1. Give an alternative operational semantics to Boolean expressions \texttt{Bexp} which specifies a simple form of “lazy evaluation” where arguments of an expression are evaluated only if they are really needed. In particular, to evaluate \( b_0 \) and \( b_1 \), we first evaluate \( b_0 \) and if it happens to be \textit{false}, the whole expression \( b_0 \) and \( b_1 \) is evaluated to \textit{false} without ever examining the subexpression \( b_1 \); otherwise (if \( b_0 \) is \textit{true}), we have to go on and evaluate also \( b_1 \). Similarly, \( b_0 \) or \( b_1 \) is evaluated by evaluating \( b_0 \) first. If it is \textit{true}, \( b_0 \) or \( b_1 \) is also evaluated to \textit{true} without inspecting \( b_1 \); otherwise we have to check the value of \( b_1 \).

Your solution (i.e., the designed rules) should properly reflect the informal description of the lazy evaluation strategy given above (so that constructing an inference tree for \( \langle b, \sigma \rangle \rightarrow t \) in your system ‘models’ the lazy evaluation of \( b \) in the state \( \sigma \)).

[5 marks]
2. Imagine introducing the \( ? : \) operator (which exists, e.g., in the programming language C) into \( \text{Aexp} \). The syntax of \( ? : \) is defined as follows:

\[
\text{if } b \in \text{Bexp} \text{ is a Boolean expression and } a_0, a_1 \in \text{Aexp} \text{ are arithmetic expressions, then } b \ ? \ a_0 : a_1 \text{ is also an arithmetic expression.}
\]

The intended meaning of \( b \ ? \ a_0 : a_1 \) is defined like this:

- if \( b \) evaluates to \( \text{true} \), then the expression \( b \ ? \ a_0 : a_1 \) has the value of \( a_0 \);
- if \( b \) evaluates to \( \text{false} \), then the expression \( b \ ? \ a_0 : a_1 \) has the value of \( a_1 \).

a) Define the operational semantics of the \( ? : \) operator (i.e., write down the inference rules for \( ? : \))  

[4 marks]

b) Prove that for all \( b \in \text{Bexp}, a_0, a_1 \in \text{Aexp}, \sigma \in \Sigma, \) and \( n \in \mathbb{Z} \) we have that

\[
\langle b \ ? \ a_0 : a_1, \sigma \rangle \rightarrow n \iff \langle \text{if } b \text{ then } x := a_0 \text{ else } x := a_1, \sigma \rangle \rightarrow \sigma[x\rightarrow n]
\]

(Here you need to use the rules designed in the previous point.)  

[4 marks]

3. a) Give an example of a finite domain \( \langle D, \subseteq \rangle \) and a continuous function \( f : D \rightarrow D \) such that \( \text{fix } f = f^3(\bot) \neq f^2(\bot) \).  

[4 marks]

b) Let \( \omega = \{0, 1, 2, 3, \ldots \} \) be the set of all natural numbers, and let \( \mathcal{P}(\omega) \) be the powerset (the set of all subsets) of \( \omega \).
   A) Prove that \( \langle \mathcal{P}(\omega), \subseteq \rangle \) is a domain (i.e., a CPO with bottom).  

[4 marks]

B) Let us consider the domain \( \langle \{ \bot, * \}, \subseteq \rangle \) where \( \bot \subseteq * \). Show that the function \( f : \mathcal{P}(\omega) \rightarrow \{ *, \bot \} \) defined by

\[
f(X) = \begin{cases} * & \text{if } X \text{ is infinite} \\ \bot & \text{if } X \text{ is finite} \end{cases}
\]

is monotonic but not continuous.  

[4 marks]

4. Give an appropriate semantic domain for the following Pascal data type which represents objects specifying a student’s name, score, and an indication as to whether the student has passed or not.

record
    name : array[1..k] of char;
    score : integer;
    pass : boolean
end
5. Consider the following declaration $d$ in Rec:

$$f(x) = \begin{cases} \text{if } x \text{ then } g(x-1) + 1 \text{ else } f(x) + 2 \\ g(x) = \begin{cases} \text{if } x \text{ then } 3 \text{ else } 2 \end{cases} \end{cases}$$

a) Is there any $n$ such that $g(f(0)) \xrightarrow{d_{na}} n$? If yes, write down the derivation and show what the $n$ is. If no, then explain why.

b) Is there any $n$ such that $g(f(0)) \xrightarrow{d_{na}} n$? If yes, write down the derivation and show what the $n$ is. If no, then explain why.

6. a) Define the following notions:
   - A) CPO
   - B) domain
   - C) continuous function

b) State and prove the fixed-point theorem

7. Consider the following process graph (for your convenience, all $b$-transitions are drawn using dashed lines so that you can recognize them easily):

![Process Graph]

a) Write down the list of all states $x$ such that Bob has a winning strategy for the game $\Gamma_2(x, 6)$
b) Write down the list of all states $x$ such that Bob has a winning strategy for the game $\Gamma_{\infty}(x, 5)$ [4 marks]

c) Is there any bisimulation colouring of the graph which uses exactly three colours? If so, give an example of such a colouring. [4 marks]

8. Let us finish with a simple question. What kind of national disaster was recently experienced both in Sweden and in the Czech Republic?