

# Mega Quiz (1/2)

## Numerical Functional Analysis, 5.0 hp

*Præparatus supervivet*

Stefan Engblom

Division of Scientific Computing  
Department of Information Technology  
Uppsala University

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1. **True/False:** Consider the Euclidean metric space  $(X = \mathbb{R}^n, d_2)$  and the map  $T : X \rightarrow X$  defined as  $Tx = Cx + b$ , where  $C$  is a real  $N \times N$  fixed matrix and  $b$  is a fixed vector. Then  $T$  is a contraction on  $(X, d_2)$  if  $C = (c_{jk})_{j,k=1,\dots,N}$  is such that

$$\sum_{k=1}^N c_{jk}^2 < 1 \quad j = 1, \dots, N$$

2. **True/False:** The set of rational numbers in itself is meager.
3. **True/False:** If  $T^n$  is a contraction for an  $n > 1$ , then  $T$  need not be a contraction.
4. **True/False:** A norm is a sublinear functional.
5. **True/False:** A bounded linear operator is closed.
6. **True/False:** Let  $X, Y$  be metric spaces and  $X \subset Y$ . Then the identity mapping  $T : X \rightarrow Y$  defined by  $x \rightarrow x$  is an open mapping.
7. **True/False:** If a closed linear operator is invertible, the inverse is also closed.
8. **True/False:** There exists no continuous function whose Fourier series diverges at a dense set of points.

9. **True/False:** A closed linear operator is bounded.
10. **True/False:** Let  $\|\cdot\|_1$  and  $\|\cdot\|_2$  be norms on a vector space  $X$  such that  $X_1 = (X, \|\cdot\|_1)$  and  $X_2 = (X, \|\cdot\|_2)$  are complete. Suppose there is a constant  $c > 0$  such that  $\forall x \in X, \|x\|_1 \leq c\|x\|_2$ . Then, the two norms are equivalent.
11. **True/False:** Let  $T : X \rightarrow X$  be such that  $Tx = \alpha x$ , with  $\alpha$  a real constant. Let  $g \in X'$ . Then  $(T^\times g)(x) = \alpha g(x)$ .