Exercise 1

Prove or disprove each of the following.

a) $C[\text{skip};c] = C[c]$.

b) $C[\text{if } b \text{ then } (c_0;c) \text{ else } (c_1;c)] = C[(\text{if } b \text{ then } c_0 \text{ else } c_1);c]$.

c) $C[\text{if } b \text{ then } (c;c_0) \text{ else } (c;c_1)] = C[c;(\text{if } b \text{ then } c_0 \text{ else } c_1)]$.

d) If $C[c_1] = C[c_2]$ then $C[\text{while } b \text{ do } c_1] = C[\text{while } b \text{ do } c_2]$.

Exercise 2

Let $g : \Sigma \to \Sigma$ defined by

$$g(\sigma) = \begin{cases} \sigma[\sigma = 0] & \text{if } \sigma(x) \geq 0 \text{ even}, \\ \text{undefined} & \text{otherwise}. \end{cases}$$

Prove that $g = C[w]$ where $w \equiv \text{while } \text{not}(x=0) \text{ do } \text{if } 0 < x \text{ then } x := x - 2 \text{ else } \text{skip}$

(Hint: When proving that $g$ is smaller than any other fixed point of $\Gamma$ use induction on $\sigma(x)$)

Exercise 3

Let $O$ and $T_\bot$ be the cpos described by the following Hasse diagrams:

$$O = \begin{array}{c} \top \downarrow \bot \end{array} \quad T_\bot = \begin{array}{c} \text{false} \downarrow \text{true} \end{array}$$

a) How many functions $f : O \to T_\bot$ are there?

b) How many continuous functions $f : O \to T_\bot$ are there?

c) Which functions $f : O \to T_\bot$ are not continuous?

d) Draw the cpos $[O \to T_\bot]$ and $[T_\bot \to O]$ as Hasse diagrams, with functions ordered pointwise: $f \sqsubseteq g$ iff $f(x) \sqsubseteq g(x)$ for all $x$. 

Exercise 4

Give a denotational semantics for the “repeat” command of exercise 1. That is, extend the definition of the function $C$ to cover $\text{repeat}$. The definition must be compositional, i.e., you cannot simply define $\text{repeat}$ in terms of $\text{while}$.

Hint: The definition will be similar to that of $\text{while}$ – using the least fixpoint of a function similar to $\Gamma$.

Exercise 5

Modify the ML implementation of the denotational semantics to include the language extension with $\text{repeat}$ above. Submit a printout of the ML file with your modifications marked using a pen. Include a printout of an ML run where you demonstrate the changes by executing some simple example programs of your own invention.