Fast Quiz #3
Numerical Functional Analysis

Præparatus supervivet

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Question 1

True/False:  \[ \|x + y\|^2 \leq \|x\|^2 + \|y\|^2. \]
True/False: One way of defining the $l^p$ inner product is

$$(x, y) = \sum_{j \geq 1} \xi_j^{p/2} \eta_j^{p/2},$$

where $x = (\xi_j)$ and $y = (\eta_j)$. 
True/False: *Unfortunately*, the norm $\|x\| = \max_{t \in [a,b]} |x(t)|$ for $x \in C[a,b]$ can not be obtained from an inner product.
Question 4

**True/False:** Let $Y$ be an open subspace of a Hilbert space $H$. Then $Y$ is complete.
True/False: Suppose $(Sv, v) = 0$ for all $v$ in a complex Hilbert space $H$, where $S$ is a bounded linear operator. Then $S = 0$. 
True/False: A bounded bilinear form $a(\cdot, \cdot) \geq 0$ defines a norm by $\|v\|^2 := a(v, v)$. 
Question 7

**True/False:** If a Hilbert space $H$ contains a total orthogonal sequence, then $H$ is separable.
Question 8

**True/False:** Let $f \in L^2[0, 1]$. Assume that $(e_k)$ is an orthonormal sequence in $L^2[0, 1]$. Put

$$\tilde{f} = \sum_{k \geq 1} (f, e_k) e_k.$$ 

Then $f = \tilde{f}$. 
True/False: If \( x \perp y \), then \( \| x + y \|^2 = \| x \|^2 + \| y \|^2 \).
Question 10

**True/False:** Suppose \((Sv, v) = 0\) for some \(v\) in a complex Hilbert space \(H\), where \(S\) is a bounded linear operator. Then \(Sv = 0\).
True/False: A bounded coercive bilinear form defines an inner product by 

\[(u, v) := a(u, v).\]