Mega Quiz (2/2)
Numerical Functional Analysis, 5.0 hp

Præparatus supervivet

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Uppsala, November, 2014

1. **True/False:** The set of rational numbers in $\mathbb{R}$ is meager.
2. **True/False:** The subset of polynomials in the metric space $C[a,b]$ is closed.
3. **True/False:** A bounded bilinear form $a(\cdot,\cdot) \geq 0$ defines a norm by $\|v\|^2 := a(v,v)$.
4. **True/False:** If $x \perp y$, then $\|x+y\|^2 = \|x\|^2 + \|y\|^2$.
5. **True/False:** A bounded linear operator T, which is a mapping from a Banach space X into a Banach space Y, is an open mapping.
6. **True/False:** Define $f_z(x) = \|x - z\|$ for arguments x in some normed vector space X. Then there is a solution to $x = \arg\max_{x\in M} f_z(x)$ for any compact subset $M \subseteq X$.
7. **True/False:** Let X be a normed space. $f(x_0) = 0$ for all $f \in X'$ implies $x_0 = 0$.
8. **True/False:** When considered on $\mathbf{R}^n$, the norms $\|\cdot\|_\infty$ and $\|\cdot\|_1$ imply the same topology, but they are not equivalent.
9. **True/False:** The function f defined by $f(x) = |x|^{1/2}/2$ satisfy a Lipschitz condition in $[-1,1]$.

10. **True/False:** *Unfortunately*, the norm $\|x\| = \max_{t\in[a,b]} |x(t)|$ for $x \in C[a,b]$ can not be obtained from an inner product.

11. **True/False:** If a linear operator is continuous at 0, it is bounded.

12. If x is an exact quantity and $\tilde{x}$ is an approximation to x, what is the error; $e = x - \tilde{x}$, or $e = \tilde{x} - x$? Does it matter? How does this *sign convention* define the term 'residual'?