Lecture 2

Transition Systems

Modeling: informal motivation

Model is a simplification of a system for understanding and analysing particular aspects

In e.g., control engineering, models describe relation between (real-valued) quantities

In protocol and software verification, models should describe possible combinations of sequences of computation steps

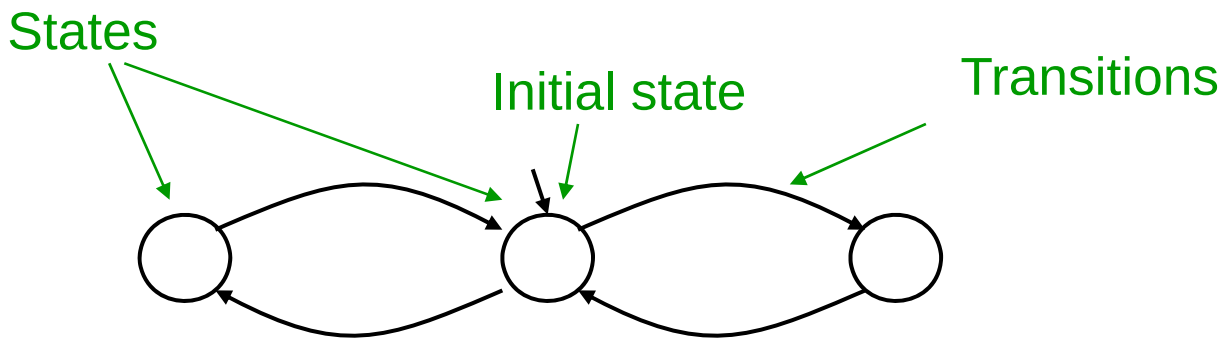
=>

Models will have discrete states and transitions between states.

Transition Systems

Model to describe the operational behavior of systems

Example: Resource Allocation System:



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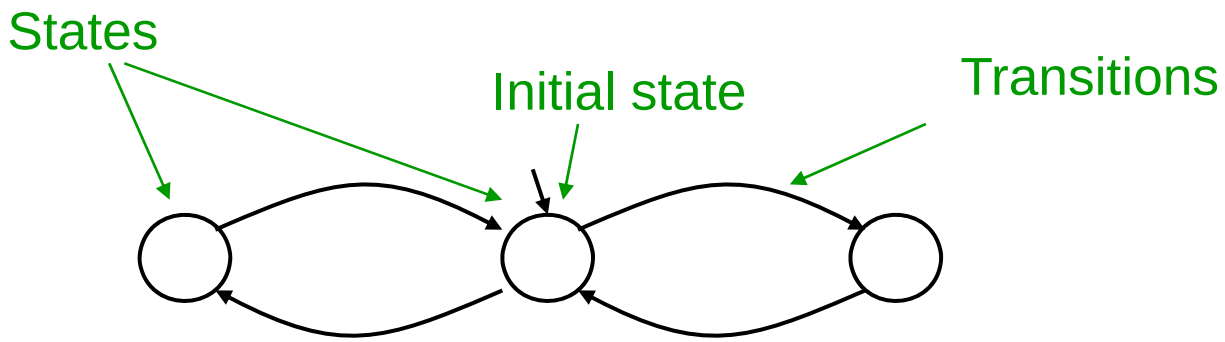

We need a mechanism to describe “what happens” in a transition system.

I.e.: A language of assertions about “current” state,

We should be able to say, e.g.,

- “The process P1 holds the resource”
- “No process holds the resource”

Each assertion can be true or false in the “current state”.

Transition Systems

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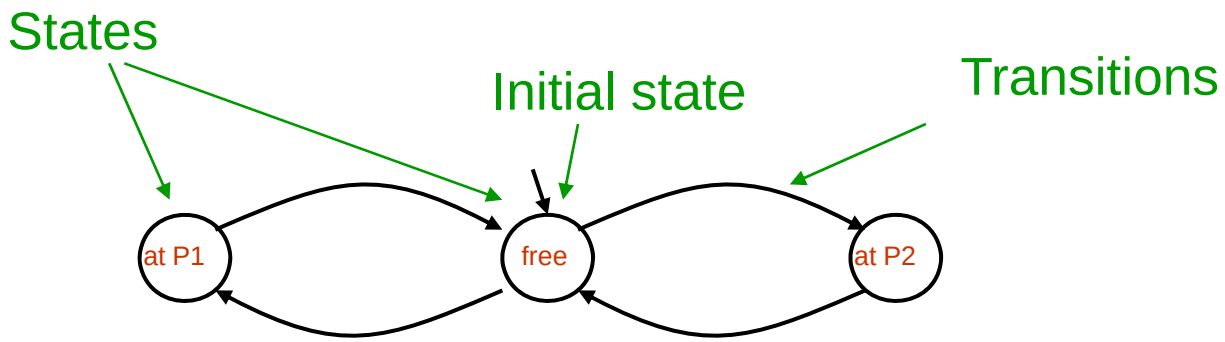

Variables: status (what can be “observed”)

Transition Systems

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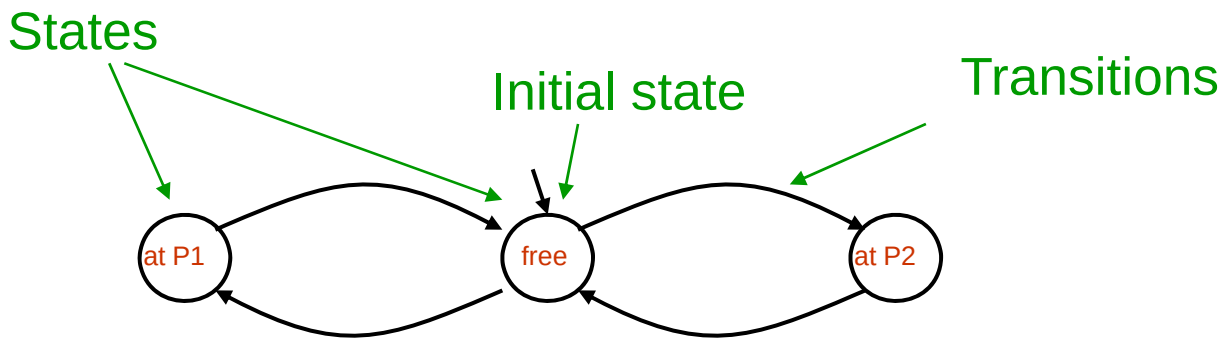

Variables: status (what can be “observed”)

Computation:

Transition Systems (formal definition)

A Transition System is a tuple $(S, S_0, \rightarrow, V, L)$ where

- S is a set of states
- S_0 is a set of initial states
- $\rightarrow$ (a subset of $S \times S$) is a transition relation
- V is a set of variables
- L : $S \rightarrow (V \rightarrow Val)$ gives a valuation of variables in each state

Write $s \rightarrow s'$ for $(s, s') \in \rightarrow$

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Write $s \rightarrow s'$ for $(s, s') \in \rightarrow$

A Computation is a finite or infinite sequence of states

s_0 s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9

such that

- s_0 is an initial state
- $(s_i, s_{i+1}) \in \rightarrow$ for all relevant i

Structured Descriptions of Transition Systems

Drawing large graphs is very inconvenient.

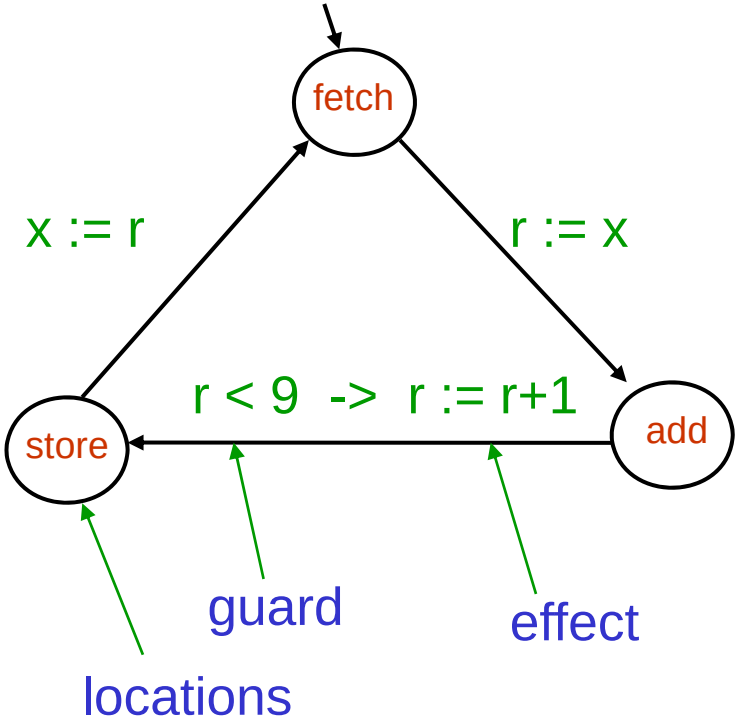
We need constructs to define transition systems conveniently

- Processes
- control structures
- (shared) variables
- communication channels
  - Can be modeled as shared variables

Adding Variables (Action Systems)

Example: simple incrementation of stored variable

Action System
Variables: x, r: integer
Initially x = 0



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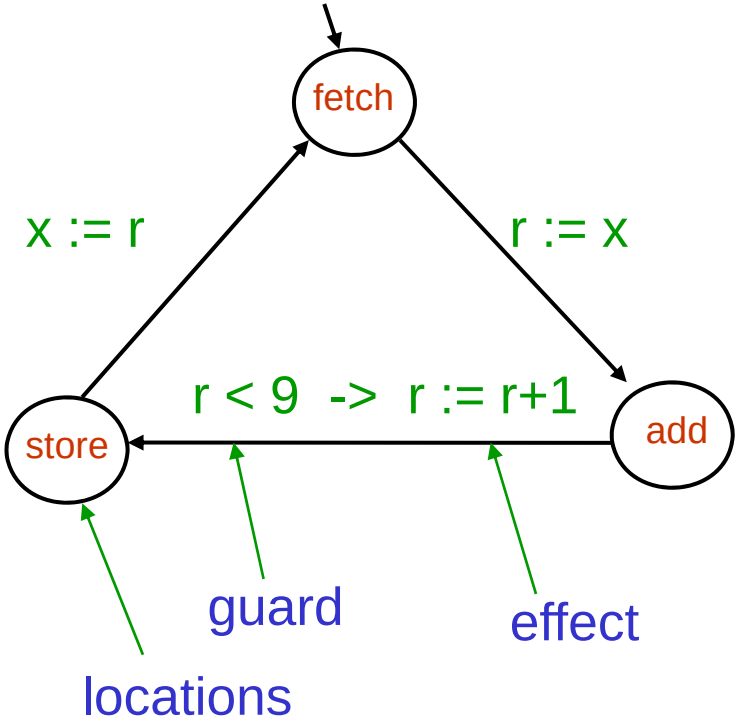


Transition system

States:

locations × assignments to variables

(add, {x -> 0 , r -> 0})

Initial States (example):

(fetch, {x -> 0 , r -> 76})

Transitions:

(add, {x -> 0 , r -> 0}) ->
(store, {x -> 0 , r -> 1})

Variables:

loc x, r

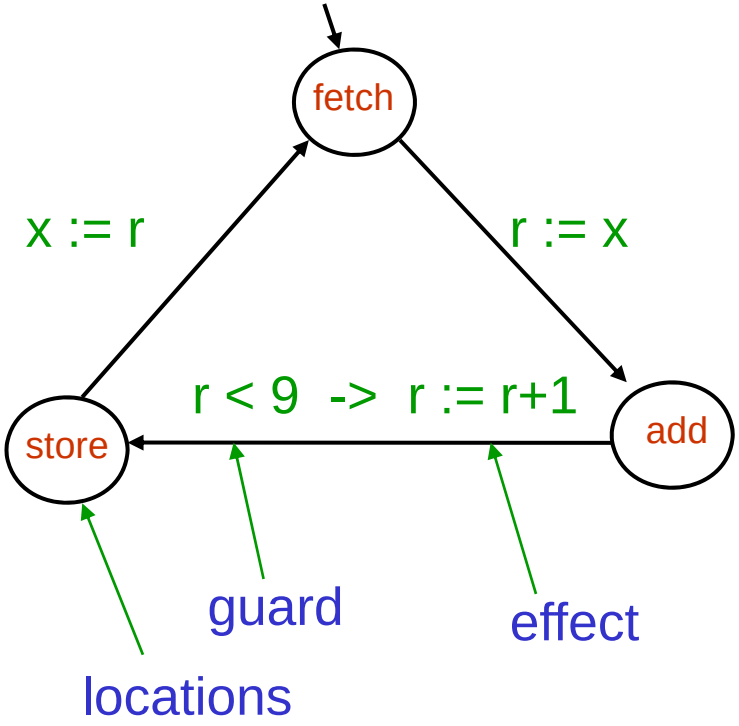
Valuation:

(add, {x -> 0 , r -> 0}) (x) = 0

(add, {x -> 0 , r -> 0}) (loc) = add

Corresponding Transition System

Action System
Variables: x, r: integer
Initially x = 0



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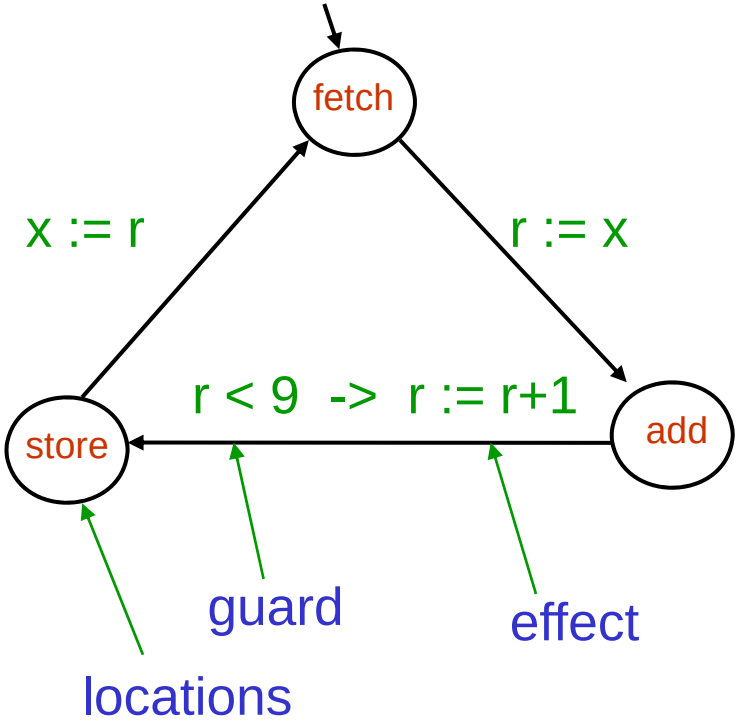
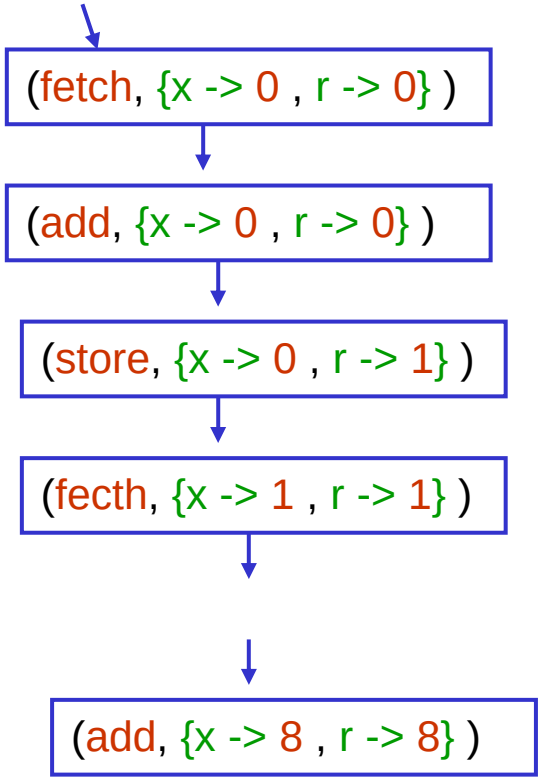
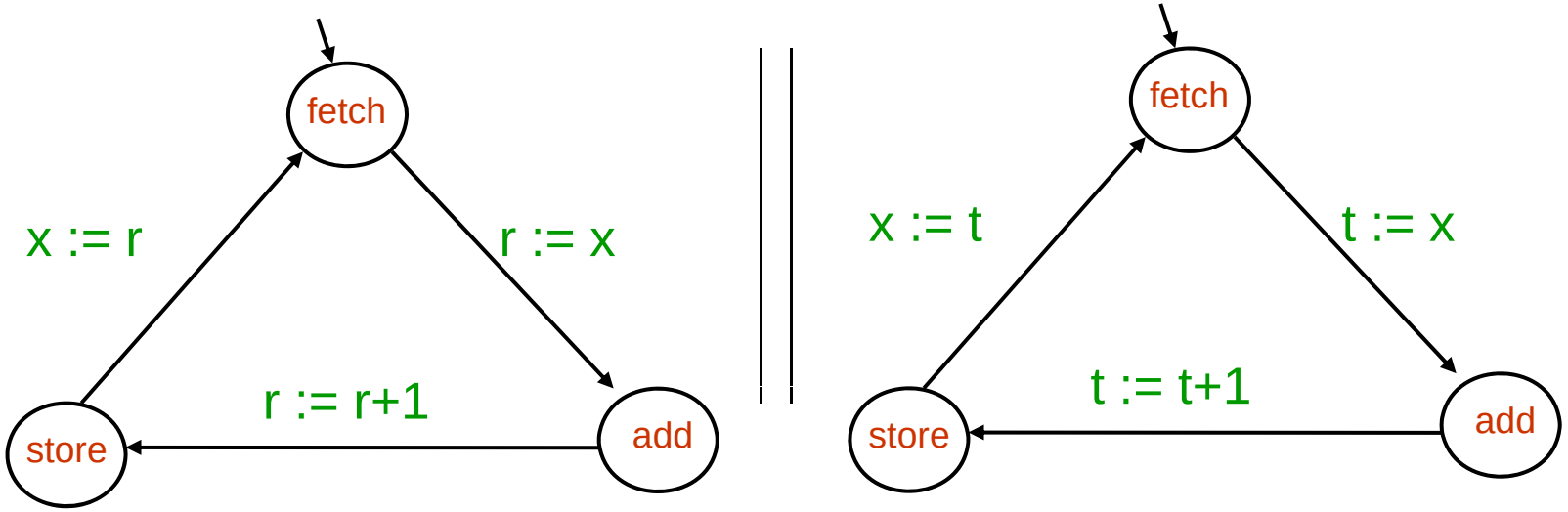




Parallel Processes

Program graphs can be put in parallel

Variables: x, r, t: integer
Initially x = 0



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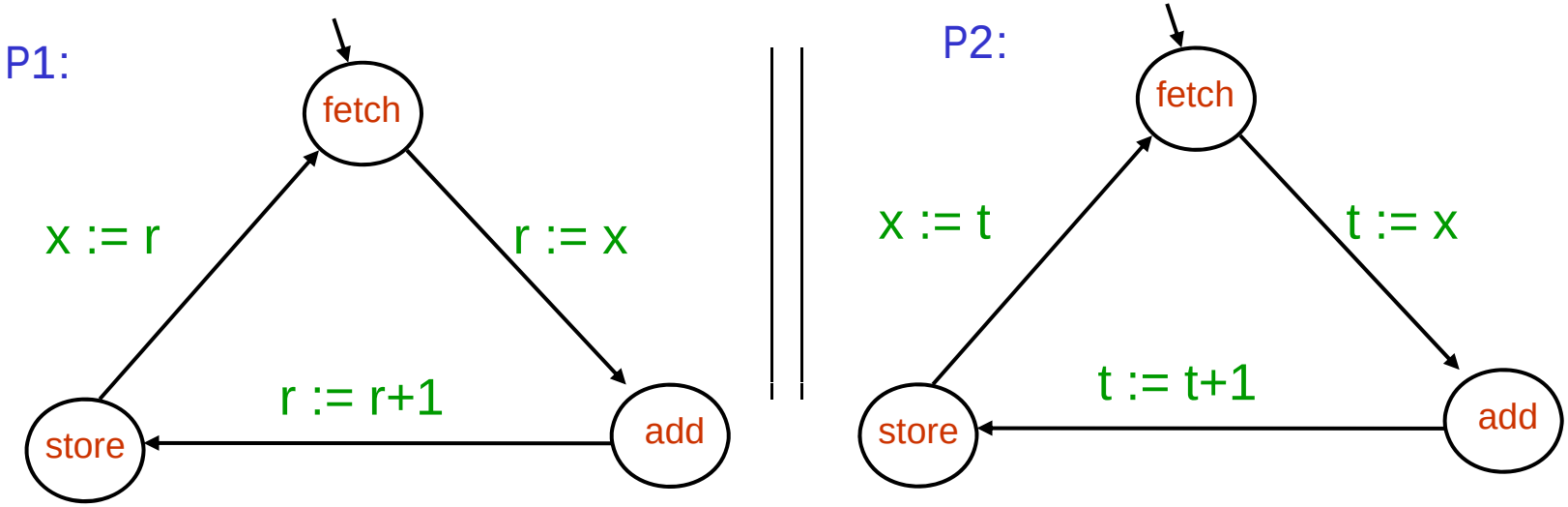
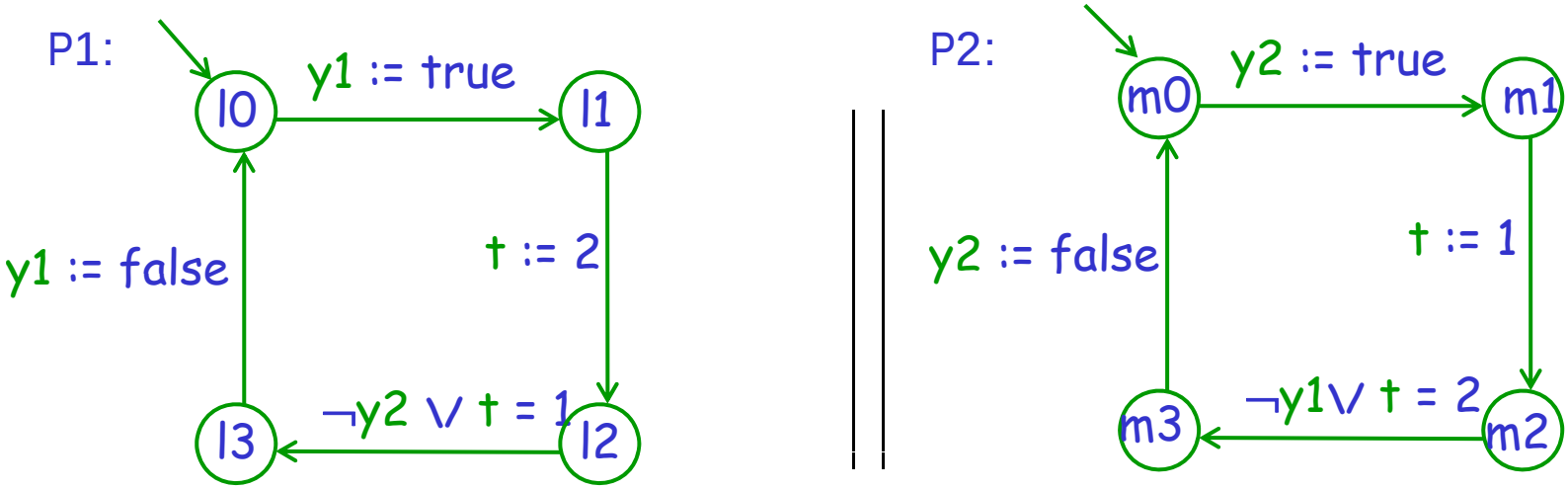


States: (add, fetch, {x -> 0 , r -> 0 , t -> 0})
Transitions: (add, fetch, {x -> 0 , r -> 0 , t -> 0})
-> (store, fetch, {x -> 0 , r -> 1 , t -> 0})
Variables: loc[P1], loc[P2], x, r , t

Peterson-Fischer Mutual Exclusion prot.

Variables: y1, y2: boolean, t: {1,2}
Initially y1 = y2 = false, t = 1



→

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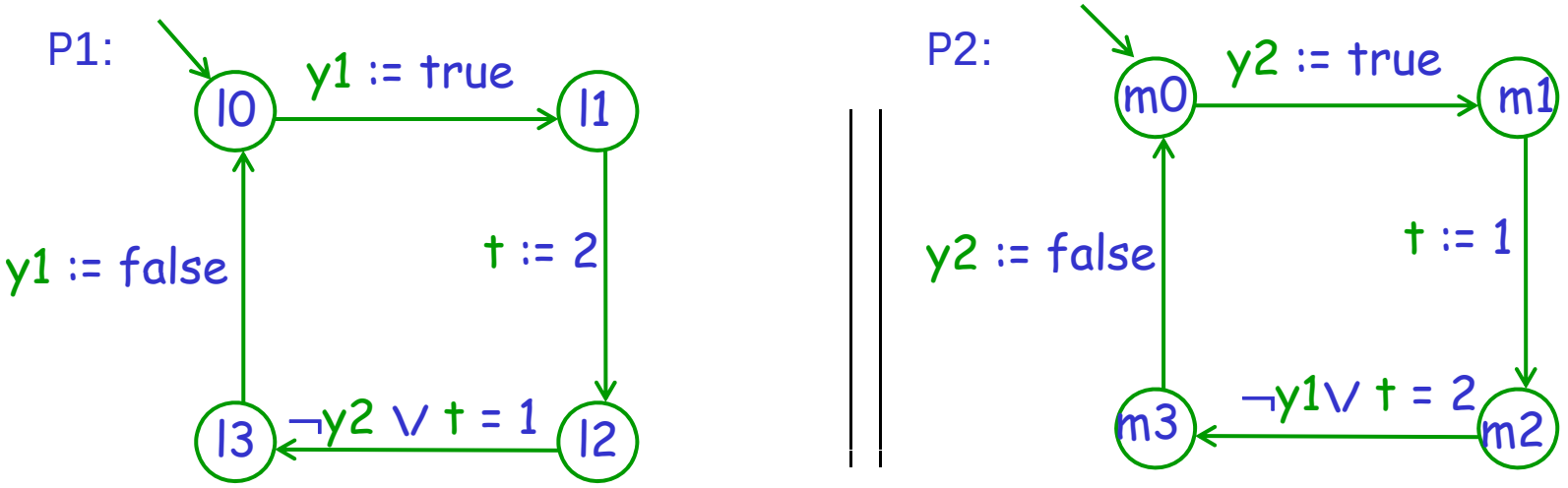

States: (l1 , m1, {y1 -> true ,y2 -> true , t -> 1})
Initial States: (l0 , m0, {y1 -> false ,y2 -> false , t -> 1})

Transition:

(l0 , m0, {y1 -> false ,y2 -> false , t -> 1}) → (l1 , m0, {y1 -> true ,y2 -> false , t -> 1})

Interleaving

Transitions of parallel components are merged or “interleaved”

- as if the model is “executed” on a single processor

The order between transitions of different components is completely arbitrary

- as if there is a scheduler with an unknown strategy
- possible that some components are scheduled “very seldom” or never at all

Particular scheduling strategies can be enforced by adding a “scheduler” into the model

- e.g., by using shared variable(s) to model the state of a scheduler

Certain minimal assumptions on scheduling can be represented using fairness (more about this later)

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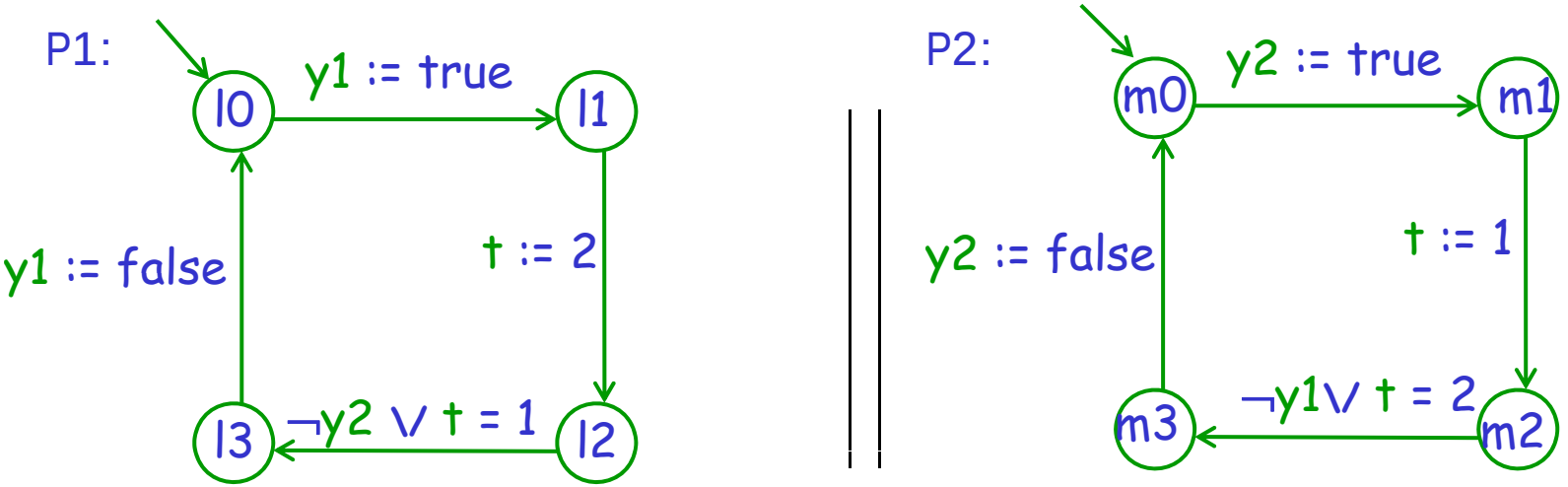

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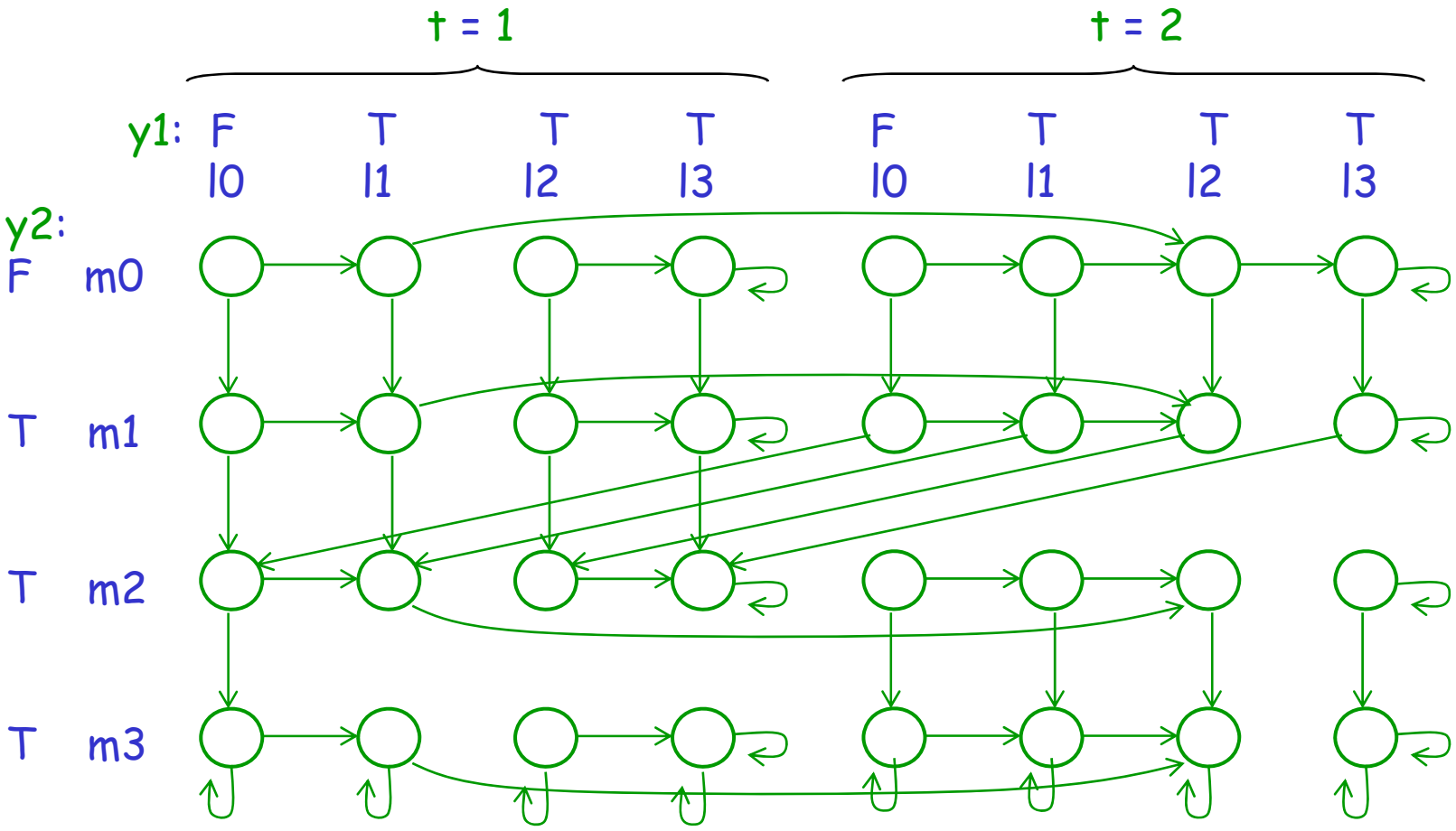
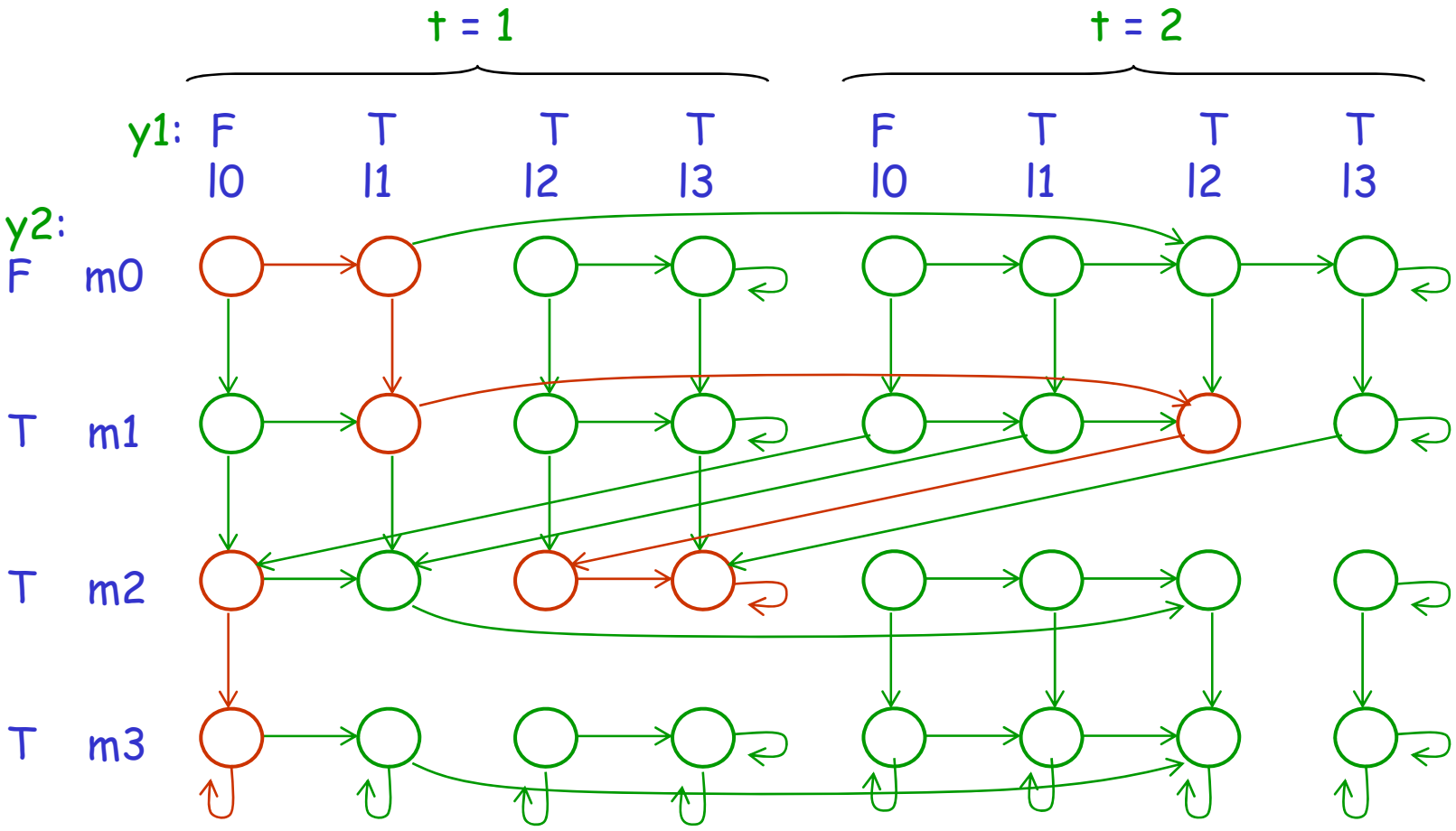
Peterson-Fischer Transition System



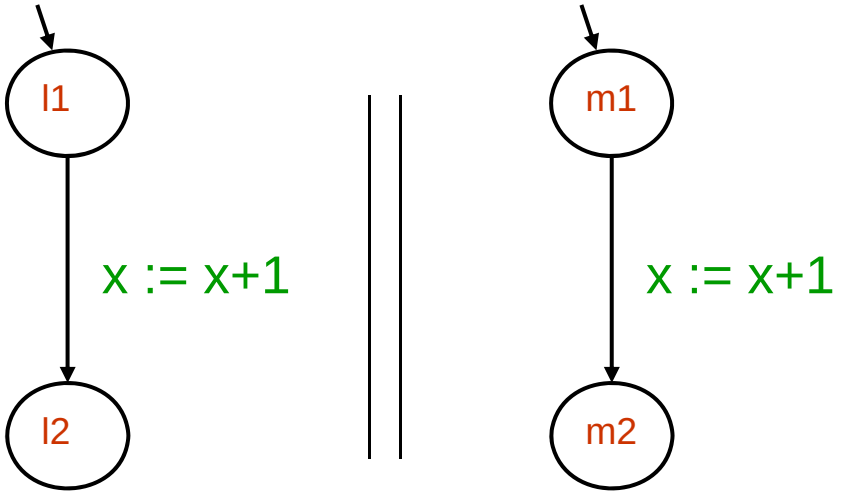
Peterson-Fischer: Computation



Atomicity

Each transition is atomic, i.e., it is an undivisible operation, which has no "intermediate states", i.e., can not be stopped "in the middle"

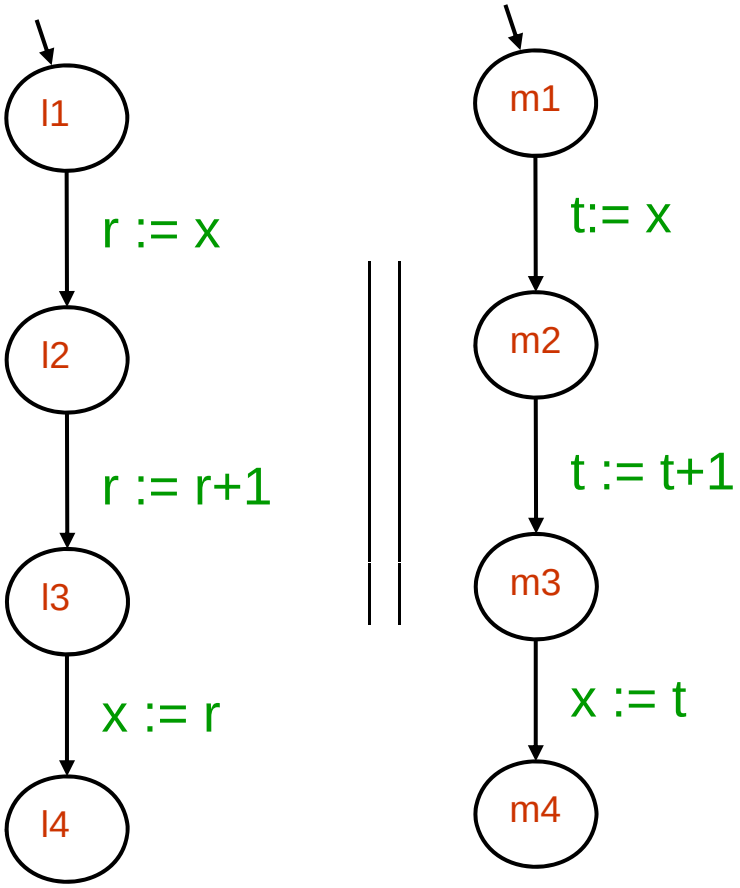
Variables: x: integer
Initially x = 0



This transition system will "terminate" with the value of x being 2

Atomicity

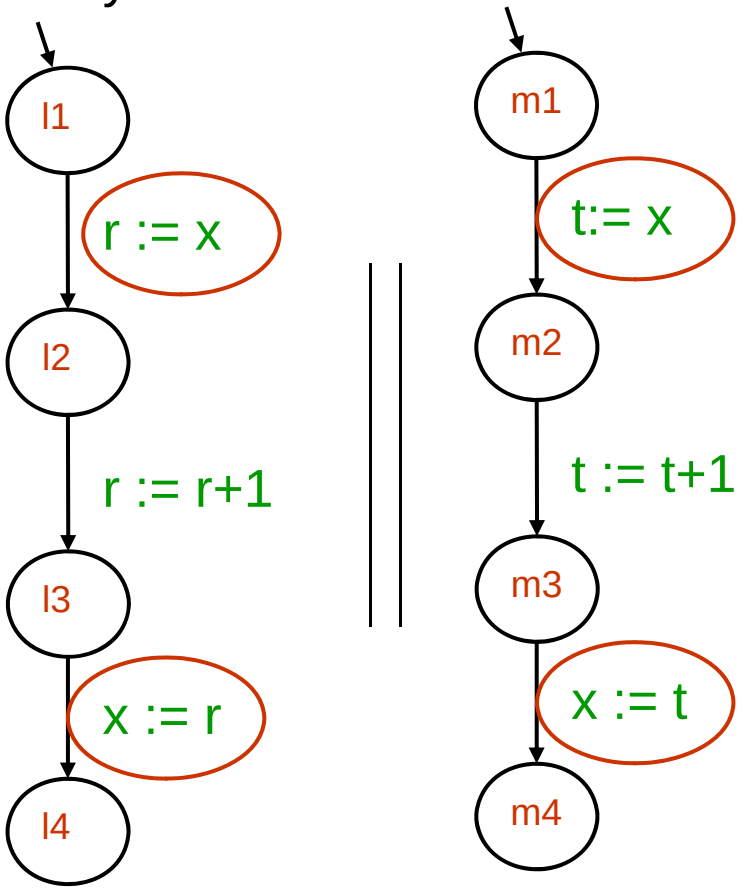
Variables: x, r, t: integer
Initially x = 0



This transition system will “terminate” with the value of x being 1 or 2

How much detail must be modeled?

Variables: x, r, t: integer
Initially x = 0



All possible sequences of ”externally interesting” transitions must be modeled

Critical event is an operation, for which is makes a difference how it is interleaved
- Reading shared varialbe
- Writing to shared variable

Recepy:
- Each transition should involve at most one critical event

Also a faithful model

Variables: x, r, t: integer
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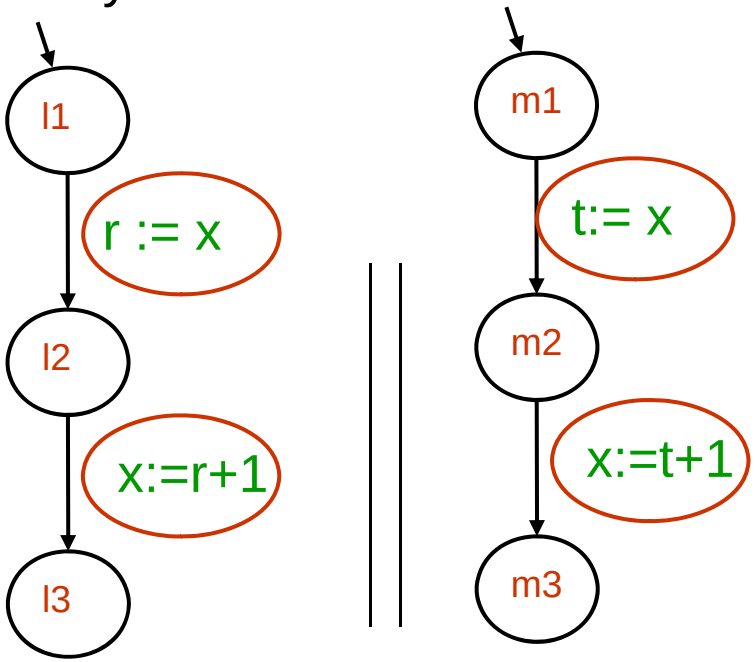

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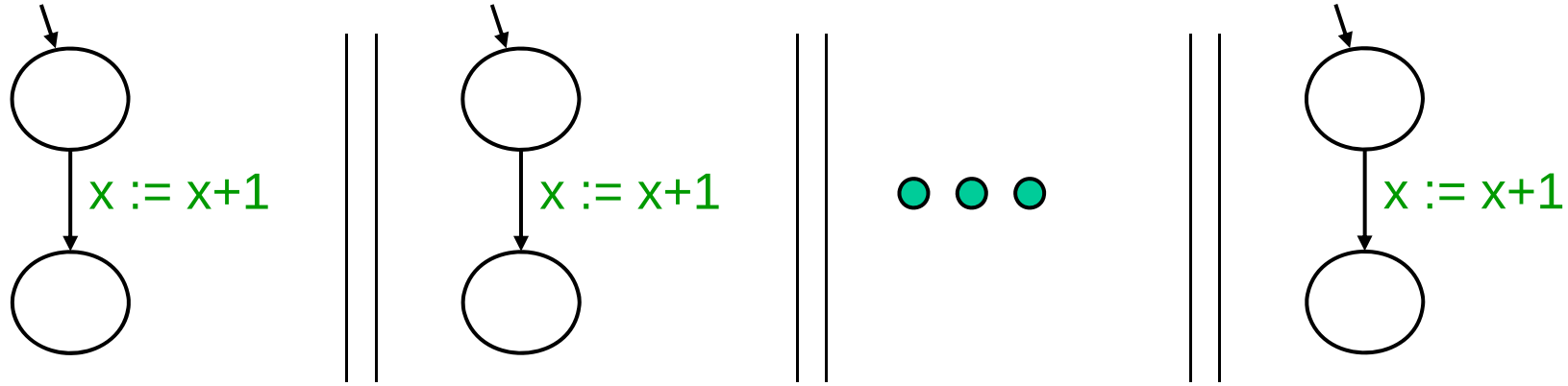
Arguments for using Interleaving

- “Principle of relativity” (simultaneity can not be observed)
- Fewer transitions to consider in analysis

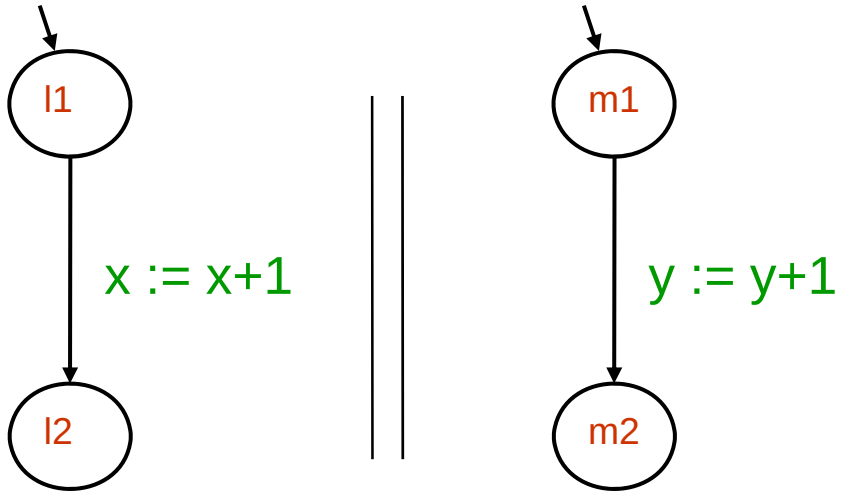


- In general: 2^n possible transitions
- With interleaving: n possible transitions

Limitations of interleaving

- Some (noninteresting) “pathological” properties are satisfied when using interleaving

Variables: x,y: integer
Initially x = y = 0



- Interleaving makes the system satisfy the (uninteresting) property:
  “there must be a (short) instant when x and y are different”

Small Idealized Example from Reality

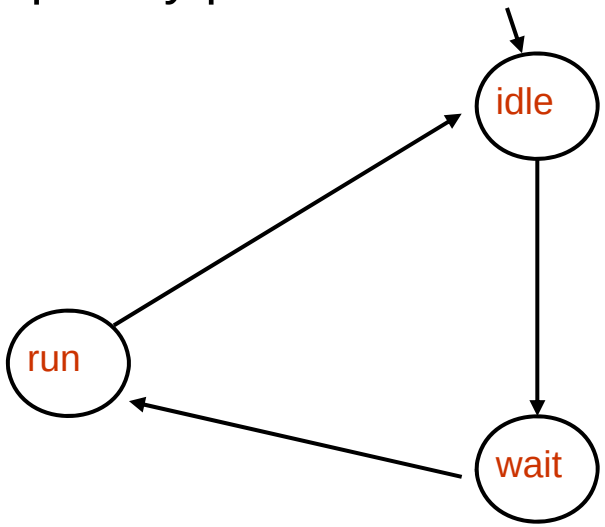
Small Example: Mars Pathfinder 1997

Typical properties of synchronization in real-time systems

- Mutual exclution
  - A process cannot access the data-bus unless it owns a mutex-lock
- Scheduling priority
  - Saving data to memory has higher priority than processing data
  - Low priority process cannot execute when high priority process is ready to execute or executes

Idealized model of processes

High priority process



Idealized model of processes

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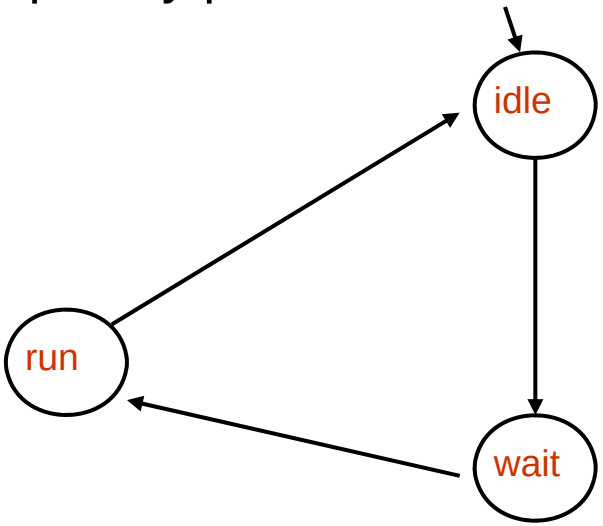
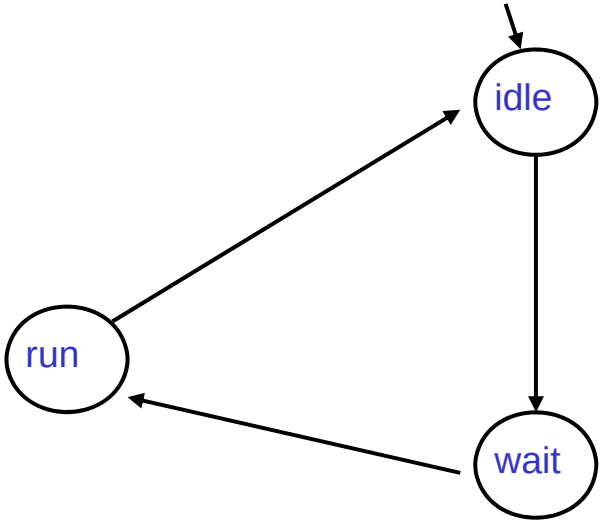


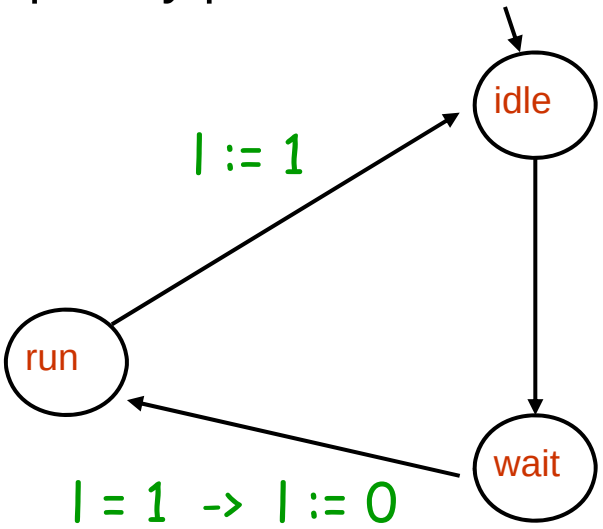
Low priority process



Idealized model of processes

Variables: l: integer
Initially l = 1

High priority process



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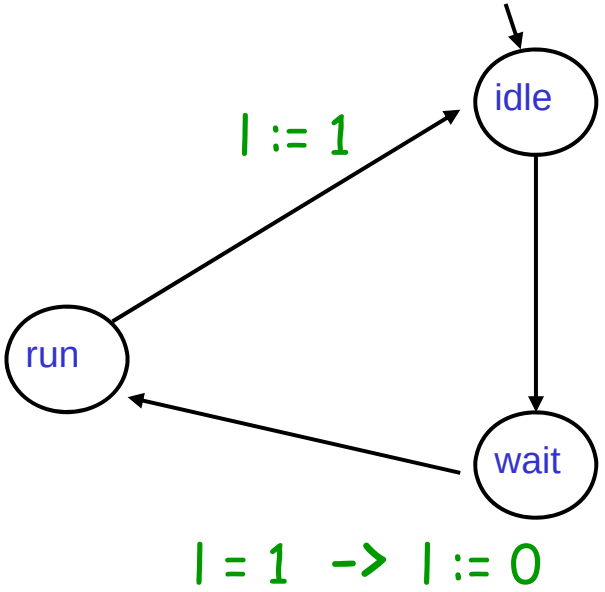

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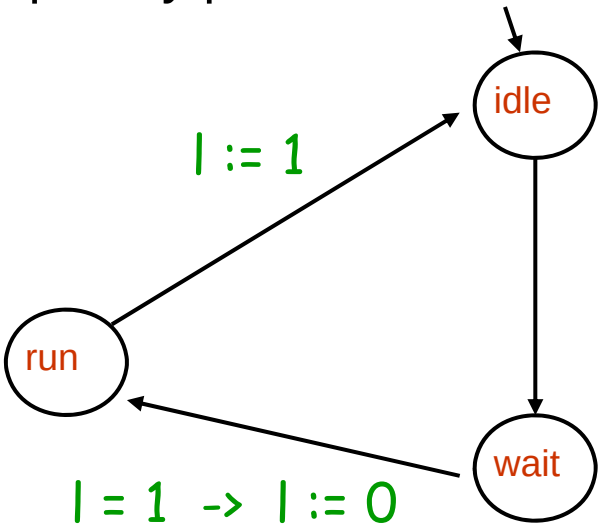

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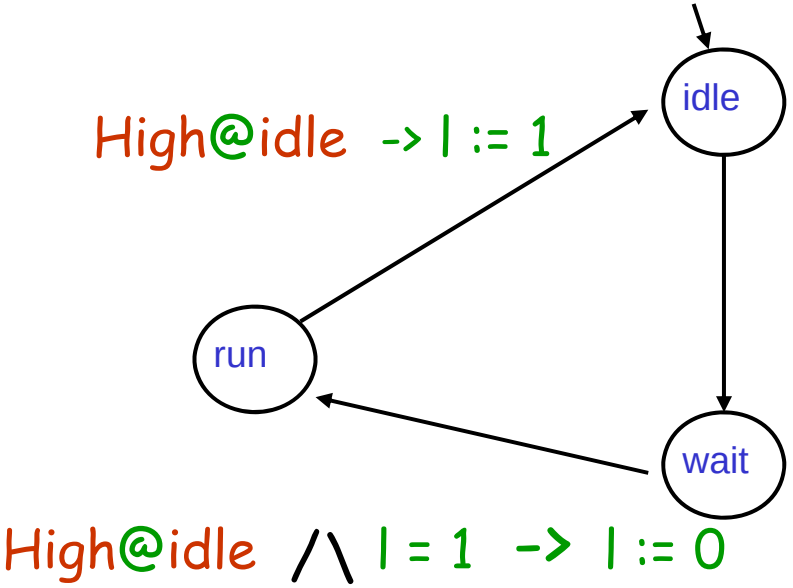
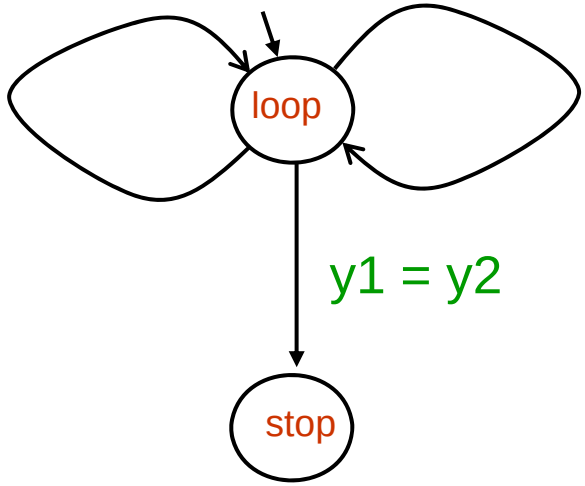


Example: GCD Computation

Action System
Variables: y1,y2: integer
Initially y1=x1, y2=x2

y1 > y2 ->
y1 := y1-y2

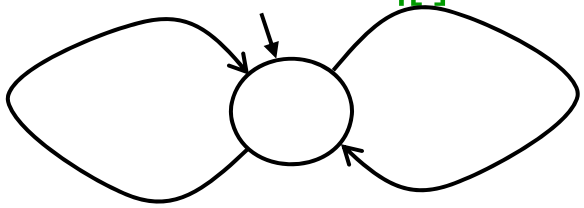
y1 < y2 ->
y2 := y2-y1



Example: Termination Detection

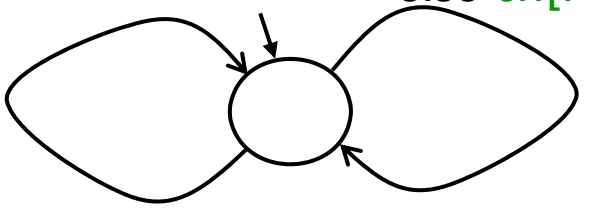
Variables: ch[N]: FIFO Channel of {black,white}
q[N]: boolean
dist = false: boolean
Processes: Q[0] || ... || Q[N-1] || P[0] || ... || P[N-1]

Q[i]:



q[i-1] -> q[i] := false ;
if i=0 then dist := true

P[i]:



ch[i]?black -> ch[i+1]!black

P[0]: