\(\exists! \rho = (\rho)\), \(\rho = (\rho)\), then \(\exists \rho = (\rho)\).

Theorem 1. If \(\exists \rho = (\rho)\), then \(\exists \rho = (\rho)\).

Proof. By induction on the depth of the formula. We prove the theorem for a formula with a single variable. Let \(\rho = (\rho)\). Then we must show that \(\exists \rho = (\rho)\).

In order to infer that \(\exists \rho = (\rho)\), we must use the rule

\[
\exists \rho = (\rho) \rightarrow \exists \rho = (\rho)
\]

We shall demonstrate that \(\exists \rho = (\rho)\) by case analysis of \(\rho = (\rho)\).

Theorem 2. If \(\exists \rho = (\rho)\), then \(\exists \rho = (\rho)\) and \(\exists \rho = (\rho)\).

Proof. By induction on the depth of the formula. We prove the theorem for a formula with a single variable. Let \(\rho = (\rho)\). Then we must show that \(\exists \rho = (\rho)\).

In order to infer that \(\exists \rho = (\rho)\), we must use the rule

\[
\exists \rho = (\rho) \rightarrow \exists \rho = (\rho)
\]

We shall demonstrate that \(\exists \rho = (\rho)\) by case analysis of \(\rho = (\rho)\).

Theorem 3. If \(\exists \rho = (\rho)\), then \(\exists \rho = (\rho)\) and \(\exists \rho = (\rho)\).

Proof. By induction on the depth of the formula. We prove the theorem for a formula with a single variable. Let \(\rho = (\rho)\). Then we must show that \(\exists \rho = (\rho)\).

In order to infer that \(\exists \rho = (\rho)\), we must use the rule

\[
\exists \rho = (\rho) \rightarrow \exists \rho = (\rho)
\]