Exercises

7.20
The continuous-time description of an unstable system has a pole in $0.005$.

1. On what time-scale will the closed loop system have to work (at least), with a well-designed controller? Are we talking about hours, minutes, seconds, milliseconds, . . . ?

2. If we are given the information that the system also contains a time-delay of 10 second, what can you say about the possibilities to control the system?

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As a control engineer, you are asked to design a (linear, time-invariant) controller, which at 5 rad/s has the following properties:

- It dampens disturbances on the output 10 times, i.e., the absolute value of the transfer function from disturbances on the output to the controlled variable is not bigger than 0.1 at this frequency.

- Noise in the measurements may at most affect the controlled variable with a tenth of the amplitude of the noise. I.e., the absolute value of the transfer function from measurement noise to the controlled variable is at most 0.1 at this frequency.

How can you easily tell you have been assigned an impossible task?
Solutions

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1. Using the rule of thumb \( \omega_0 > 2p_1 \), where \( \omega_0 \) is the lower bandwidth and \( p_1 \) is the unstable pole, a bandwidth of at least 0.01 rad/s is necessary. That corresponds to a timescale of \( \frac{1}{\omega_0} = 100 \text{ s} \approx 1 \text{ min} \).

2. For systems with time-delays, the bandwidth \( \omega_B \) is approximately limited by \( \omega_B < \frac{1}{T} \), where \( T \) is the time delay. Hence cannot the bandwidth exceed 0.1 rad/s due to the time delay, which is compatible with the (approximate) lower limit of 0.01 rad/s.

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The sensitivity function \( S(i\omega) \) and the complementary sensitivity function \( T(i\omega) \) always fulfill the relation \( S(i\omega) + T(i\omega) = 1 \) for all \( \omega \). This gives \( 1 - |S(i\omega) + T(i\omega)| \leq |S(i\omega)| + |T(i\omega)| \). However, you would need \( |S(i\delta)| \leq 0.1 \) and \( |T(i\delta)| \leq 0.1 \), which cannot satisfy \( S + T = 1 \).