Numerical Functional Analysis, 5.0 hp
Suggested subjects for mini-essays

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1 Format

Although there is a LaTeXtemplate to use, the actual content of the mini-essay is free. The best essays are likely the results of an author taking a true interest in the subject!

Write about a theorem and its use in some theory, or write about a 'nice' proof of a theorem, or about a result which has a beautiful proof thanks to some fundamental results of functional analysis. Or walk through a technical paper you have read with (numerical) functional analysis in mind.

Try to mix a summarizing discussion with some more detailed parts; I want top-down *and* bottom-up!

The length of the mini-essay should stay in the range 2–6 pages (not counting the title page).

2 Suitable subjects

Some more or less concrete suggestions include

- The Metric Lax and Applications *(I have a sketch of the proof written down)*
- The Normed Nonlinear Lax and Applications *(I have some slides from a useful talk)*
- "Traditional" proof Lax-Milgram

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- Fixed point proof of Lax-Milgram *(A nice application of Banach's fixed point theorem)*
- Convergence of inexact Newton algorithms in normed spaces *(There are papers about this but I am unaware of a 'final' theory)*
- Construction of Sobolev spaces
- 5 Big Theorems of linear functional analysis *(But you may not take the theorem you presented during the course!)*
- Other theorems: Baire's Theorem, Banach closed range Theorem, Banach-Saks-Mazur Theorem, Weak convergence and weak * convergence, Banach-Eberlein-Smulian Theorem *(or check Wikipedia!)*
- Theorems of Nonlinear Functional Analysis: Ekeland's variational principle, Brouwer's fixed point Theorem, Schauder's fixed point Theorem, Minty-Browder Theorem, Borsuk's and Borsuk-Ulam Theorems.
- Fredholm theory: integral operators of the type $A = I + K$ with K compact, but also for $A = k^2 I + \Delta$ (i.e. Helmholtz); here H^1 is compact in L^2 since the domain is bounded.
- The Babuska theory for non-coercive bilinear forms; roughly, if the bilinear form $a : V \times U \to K$ is continuous and there is an $\alpha > 0$ such that

  $$\alpha \|u\| \leq \sup_v |a(v,u)| \quad \forall u \in U,$$
  $$0 < \sup_u |a(v,u)| \quad \forall v \in V.$$

  then a is an isomorphism of U onto U'. One can also discuss only the case of the Stokes problem (a special case).

There are no ends to the possibilities!