Control Design (F, IT)
Computer Controlled Systems (STS, W), 2007

Computer exercise 1

Introduction to multi-variable control: Hints
1 Solution to the preparation exercises

Ex 2.1
The system is described by

\[ \dot{x}(t) = Ax(t) + Bu(t), \quad (1) \]
\[ y(t) = Cx(t) + w(t). \quad (2) \]

The system is controlled by the controller

\[ u(t) = -L\hat{x}(t) + mr(t). \quad (3) \]

Observer:

\[ \hat{x}(t) = A\hat{x}(t) + Bu(t) + K\{y(t) - C\hat{x}(t)\}. \quad (4) \]

Under static condition (nothing is changing) \( \dot{x}(t) = 0 \) and \( \hat{x}(t) = 0 \). We are interested in the static gain from \( r(t) \) to \( y(t) \). Hence set the other input \( w(t) = 0 \). Using these observations and subtracting (1) from (4) we get

\[ (A - KC)\{\dot{x}(t) - x(t)\} = 0. \quad (5) \]

Since \( A - KC \) is not a singular matrix in general,

\[ \hat{x}(t) = x(t). \quad (6) \]

Note that the above equality may not hold if \( A - KC \) is a singular matrix, which would mean that the estimation error may not be zero at steady state (static condition). Using (1), (3) and (6) we get

\[ x(t) = -(A - BL)^{-1}Bmr(t) \quad \Rightarrow \quad y(t) = -C(A - BL)^{-1}Bmr(t). \quad (7) \]

Hence to get unit static gain we must have

\[ m = -\{C(A - BL)^{-1}B\}^{-1}. \quad (8) \]

Ex 2.2

>From (1) and (3) we get

\[ \dot{x}(t) = Ax(t) - BL\hat{x}(t) + Bmr(t). \quad (9) \]

>From (2), (3) and (4) we get

\[ \dot{x}(t) = KCx(t) + (A - BL - KC)\hat{x}(t) + Bmr(t) + Kw(t). \quad (10) \]
Hence the required state space form is given by

$$
\begin{bmatrix}
\dot{x}(t) \\
\dot{\hat{x}}(t)
\end{bmatrix} =
\begin{bmatrix}
A & -BL \\
KC & A - BL - KC
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\dot{\hat{x}}(t)
\end{bmatrix} +
\begin{bmatrix}
B_{m} \\
B_{m}
\end{bmatrix} r(t) +
\begin{bmatrix}
0 \\
K
\end{bmatrix} w(t)
$$

$$
y(t) = \begin{bmatrix}
C & 0
\end{bmatrix} \begin{bmatrix}
x(t) \\
\dot{\hat{x}}(t)
\end{bmatrix} + w(t).
$$

(11)

Ex 2.3

$$K = \text{acker}(A^{'}, C^{'}, p_{obs})'$$

Ex 2.4

$$F = [A, -B*L; K*C, A - B*L - K*C], G = [B; E]*m, \quad H = [C \text{ zeros(size(C))}], S = \text{ss}(F,G,H,0)$$

Ex 2.5

$$S_n = \text{ss}(F, [\text{zeros(size(K))}; K], H, 1)$$

Ex 2.6 Let \( C = [c_1 \ldots c_n] \) and \( B = [b_1 \ldots b_n]^T \) (Note the difference in dimensions of \( B \) and \( C \)). Then

$$g(t) = \sum_{k=1}^{n} c_k b_k e^{p_k t}.$$  

(12)

If \( p_1 \) is the dominating pole, for \( t >> 0 \)

$$g(t) \approx c_1 b_1 e^{p_1 t} \quad \Rightarrow \quad \log [g(t)] = p_1 t + \log(c_1 b_1),$$

(13)

which means that the slope of the semilog curve for large \( t \) gives the dominating pole.
% **************************** Closed loop system ****************************
g1 = tf([2 1],[1 2 1 0]); % Open loop system G1
f1 = tf(1,1); % Unity feedback
h1 = feedback(g1,f1); % closed loop system
step(h1); % step response

% **************************** Stability margins ****************************
bode(g1); % bode plot
[M ph w] = bode(g1); % frequency response data
[Gm Pm Gc Pc] = margin(M,ph,w); % Gain and phase margin:
Pc = phase cross over

% **************************** Pole placement ****************************
K = (place(A’,C’,p_o)’); % Need to transpose matrices.
L = acker(A,B,p,s); % No need to transpose matrices.

% **************************** Closed loop system ****************************
F = [A -B*L; K*C A-B*L-K*C]; %
m = inv(C*inv(B*L-A)*B); % Set the static gain = 1
G = [B;B]*m; % State space description
H = [C zeros(size(C))]; % of the closed loop system
Gr = ss(F,G,H,0); % You have done it before.
step(Gr); % Step response of Gr

Table 1: Sample code for system 1.

% **************************** State feedback and pole placement ****************************

2 Difficulties in control

A brief outline of the Matlab code for forming the closed loop system for system 1 is given in Table 1. You can modify the same set of codes to study the other systems also. Next compensators are required to be designed. Follow the instructions given in the instruction sheet. Create the LTI object for the compensated system. Study the compensated closed loop system properties using bode and Nyquist plots. You may need to cascade two systems. Use

\[ h = \text{series}(g1,c1); \text{ or } h = g1*c1; \]

for that. Next we study state space feedback control of the systems. To create the state space description of open loop system use

\[ s1 = \text{ss}(g1), \text{ [A, B, C, D] = ssdata(s1); } \]

Next choose your pole! Let \( p_s \) be the vector of controller poles and \( p_o \) be the vector of observer poles. Observer poles has to be faster than the system poles. The set of codes given in Table 2 may be helpful.