Exam in Automatic Control III
Reglerteknik III 5hp

Date: December 17, 2012

Venue: Polacksbacken, skrivsal

Responsible teacher: Hans Norlander.

Aiding material: Textbooks (by Glad/Ljung), calculator, mathematical handbooks, copies of OH slides.

Preliminary grades: 13p for grade 4, 21p for grade 5. Maximum score is 30p.

Use separate sheets for each problem, i.e. no more than one problem per sheet. Write your exam code on every sheet.

Important: Your solutions should be well motivated unless else is stated in the problem formulation! Vague or lacking motivations may lead to a reduced number of points.

Your solutions can be given in Swedish or in English.

Good luck!

Problem 1

(a) Determine the poles and zeros, including multiplicity, for the system

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} x(t).$$

Also show that the system is a minimal realisation. **(3p)**

(b) The system

$$\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 1 & -2 \end{bmatrix} x(t)$$

is both unstable and has a non-minimum phase zero (in the RHP). A seemingly clever control engineer proposed the observer based state feedback control law $u(t) = -L\hat{x}(t) + y_{ref}(t)$, with the feedback and observer gains

$$L = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} 40 \\ 17 \end{bmatrix},$$

as a remedy for these flaws. Verify that this controller indeed yields the closed loop system

$$Y(s) = \frac{1}{s+1} Y_{ref}(s).$$

(2p)

(c) Is the closed loop system in (b) internally stable? **(3p)**

Problem 2

(a) Find all equilibria (stationary points) for the system

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -x_1 - x_2(1 - x_1^2 - x_2^2).$$

Characterize the equilibria in terms of stability and behaviour. **(3p)**

(b) The Lorenz attractor is a well known system with a chaotic behaviour, and a commonly used version (the original) is

$$\dot{x}_1 = 10(x_2 - x_1) + u,$$
$$\dot{x}_2 = x_1\left(\frac{8}{3} - x_3\right) - x_2,$$
$$\dot{x}_3 = x_1 x_2 - 28x_3,$$

where the input u is added here merely for the purpose of this problem. Assume that x_2 is measured, and thereby available for feedback. Find a feedback $u = u(x_2)$ that makes the origin $x = 0$ an asymptotically stable stationary point. The stability must be proven. **(4p)**

Problem 3 The system

$$\begin{cases} \dot{x} = Ax + Bu, \\ \bar{z} = Cx, \\ y = Cx + w, \end{cases} \qquad \Leftrightarrow \qquad G(s) = C(sI - A)^{-1}B,$$

where w is a measurement disturbance, is to be controlled by an $\mathcal{H}_\infty$ controller, designed using the frequency weightings

$$W_S(s) = \frac{K_S}{s + \alpha_S}, \quad W_T(s) = K_T \frac{s + \beta_T}{s + \alpha_T}, \quad W_u(s) = K_u.$$

(a) Give a state space model representing the extended open loop system (i.e. incorporating the frequency weightings). **(2p)**

(b) The following specifications should be fulfilled:

1. The controller $F_y(s)$ should have integral action.
2. The bandwidth of the closed loop system should be approximately 2 rad/s.
3. The effect of w on $\bar{z} = Cx$ should never be amplified more than 50 %, and it should be attenuated at least 100 times for frequencies $\omega \geq 314$ rad/s (= 50 Hz).
4. For the input $|u| < 4$ should hold.

Suggest appropriate values of the parameters $K_S, \alpha_S, K_T, \beta_T, \alpha_T, K_u$ in the frequency weightings, so that the specifications 1–4 are fulfilled if

$$\| W_S S \|_\infty < 1, \quad \| W_T T \|_\infty < 1, \quad \| W_u G_{wu} \|_\infty < 1. \tag{1}$$

(3p)

(c) It turns out that $G(s)$ has a pole in $s = +1$. Discuss the consequencies of this — is it possible obtain (1) with the frequency weightings you have chosen in (b)? **(2p)**

Problem 4

(a) Consider the square (m inputs and m outputs) system

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx + Du \end{cases} \quad \Leftrightarrow \quad G(s) = C(sI - A)^{-1}B + D,$$

where $\det D \neq 0$. Show that the state space model

$$\begin{cases} \dot{x} = (A - BD^{-1}C)x + BD^{-1}y, \\ u = -D^{-1}Cx + D^{-1}y, \end{cases}$$

is a representation of the *inverse* system $U(s) = G^{-1}(s)Y(s)$. **(1p)**

Now consider the strictly proper nth order system

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx. \end{cases}$$

Associated to the system is the *Hamiltonian* matrix,

$$H(\alpha) = \begin{bmatrix} A & \alpha BB^T \\ -C^TC & -A^T \end{bmatrix},$$

which is of theoretical interest for several reasons.

(b) Let λ denote any eigenvalue of $H(\alpha)$. Show that also $-\lambda$ is an eigenvalue of $H(\alpha)$. (This means that the eigenvalues of $H(\alpha)$ are symmetric about the imaginary axis.) *Hint:* With $H(\alpha)v = \lambda v$, show that $w^TH(\alpha) = -\lambda w^T$ with $w = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} v$. **(1p)**

(c) Let Λ, V_1 and V_2 be real-valued $n \times n$ matrices such that

$$H(-1)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}\Lambda.$$

Show that $X = V_2V_1^{-1}$ is a solution to the algebraic Riccati equation (ARE)

$$0 = A^TX + XA - XBB^TX + C^TC$$

(which is one of the AREs associated to the Glover-McFarlane robust loop shaping procedure). **(3p)**

(d) Let $G(s) = C(sI - A)^{-1}B$ be the transfer function for the strictly proper system above. Show that for all $\gamma > \| G \|_\infty$ the Hamiltonian $H(\gamma^{-2})$ will have no eigenvalues on the imaginary axis.

Hints: Use e.g.

- the results in (a) and (b)
- $\| G \|_\infty < \gamma \Leftrightarrow \gamma^2 I - G^*(i\omega)G(i\omega) > 0, \forall\omega$ (where $G^*(i\omega) = G^T(-i\omega)$)
- The zeros of $G(s)$ are the poles of $G^{-1}(s)$

(3p)

Solutions to the exam in Automatic Control III, 2012-12-17:

1. (a) First note that the A-matrix is lower triangular, so its eigenvalues are found in the diagonal. The eigenvalues are -1, 0 and 0. Since it is a minimal realisation (shown below) the eigenvalues are also the poles of the system (Def. 3.4).
According to Def. 3.5 the zeros are the s for which the matrix

$$M(s) = \begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 & 1 & 0 \\ 0 & s & 0 & 0 & 1 \\ 0 & -1 & s & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

loses rank. Since $M(s)$ is quadratic here it loses rank exactly when $\det M(s) = 0$, and

$$\det M(s) = s + 1,$$

so there is one zero in -1.
A minimal realisation is both controllable and observable. This can be checked e.g. by the PBH rank tests (Thm 3.3):
The system is controllable iff the matrix $\begin{bmatrix} A - \lambda I & B \end{bmatrix}$ has full rank for all λ (and it suffices the check for the eigenvalues of A). Here

$$\begin{bmatrix} A - \lambda I & B \end{bmatrix} = \begin{bmatrix} -1-\lambda & 0 & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 0 & 1 \\ 0 & 1 & -\lambda & 0 & 0 \end{bmatrix},$$

which will have full rank since columns 2, 4 and 5 are linearly independent for all λ. The system is observable iff the matrix $\begin{bmatrix} A - \lambda I \\ C \end{bmatrix}$ has full rank for all λ. Here

$$\begin{bmatrix} A - \lambda I \\ C \end{bmatrix} = \begin{bmatrix} -1-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ 0 & 1 & -\lambda \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix},$$

which has full rank for all λ, since rows 4 and 5 are linearly independent with row 2 for $\lambda \neq 0$ and with row 3 for $\lambda \neq -1$. Thus, both observable and controllable $\Rightarrow$ minimal realisation. There is one pole in -1 and a double pole in the origin, and there is one zero in -1. (Note that there are both a pole and a zero in -1 which do not cancel each other — this is only possible for MIMO systems.)
(b) We have (also when using an observer)

$$G_c(s) = C(sI - A + BL)^{-1}B = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} s-1 & -2 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \frac{1}{s^2 - s - 2} \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} s & 2 \\ 1 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{s-2}{(s+1)(s-2)} = \frac{1}{s+1}.$$

(c) For internal stability we need to check S, SG and F_yS (and S_u, but for SISO systems $S_u = S$). We need $G(s)$ and $F_y(s)$:

$$G(s) = C(sI - A)^{-1}B = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} s-2 & -1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{s-2}{s^2 - 2s - 1},$$

$$F_y(s) = L(sI - A + BL + KC)^{-1}K = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} s+39 & -82 \\ 16 & s-34 \end{bmatrix}^{-1} \begin{bmatrix} 40 \\ 17 \end{bmatrix}$$

$$= \frac{23s + 11}{s^2 + 5s - 14} = \frac{23s + 11}{(s-2)(s+7)}.$$

The loop gain is then

$$G(s)F_y(s) = \frac{s-2}{s^2 - 2s - 1} \cdot \frac{23s + 11}{(s-2)(s+7)} = \frac{23s + 11}{(s^2 - 2s - 1)(s+7)},$$

and the sensitivity function becomes

$$S(s) = \frac{1}{1 + G(s)F_y(s)} = \frac{1}{1 + \frac{23s+11}{(s^2-2s-1)(s+7)}} = \frac{(s^2 - 2s - 1)(s+7)}{(s^2 - 2s - 1)(s+7) + 23s + 11}$$

$$= \frac{(s^2 - 2s - 1)(s+7)}{s^3 + 5s^2 + 8s + 4} = \frac{(s^2 - 2s - 1)(s+7)}{(s+1)(s+2)^2},$$

which clearly is stable (can be checked e.g. by Routh's algorithm if the poles are not found). Also $S(s)G(s)$ is stable since $(s^2 - 2s - 1)$ is cancelled. However,

$$F_y(s)S(s) = \frac{23s + 11}{(s-2)(s+7)} \cdot \frac{(s^2 - 2s - 1)(s+7)}{(s+1)(s+2)^2} = \frac{(23s + 11)(s^2 - 2s - 1)}{(s-2)(s+1)(s+2)^2},$$

which is unstable. Hence, the closed loop system is *not* internally stable.

2. (a) For a stationary point $\dot{x}_1 = \dot{x}_2 = 0$ must hold. $\dot{x}_1 = 0 \Rightarrow x_2 = 0$, and using this in $\dot{x}_2 = 0 \Rightarrow 0 = -x_1$. The origin, $x_1 = x_2 = 0$, is the only equilibrium. Linearize the system around $x = 0$:

$$\dot{x} = Ax, \quad A = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -1 + 2x_1x_2 & -1 + 3x_2^2 \end{bmatrix}_{x=0} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}.$$

A has eigenvalues $-0.5 \pm i\sqrt{0.75}$, meaning that $x = 0$ is a stable focus.
(b) Try with a linear feedback, $u = -Kx_2$, giving the closed loop system

$$\dot{x}_1 = 10(x_2 - x_1) - Kx_2,$$
$$\dot{x}_2 = x_1\left(\frac{8}{3} - x_3\right) - x_2,$$
$$\dot{x}_3 = x_1x_2 - 28x_3.$$

Linearizing the system about the origin gives

$$A_c = \frac{\partial f_c}{\partial x} = \begin{bmatrix} -10 & 10-K & 0 \\ 8/3 & -1 & -x_1 \\ x_2 & x_1 & -28 \end{bmatrix}_{x=0} = \begin{bmatrix} -10 & 10-K & 0 \\ 8/3 & -1 & 0 \\ 0 & 0 & -28 \end{bmatrix}.$$

The eigenvalues are the zeros of the characteristic polynomial

$$\begin{aligned}\det(sI - A_c) = \det \begin{bmatrix} s+10 & -10+K & 0 \\ -8/3 & s+1 & 0 \\ 0 & 0 & s+28 \end{bmatrix} \\ = (s+28)\left((s+10)(s+1) + \frac{8}{3}(-10+K)\right) \\ = (s+28)\left(s^2 + 11s + (8K-50)/3\right),\end{aligned}$$

and for asymptotical stability (locally) it suffices to have the eigenvalues strictly in the left half plane (Thm 12.1). This is fulfilled if $8K - 50 > 0$, so choose e.g. $K = 10$. Stability follows according to Theorem 12.1.

3. **(a)** Introduce $Z_1(s) = W_u(s)U(s) = K_u U(s)$, $Z_2(s) = W_T(s)\bar{Z}(s) = K_T \frac{s+\beta_T}{s+\alpha_T} CX(s)$ and $Z_3(s) = W_S(s)Y(s) = \frac{K_S}{s+\alpha_S}(CX(s) + W(s))$. We need some extra states to account for the dynamics in $W_S(s)$ and in $W_T(s)$. Introduce $x_S = z_3 \Rightarrow sX_S(s) = -\alpha_S X_S(s) + K_S CX(s) + K_S W(s) \Leftrightarrow \dot{x}_S = -\alpha_S x_S + K_S Cx + K_S w$. Furthermore, notice that $W_T(s) = K_T\left(1 + \frac{\beta_T - \alpha_T}{s+\alpha_T}\right)$. By choosing $X_T(s) = \frac{\beta_T - \alpha_T}{s+\alpha_T} CX(s)$ we get

$$\begin{aligned} z_2 = K_T(Cx + x_T) \quad \text{and} \quad sX_T(s) &= -\alpha_T X_T(s) + (\beta_T - \alpha_T)CX(s) \\ \Leftrightarrow \quad \dot{x}_T &= -\alpha_T x_T + (\beta_T - \alpha_T)Cx. \end{aligned}$$

One possible state space representation for the extended open loop system is then

$$\begin{aligned}
\begin{bmatrix} \dot{x} \\ \dot{x}_S \\ \dot{x}_T \end{bmatrix} &= \begin{bmatrix} A & 0 & 0 \\ K_S C & -\alpha_S & 0 \\ (\beta_T - \alpha_T)C & 0 & -\alpha_T \end{bmatrix} \begin{bmatrix} x \\ x_S \\ x_T \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ K_S \\ 0 \end{bmatrix} w, \\
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ K_T C & 0 & K_T \end{bmatrix} \begin{bmatrix} x \\ x_S \\ x_T \end{bmatrix} + \begin{bmatrix} K_u \\ 0 \\ 0 \end{bmatrix} u, \\
y &= \begin{bmatrix} C & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_S \\ x_T \end{bmatrix} + w.
\end{aligned}$$

(b) First note that (1) is equivalent to

$$|S(i\omega)| < 1/|W_S(i\omega)|, \quad |T(i\omega)| < 1/|W_T(i\omega)|, \quad |G_{wu}(i\omega)| < 1/|W_u(i\omega)|$$

for all ω.

1. Integral action in $F_y(s) \Rightarrow S(0) = 0$, which is obtained if $W_S(s)$ has a pole in the origin $\Rightarrow$ choose $\alpha_S = 0$.

2. To get $\omega_B \approx 2$ rad/s, try to get $|T(i2)| \geq 1/\sqrt{2}$ and/or $|S(i2)| \leq 1/\sqrt{2}$.

3. We should have $|T(i\omega)| \leq 1.5$ for all ω and $|T(i\omega)| < 0.01$ for $\omega \geq 314$ rad/s.

4. Make $|G_{wu}(i\omega)| < 4$ for all $\omega \Rightarrow$ choose $W_u(s) = K_u = 0.25$.

To fix 2. choose K_S so that $1/|W_S(i2)| \leq 1/\sqrt{2}$:

$$\frac{1}{|W_S(i\omega)|} = \frac{\omega}{K_S}, \quad \text{choose e.g.} \quad K_S = 3 \quad \Rightarrow \quad \frac{1}{|W_S(i2)|} = \frac{2}{3} < \frac{1}{\sqrt{2}}.$$

To fix 3., notice that with $\alpha_T > \beta_T$

$$\frac{1}{K_T} \leq \frac{1}{|W_T(i\omega)|} = \frac{\sqrt{\omega^2 + \alpha_T^2}}{K_T\sqrt{\omega^2 + \beta_T^2}} \leq \frac{\alpha_T}{K_T\beta_T}$$

The asymptotic amplitude curve is $\frac{\alpha_T}{K_T\beta_T}$ up to $\omega = \beta_T$, then decaying with slope -1 up to $\omega = \alpha_T$, and for higher frequencies it is $\frac{1}{K_T}$. We then must have $1/K_T < 0.01$ and $\alpha_T/K_T\beta_T \leq 1.5$. Choose e.g. $K_T = 150$ and $\alpha_T = 300$. Then $\beta_T \geq \alpha_T/1.5K_T = 300/225 = 4/3$ must hold. With the choice $\beta_T = 4/3$ we have

$$\frac{1}{|W_T(i314)|} = \frac{\sqrt{314^2 + 300^2}}{150\sqrt{314^2 + (4/3)^2}} \approx 0.0092, \quad \frac{1}{|W_T(0)|} = \frac{300}{150 \cdot (4/3)} = 1.5$$

$$\text{and} \quad \frac{1}{|W_T(i2)|} = \frac{\sqrt{2^2 + 300^2}}{150\sqrt{2^2 + (4/3)^2}} \approx 0.83 > \frac{1}{\sqrt{2}}.$$

(The latter inequality is to make sure that 2. is not violated.)
(c) $W_T(1) = 150\frac{1+4/3}{1+300} \approx 1.16$. According to Theorem 7.6 $\| W_T T \|_\infty \leq 1$ is possible only if $|W_T(p)| \leq 1$ for every right half plan pole p. Obviously that does not hold in this case, so (1) cannot be met with the $W_T(s)$ in (b).

4. (a) From the output equation we directly get $u = -D^{-1}Cx + D^{-1}y$, which, when plugged into the state equation, gives

$$\dot{x} = Ax + B(-D^{-1}Cx + D^{-1}y) = (A - BD^{-1}C)x + BD^{-1}y.$$

(b) With $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ we get

$$H(\alpha)v = \lambda v \quad \Leftrightarrow \quad \begin{cases} Av_1 + \alpha BB^T v_2 & = \lambda v_1, \\ -C^T C v_1 - A^T v_2 & = \lambda v_2. \end{cases}$$

Using the hint we try with $w = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -v_2 \\ v_1 \end{bmatrix}$. Now

$$w^T H(\alpha) = \mu^T \quad \Rightarrow \quad \begin{cases} -v_2^T A - v_1^T C^T C & = \mu_1^T, \\ -v_2^T \alpha BB^T - v_1^T A^T & = \mu_2^T. \end{cases}$$

Comparing with the expressions above (take the transpose) we get

$$\begin{cases} \mu_1 & = -C^TCv_1 - A^Tv_2 = \lambda v_2 = -\lambda(-v_2), \\ \mu_2 & = -\alpha BB^Tv_2 - Av_1 = -\lambda v_1 \end{cases} \quad \Rightarrow \quad \mu = -\lambda w,$$

which proves the statement.

(c) Λ, V_1 and V_2 are defined by

$$AV_1 - BB^TV_2 = V_1\Lambda, \tag{2}$$

$$-C^TCV_1 - A^TV_2 = V_2\Lambda. \tag{3}$$

Multiplying (2) with V_1^{-1} from the left yields $\Lambda = V_1^{-1}AV_1 - V_1^{-1}BB^TV_2$, which, when used in (3), gives

$$-C^TCV_1 - A^TV_2 = V_2V_1^{-1}AV_1 - V_2V_1^{-1}BB^TV_2.$$

Multiplying this expression with V_1^{-1} from the right and then moving all terms to the right hand side results in the ARE

$$0 = A^TV_2V_1^{-1} + V_2V_1^{-1}A - V_2V_1^{-1}BB^TV_2V_1^{-1} + C^TC$$

with $X = V_2V_1^{-1}$. (See also page 274 in the textbook by Glad/Ljung.)

(d) Start with $\| G \|_\infty < \gamma \Leftrightarrow$

$$\gamma^2 I - G^T(-i\omega)G(i\omega) > 0.$$

This can be interpreted as that the system with transfer function $\Omega(s) = \gamma^2 I - G^T(-s)G(s)$ has no zeros on the imaginary axis. This in its turn means that the inverse $\Omega^{-1}(s)$ has no poles on the imaginary axis. We now need to show that $H(\gamma^{-2})$ is the A-matrix for a state space representation of $\Omega^{-1}(s)$. First notice that $G^T(-s) = B^T(-sI - A^T)^{-1}C^T = -B^T(sI - (-A^T))^{-1}C^T$ which has the state space representation

$$\dot{z} = -A^Tz - C^Ty,$$
$$\nu = B^Tz.$$

Combining this with the state space model of $G(s)$ (by setting $\nu = G^T(-s)y = G^T(-s)G(s)u$ and $\mu = \gamma^2 u - \nu = \Omega(s)u$), $\Omega(s)$ can be represented as

$$\dot{x} = Ax + Bu,$$
$$\dot{z} = -A^Tz - C^Ty = -C^TCx - A^Tz,$$
$$\mu = -B^Tz + \gamma^2 u.$$

Using the same technique as in (a) a state space representation of the inverse system is obtained by first noting that $u = \gamma^{-2}B^Tz + \gamma^{-2}\mu$, resulting in

$$\dot{x} = Ax + B(\gamma^{-2}B^Tz + \gamma^{-2}\mu) = Ax + \gamma^{-2}BB^Tz + \gamma^{-2}B\mu,$$
$$\dot{z} = -C^TCx - A^Tz,$$
$$u = \gamma^{-2}B^Tz + \gamma^{-2}\mu.$$

Introducing $\xi = \begin{bmatrix} x \\ z \end{bmatrix}$ the state space model for $u = \Omega^{-1}(s)\mu$ can be written as

$$\begin{aligned} \dot{\xi} &= \begin{bmatrix} A & \gamma^{-2}BB^T \\ -C^TC & -A^T \end{bmatrix} \xi + \begin{bmatrix} \gamma^{-2}B \\ 0 \end{bmatrix} \mu, \\ u &= \begin{bmatrix} 0 & \gamma^{-2}B^T \end{bmatrix} \xi + \gamma^{-2}\mu. \end{aligned}$$

Indeed, the A-matrix is the Hamiltonian $H(\gamma^{-2})$.