System Identification

Computer lab 1

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Nonparametric techniques

▶ Correlation analysis (Matlab command: cra)
  ▶ Impulse response

\[ y_t = \sum_{k=0}^{\infty} h_k u_{t-k} + v_t \]

where \( v \) is white noise uncorrelated with \( u \). The aim is to obtain \( h_1, \ldots, h_k \) that fits the data.

▶ Impulse response estimation

\[ h_k = \frac{r_{yu}}{\lambda_n} \]

where the crosscovariance function \( r_{yu} \) can be computed from \( n \) datapoints

\[ r_{yu} = \frac{1}{n} \sum_{t=1}^{n} y_t u_{t-\tau} \]

and where \( \lambda_n \) is the input variance

\[ \lambda_n = \frac{1}{n} \sum_{t=1}^{n} u_t^2 \]
Nonparametric techniques

- Correlation analysis

\[ r_{yu} = \sum_{t=1}^{n} y_t u_{t-\tau} \]

- Wiener-Hopf equation

\[
\begin{pmatrix}
  r_u(0) & r_u(1) & \ldots & r_u(\tau) & \ldots \\
  r_u(1) & r_u(0) & \ldots & r_u(\tau - 1) & \ldots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
  r_u(\tau) & r_u(\tau - 1) & \ldots & r_u(0) & \ldots \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
\end{pmatrix} \begin{pmatrix}
  h_0 \\
  h_1 \\
    \vdots \\
  h_\tau \\
    \vdots \\
\end{pmatrix} = \begin{pmatrix}
  r_{uy}(0) \\
  r_{uy}(1) \\
    \vdots \\
  r_{uy}(\tau) \\
    \vdots \\
\end{pmatrix}
\]
Nonparametric techniques

- **Spectral analysis** Estimating transfer function estimation from spectra

\[ Y(\omega) = G(i\omega)U(\omega) + V(\omega) \]

where \( u \) and \( v \) are uncorrelated.

- **Transfer function estimation**

\[ G(i\omega) = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)} \]

where \( \Phi_{yu}(\omega) \) is the crossspectrum (discrete Fourier transforms of crosscovariance) and \( \Phi_u(\omega) \) is the input spectrum (discrete Fourier transform of input covariance).
Nonparametric techniques

Spectral analysis algorithm Matlab command: spa

1. Cross-covariance and input covariance

\[ r_{yu}(\tau) = \frac{1}{n} \sum_{t=1}^{n} y_t u_{t-\tau}, \quad r_u(\tau) = \frac{1}{n} \sum_{t=1}^{n} u_t u_{t-\tau} \]

2. Fourier transform of the covariance and cross-covariance

\[ \Phi_u(\omega) = \sum_{\tau=-M}^{M} r_u(\tau) W_M(\tau) e^{-i\omega\tau} \]

\[ \Phi_{yu}(\tau) = \sum_{\tau=-M}^{M} r_{yu}(\tau) W_M(\tau) e^{-i\omega\tau} \]

where \( W_M(\tau) \) is the Hann window with lag size of \( M \). You can specify \( M \) to control the frequency resolution of the estimate.

3. Compute the transfer function \( G(i\omega) \)

\[ G(i\omega) = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)} \]
Filtered white noise

\[ w_t = \frac{a}{1-0.8q^{-1}} e_t \] where \( e_t \) is white noise.
The end