Uppsala University
Department of Information Technology
Division of Systems and Control
September 8, 2015

Automatic control III

Homework assignment 1
2015

Deadline (for this assignment): Tuesday September 22, 23.59

All homework assignments are compulsory and form an important part of the examination.

This assignment is to be solved in groups, with up to 4 students. The solution has to be handed in in written form as a pdf file. All group members are responsible for the entire solutions. There will also be an oral examination (October 15), based on this assignment and assignment 2. Each group member will be given a few individual questions related to the assignment and your solutions.

Assignment 2 (not in this document) is to be solved in the same groups, and **assignment 3** (not in this document) is to be solved individually.

Instructions on how to hand in the assignments are found on the course homepage. The solutions should be clearly presented, with well motivated reasoning and clearly stated answers. The solutions should be carefully written, with satisfactory equation typesetting, equation numbering, complete sentences, etc. However, no introduction, abstract, etc is needed, but just solutions to the problems.

Before handing in your solutions, make sure (for each exercise) that

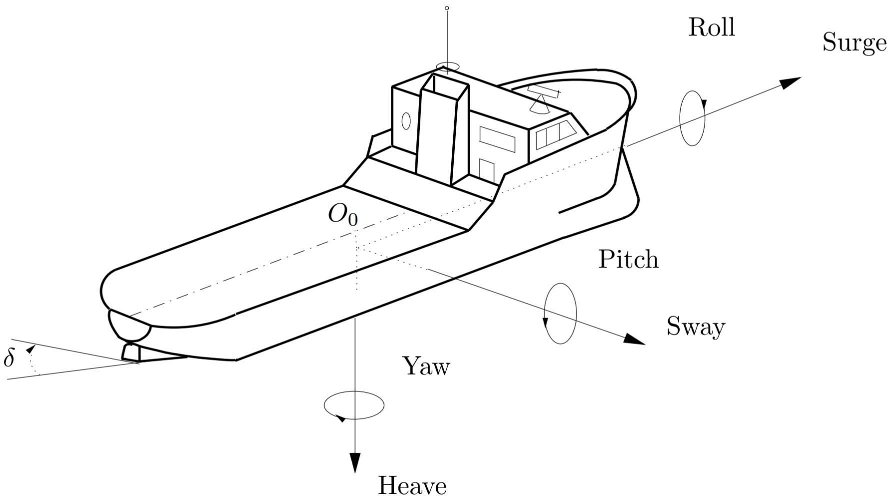
(I) You have answered all questions

(II) Your answer is reasonable

Solutions not fulfilling (I) and (II) will be rejected.

Problem I Poles and zeros

The model of Son och Nomoto[1] describes the dynamics of a ship. It is used to design autopilots, which keeps the direction of the ship and stabilizes the rolling at the same time.



The motion and rotation of a ship[2]

For a given ship the model can be simplified into the following transfer function matrix

$$G(s) = \begin{bmatrix} \dfrac{2s+2}{s^3+3.2s^2+0.6s} \\ \\ \dfrac{2.5s^2-5s-20}{(s^2+3.2s+0.6)(s^2+0.4s+13)} \end{bmatrix}. \tag{1}$$

The system input is the rudder angle $u(t)$, and the outputs are the yaw angle (direction) $y_1(t)$ and the roll angle (heeling[3]) $y_2(t)$.

(a) Your colleague has transformed the system into a state space description with the states roll angle, roll rate, sway velocity, yaw rate and yaw angle. Is is assumed that the yaw angle and the roll angle are measured, and the rudder angle is known. Your colleague claims that it is possible to control the state vector to arbitrary values using this model. Is he/she correct? Motivate your answer using control theory.

(b) When going straight in a certain direction, the rudder angle is instantly set to a certain, constant value. Will the ship heel to the same side, from the beginning until the rudder angle is changed, or will the roll angle change sign during the maneuver? The impact from wind and waves is neglected. Motivate your answer using control theory.

[1]T.I. Fossen and T. Lauvdal, "Nonlinear Stability Analysis of Ship Autopilots in Sway, Roll and Yaw," *Proc. Int. Conf. Manoevring and Control of Marine Craft*, Southhampton, UK, 1994.

[2]T. Pèrez and M. Blanke, *Mathematical Ship Modeling for Control Applications*, Technical University of Denmark, Technical Report, 2002.

[3]Swedish: krängning

Problem II Distillation column

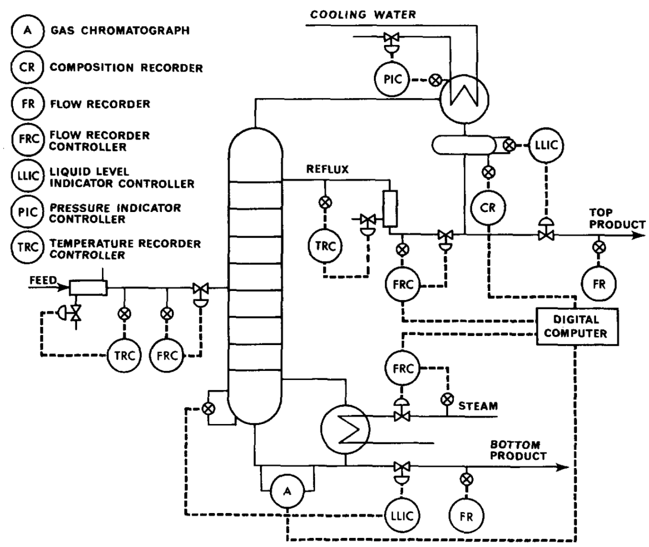
An approximate model of a binary distillation column[4] is

$$G(s) = \begin{bmatrix} \dfrac{13}{17s+1} & \dfrac{-19}{21s+1} & \dfrac{4}{15s+1} \\ \\ \dfrac{7}{12s+1} & \dfrac{-19}{15s+1} & \dfrac{5}{12s+1} \end{bmatrix}. \tag{2}$$

The inputs are u_1 reflux rate, u_2 steam rate and u_3 feed rate, whereas the outputs are y_1 distillate purity and y_2 bottoms purity.

(a) Calculate the poles and zeros (with multiplicity) for the transfer function $G(s)$.

(b) What is the largest gain of all individual elements of $G(s)$?
What is the gain of the (MIMO) system $G(s)$?
Comment on any differences!
(Hint: MATLAB commands **bodemag** and **sigma** might be useful.)

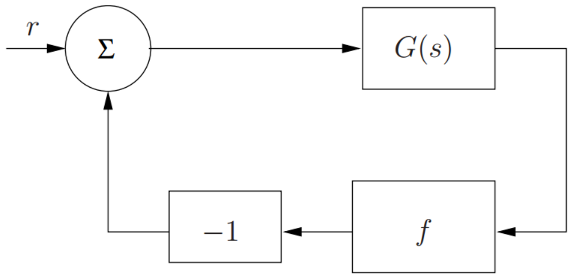


A schematic diagram of a binary distillation column

[4]R. K. Wood and M. W. Berry, *Terminal composition control of a binary distillation column*, Chemical Engineering Science, 1973, vol 28, pp. 1707-1717.

Problem III Small gain theorem

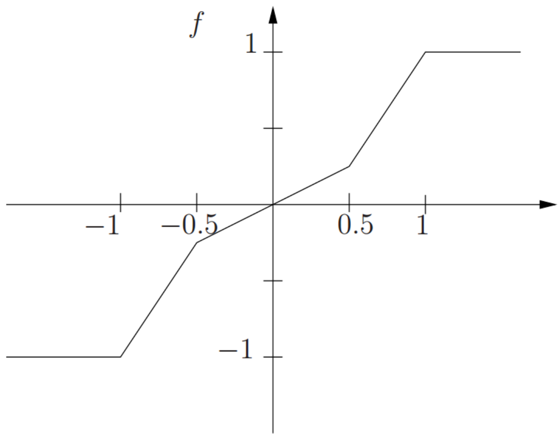
A nonlinear system is described by the block diagram



where $G(s)$ is a linear dynamical system given by

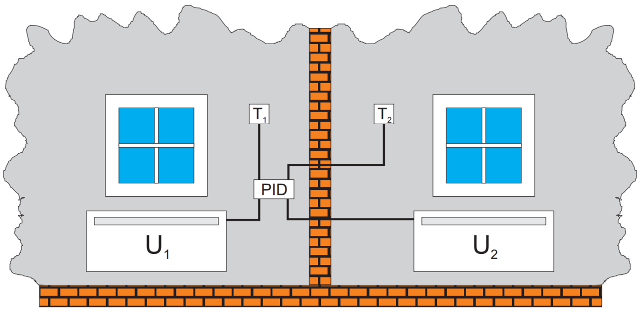
$$G(s) = \frac{1}{s(s+3)} \tag{3}$$

and f is a static nonlinearity defined as



Using the small gain theorem, can stability be guaranteed?

Problem IV RGA and IMC



You are assigned the task to design a control system for the heating of two rooms, according to the figure above. The temperatures T_1 and T_2 are measured, and the control signals are the radiator effects U_1 and U_2. U_1 and U_2 are assumed to take both negative and positive values (i.e., they can both heat and cool). After some scalings and simplifactions the transfer function can be written as

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \dfrac{5s+0.025}{s^2+0.1s+0.002} & \dfrac{10^{-2}}{s^2+0.1s+0.002} \\[2ex] \dfrac{10^{-2}}{s^2+0.1s+0.002} & \dfrac{5s+0.025}{s^2+0.1s+0.002} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{4}$$

(a) For complexity reasons, you are asked to use a solution based on regular SISO PID-controllers. Unfortunately, the heating in one room affects the other, so you cannot straightforwardly use these controllers. Describe a systematic way to handle this multivariable control problem using two PID controllers. You should clearly present your solution with block diagrams and all numerical values computed. In particular, the connection between the controlled system $G(s)$ and the two PIDs should be clear. However, you do not have to choose the PID parameters.

(b) Consider the transfer function (4). Design a multivariable IMC controller for the system in (4) such that the closed loop system fullfills

(I) A static gain I.

(II) A rise time of 10 ± 1 minutes for a step in each reference signal separately

(III) The order of the denominator polynomial should not be greater than 3 for any element in Q .

Simulate the closed loop system in Simulink. Enclose a figure of the Simulink diagram and plots verifying the requirements (a) and (b). Try also to introduce a plant model mismatch (i.e., make a small change in G_0 but not G) or a disturbance signal on the output of G_0 and evaluate how robust your controller is.

Hint: Your Simulink diagram should look similar to this: