Fast Quiz #2
Numerical Functional Analysis

Præparatus supervivet

Stefan Engblom

Division of Scientific Computing
Department of Information Technology
Uppsala University

Uppsala, May, 2019
True/False: In a normed vector space \((X, \| \cdot \|)\), the map \(x : X \to \mathbb{R}\) defined by \(x \mapsto \sqrt{1 + \|x\|^2}\) is continuous.
True/False: When considered on $\mathbb{R}^n$, the norms $\| \cdot \|_{\infty}$ and $\| \cdot \|_1$ imply the same topology, but they are not equivalent.
Question 3

True/False: All subspaces of the (sequence-) $l^2$-space are complete.
**True/False:** Define \( f_z(x) = \|x - z\| \) for arguments \( x \) in some normed vector space \( X \). Then there is a solution to \( x = \arg \max_{x \in M} f_z(x) \) for any compact subset \( M \subseteq X \).
True/False: All linear operators on $C[0, 1]$ are continuous.
Question 6

**True/False:** If there is a Schauder basis, then the space is separable.
Question 7

**True/False:** \( L^2[0, 1] \) is separable.
Question 8

**True/False:** The convergence of a Schauder expansion is always in the absolute sense.
Question 9

**True/False:** If the space is separable, then there is a Schauder basis.
Question 10

**True/False:** A closed and bounded subset of a metric space is compact.