Lecture Notes for a Course on System Identification, v2013

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Contents

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Abstract

System identification in a narrow sense is concerned with tasks of parameter estimation based on observations originating from a dynamical system. System identification in a broad sense deals with many subtleties coming up when designing, conducting and interpreting results from such an experiment. The purpose of this text is to survey the collection of ideas, insights, theorems, intuitions, language and practical experiences which constitute the art of system identification. In this course we will guide you along the basic methods and concepts, as well as show you the many connections to related fields of applied science.

Part I

Basics of System Identification

Chapter 1

Aim

> "Say we study an unknown dynamic system. How to design, conduct, process and interpret the results from an experiment applied to this system such that we will get an accurate model of its internal working?"

This question expresses the goal of system identification techniques. The actual answer to this question lies in how the different terms are translated mathematically. This course intends to illustrate the spectrum of choices one can make here. In order to bootstrap this course, let us give a working ('intuitive') definition of those:

'Unknown': Here we assume that a precise description of the system is not available. This might be as (i) the system is too complex to derive completely from (physical) laws; (ii) the system might behave different from what would be expected from a theoretical description (e.g. due to aging, unaccounted effects, or conversion from continuous to discrete time); (iii) we only need to have a 'good' approximation of the system which serves our purpose well.

'Dynamic System': The actual behavior relating input signal to output signal, often using a physical process. The keyword 'dynamic' is understood here as follows: 'output values depend on present as well as past inputs given to the system'.

'Model': A mathematical representation of the studied system. Many different flavors of models exists (see next chapter): this course will study mathematical models relating input and output signals using equations. (Logic, software, language or intuition can be used alternatively).

'Experiment': A datum collected when feeding the system with a predesigned input signal. This book will often be concerned with signals taking continuous values in $\mathbb{R}$. (Alternatively, values of signals can be binary, categorical, strictly positive, or taking elements in other structured sets).

'Design': How to choose the input signals in order to optimize the result of the overall analysis?

'Process': In which shape should the signals be (pre-)processed before submitting to analysis?

'Interpret': In what sense is the identified model accurate and reliable, and which results might be due to unknown disturbances, noisy measurements or artificial model structures?

In a specific sense, system identification is concerned with coming with an accurate model given the input-output signals recorded during working of the studied system.

Hence, it becomes plain that system identification is closely related to other fields of mathematical modeling. We mention here the various domains concerned with parameter estimation including statistical inference, adaptive filtering and machine learning. Historically, system identification originates from an engineering need to form models of dynamical systems: it then comes as no surprise that traditionally emphasis is laid on numerical issues as well on system-theoretical concerns.

Progress in the field has much been reinforced by introducing good software to execute the various algorithms. This makes the methodology semi-automatic: that is a user needs still have a conceptual overview on what is to be done, but the available software tools take care of most technical details. In this course, the use of the MATLAB System Identification toolbox is discussed in some detail.

System Identification as a field came only in existence in the 60s, while its roots can be traced back to the Least Squares techniques, other techniques of statistical inference. The field however originated from an engineering need to have good models for use in automatic control. 'Modern' system identification is often attributed to the work of Åström and Bohlin [1], while contemporary system identification is often associated to the work as reviewed in Ljung [4] or Söderström and Stoica [5]. The latter work will be the basis of this course.

1.1 Systems & Models

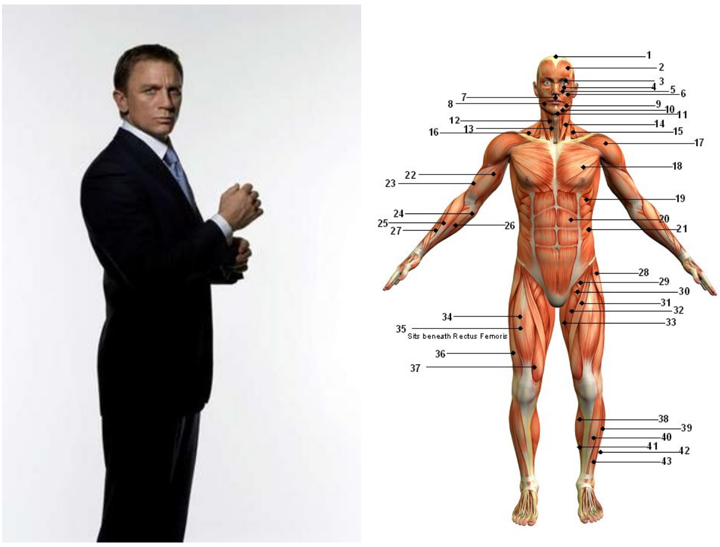
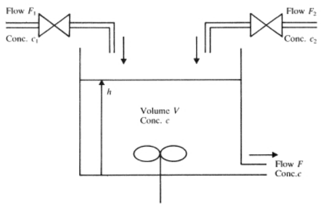


Figure 1.1: As a medical doctor you get to study the human body. As a doctor you work with an internal representation (=model) of how the body (=system) works.

Definition 1 (System or Model) *The overall behaviors you get to study is referred to as the* system. *The internal (mental) representation you as an researcher use in order to study the system, is called the* model.



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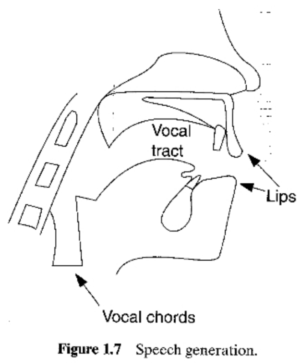

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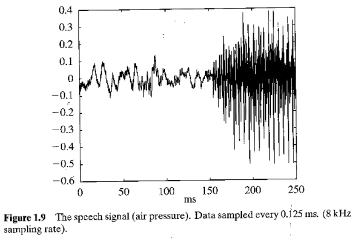


(c)

Figure 1.2: (a) Schematic representation of a stirred thank process. (b) representation of a speech apparatus. (c) Example of a generated acoustic signal with two different filters shaped by intention ('k' versus 'a')

Let us begin by describing some of the systems which are being studied in the context of this course.

Stirred Thank: The following is a prototypical example in the context of process control. Consider a biochemical reactor, where two different substances go in via respective pipelines. Both inflows comes at a certain flow-rate and have a certain concentration, either of which can be controlled by setting valves. Then the substances interacts inside the stirred tank, and the yield is tapped from the tank. Maybe the aim of such process is to maximize the concentration of the yield at certain instances. A mathematical approach to such automatic control however requires a mathematical description of the process of interest. That is, we need to set up equations relating the setting of the valves and the output. Such model could be identified by experimenting on the process and compiling the observed results into an appropriate model.

Speech: Consider the apparatus used to generate speech in the human. In an abstract fashion, this can be seen as a white noise signal generated by the glottis. Then the mouth are used to filter this noise into structured signals which are perceived by an audience as meaningful. Hence, this apparatus can be abstracted into a model with unknown white noise input, a dynamical system shaped by intention, and an output which can be observed. Identification of the filter (dynamical system) can for example be used to make a artificial speech.

Lateron in this course, you will be asked to study how techniques of system identification can be applied in a range of different applications.

Industrial: The prototypical example of an engineering system is an industrial plant which is fed by an inflow of raw material, and some complicated process converts it into the desired yield. Often the internal mechanism of the studied process can be worked out in some detail. Nevertheless, it might be more useful to come up with a simlpler model relating input-signals to output-signals directly, as it is often (i) easier (cheaper) to develop, (ii) is directly tuned to our need, and (iii) makes abstraction of irrelevant mechanisms in the process, and (iv) might better handle the unforeseen disturbances.

Figure 1.3: Illustrative examples: (a) A petrochimical plant. (b) An application of a model for acoustic signals. (c) An example of an econometric system. (d) Signals arising from TV can be seen as coming from a system.

Acoustic: The processing of acoustical signals can be studied in the present context. Let us for example study the room which converts an acoustic signal (say a music signal) into an acoustic signal augmented with echo. It is then often of interest to compensate the signal sent into the room for this effect, so as to 'clean' the perceived signal by the audience. In this example, the room is conceived as the dynamical system, and it is of interest to derive a model based on acoustic signals going into the room, and the consequent signals perceived by an audience.

Econometric: The following example is found in a financial context. Consider the records of the currency exchange rates. This multivariate time-series is assumed to be driven by political, socio-economic or cultural effects. A crude way to model such non-measurable effects is as white noise. Then the interesting bit is how the exchange rates are interrelated: how for example a injection of resources in one market might alter other markets as well.

Multimedial: Finally, consider the sequence of images used to constitute a cartoon on TV say. Again, consider the system driven by signals roughly modeling meaning, and outputting the values projected in the different pixels. It is clear that the signals of neighboring pixels are inter-related, and that the input signal is not as high-dimensional as the signals projected on the screen.

1.2 The System Identification Procedure

Let us sketch a prototypical example of the different steps taken in a successful system identification experiment. The practical way to go ahead is typically according to the following steps, each one raising their own challenges:

1. Description of the task. What is a final desideratum of a model? For what purpose is it to be used? How will we decide at the end of the day if the identified model is satisfactory? On which properties to we have to focus during the identification experiments?

2. Look at initial Data. What sort of effects are of crucial importance to capture? What are the challenges present in the task at hand. Think about useful graphs displaying the data. Which phenomena in those graphs are worth pinpointing?

3. Nonparametric analysis. If possible, do some initial experiments: apply an pulse or step to the system, and look at the outcome. Perhaps a correlation or a spectral analysis are possible as well. Look at where random effects come in. If exactly the same experiment is repeated another day, how would the result differ? Is it possible to get an idea of the form of the disturbances?

4. Design Experiment. Now that we have acquired some expertise of the task at hand, it is time to set up the large identification experiment. At first, enumerate the main challenges for identification, and formalize where to focus on during the experiment. Then design an experiment so as to maximize the information which can be extracted from observations made during the experiment. For example. make sure all the dynamics of the studied system are sufficiently excited. On the other hand, it is often paramount to make sure that the system remains in the useful 'operation mode' throughout the experiment. That is, it is no use to inject the system with signals which do not apply in situations where/when the model is to be used.

5. Identify model. What is a good model structure? What are the parameters which explain the behavior of the system during the experiment.

6. Refine Analysis: It is ok to start off with a hopelessly naive model structure. But it is then paramount to refine the model structure and the subsequent parameter estimation in order to compensate for the effects which could not be expressed in the first place. It is for example common practice to increase the order of the dynamical model. Is the noise of the model reflecting the structure we observe in the first place, or do we need more flexible noise models? Which effects do we see in the data but are not captured by the model: time-varying, nonlinear, aging, saturation,... ?

7. Verify Model: is the model adequate for the purpose at hand? Does the model result in satisfactory results as written down at the beginning? Is it better than a naive approach? Is the model accurately extracting or explaining the important effects? For example, analyze the residuals left over after subtracting the modeled behavior from the observed data. Does it still contain useful information, or is it white? Implement the model for the intended purpose, thus it work satisfactory?

A flowchart of the typical system identification experiment is shown in Fig. (1.2.a). The identification procedure can be compared to an impedance meter (see Fig. (1.2.b)) which measures the impedance of a system by comparing the current measured at input with the corresponding voltage at the output line of a system. Similarly, system identification tries to figure out the dynamics of a system by relating input signal to corresponding output, i.e. from observed input-output behavior.

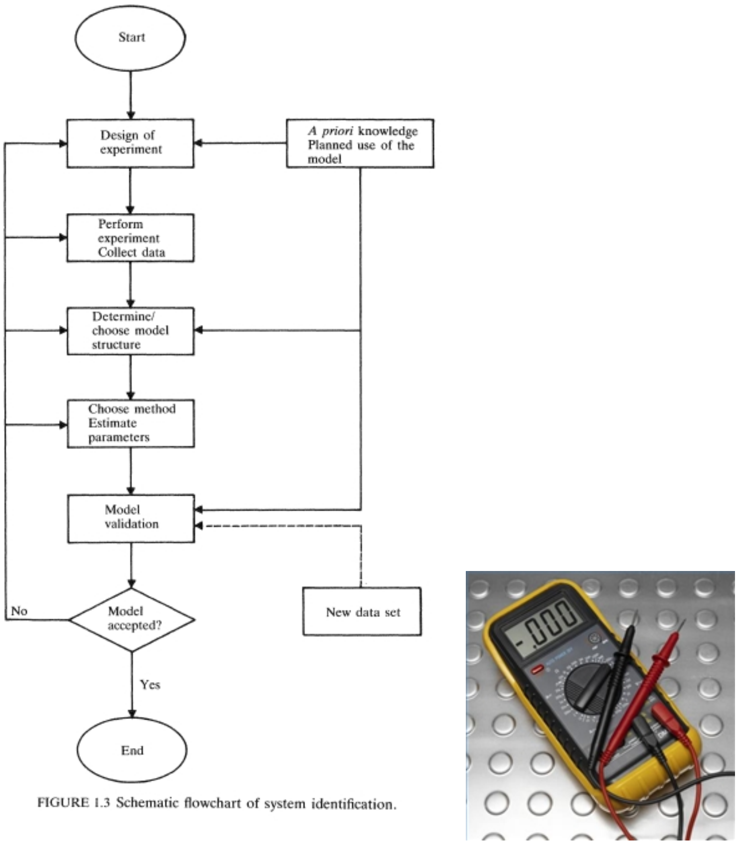


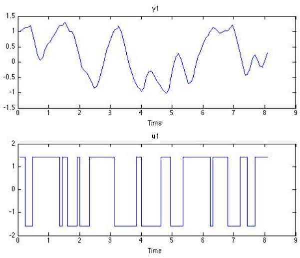
FIGURE 1.3 Schematic flowchart of system identification.

(a) (b)

1.3 A simple Example

The code of a simple identification experiment in MATLAB is given. The task is to identify a model for a hairdryer. This system fans air through a tube which is heated at the inlet. The input signal $u(t)$ reflects the power of the heating device. The output signal $y(t)$ reflects the temperature of the air coming out. The data used for identification is displayed as follows.

```
>> load dryer2
>> z2 = [y2(1:300) u2(1:300)];
>> idplot(z2, 200:300, 0.08)
```



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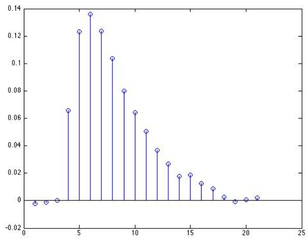

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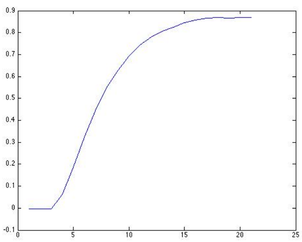

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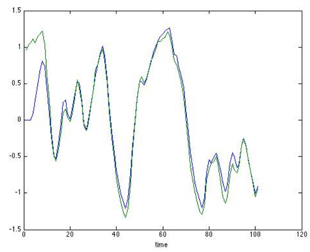

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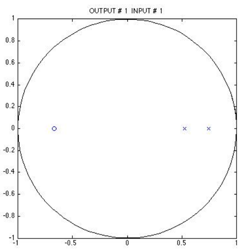

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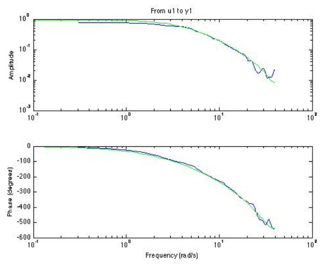

(h)

Figure 1.4:

At first, the structural properties of the system are displayed using nonparametric tools as follows

```
>> z2 = dtrend(z2);
>> ir = cra(z2);
>> stepr = cumsum(ir);
>> plot(stepr)
```

Inspection of those properties suggest the following parametric model relating input to output signals.

$$y(t) + a_1 y(t-1) + a_2 y(t-2) = b_1 u(t-3) + b_2 u(t-4) \tag{1.1}$$

Here $\{a_1, a_2, b_1, b_2\}$ are to be estimated based on the collected data as given before.

```
>> model = arx(z2, [2 2 3]);
>> model = sett(model,0.08);
>> u = dtrend(u2(800:900));
>> y = dtrend(y2(800:900));
>> yh = idsim(u,model);
>> plot([yh y]);
```

The dynamics of this estimated model are characterized as the poles and zeros of the system. This is given as

```
>> zpth = th2zp(model);
>> zpplot(zpth);
```

The transfer function corresponding to the estimated model, and derived from the non-parametric method is compared as follows

```
>> gth = th2ff(model);
>> gs = spa(z2); gs = sett(gs,0.08);
>> bodeplot([gs gth]);
```

Many open questions on the studied system remain after this simple analysis. For example 'Is the estimated model accurate enough (model validation)?', 'Is the model structure as given in eq. (1.1) appropriate?' 'If we can freely choose the input signals, what would the optimal inputs be (experiment design)?', ...

1.4 Notation

> Hardly any non-mathematician will admit that mathematics has a cultural and aesthetic appeal, that it has anything to do with beauty, power or emotion. I categorically deny such cold and rigid view. [N.Wiener, 1956]

Mathematical manuscripts tend to scare people away because of their intrinsic technical notation. However, mathematical notation is carefully crafted over the years to express ideas, inventions, truths, intuitions and reasonings of exact sciences better than any spoken language could do. Similar to spoken languages, mathematical notation comes in many different dialects, obstructing the fluent interaction between different groups of researchers. Dialects may differ in subtle issues as e.g.

the use of capital symbols for certain quantities or in indexing systems, or in capital difference as the use of operators versus explicit representations of transforms. As for any compiler, ideas might not work out properly if the syntax is not as intended. As such, it really pays off to get familiar with different notational conventions.

In this course we adapt the following notation.

(Constant): A constant quantity will be denoted as a lower-case letter in an equation. For example scalars (e.g. c, a, b), indices (as e.g. $i, j, k, n, \dots$), functions (as e.g. $f, g, \dots$) and so on.

(Vector): A vector - an array of scalars taking value in $\mathbb{R}^d$ - is denoted as a boldface lowercase letter (e.g. $\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$).

(Matrix): A matrix - a tableau of scalars taking values in $\mathbb{R}^{n\times d}$ - is denoted as a boldface capital letters (e.g. $\mathbf{A}, \mathbf{B}, \mathbf{M},$)

(Random variable): Random quantities will be noted as capital letters in equations. Random quantities need a proper stochastic setting to be well-defined (see chapter 4), in practice they can be recognized as they can take different values (realizations).

(Set): Sets of elements are denoted using curled brackets: e.g. $\{a, b, c\}$ is a set of three constant values. Sets are referred to using mathbb letters, e.g. $\mathbb{R}, \mathbb{N}, \mathbb{C}, \dots$.

(Operator): An operator is a mapping of a function to another. An operator is denoted as a calligraphic letter, e.g. $\mathcal{L} : \{f\} \to \{f\}$ is a mapping from the set of functions into the same set.

(Reserved): In order to ease notational load, a number of symbols connect (in this course) to a given meaning. An overview of those is given in Table (1.1).

Symbol	Type	Meaning
$\mathbb{R}, \mathbb{N}, \mathbb{C}$	Set	Set of real, integer or complex values.
n	Scalar	Number of observations.
θ	Vector	Unknown parameters of the problem to be estimated.
τ	Scalar	Delay of the model/system.
$i, j = 1, \dots, n$	Index	Index ranging over the n observations.
$\mathbf{i}$	Imaginary number	$\mathbf{i} = \sqrt{-1}$
d	Scalar	Order of Input Response, or dimension.
$u, \mathbf{u}$	function and vector	Continuous and discrete input signal.
$y, \mathbf{y}$	function and vector	Continuous and discrete output signal.
$\hat{\cdot}, \cdot_n$	Operator	An estimate, that is $\hat{f}$ and f_n are both an estimate of the quantity f.

Table 1.1: Enumeration of symbols taking a fixed meaning in this text.

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