A. Dq notes: problem 2, p114
B. Order reduction: Prove that the stage order of an SDIRK method is as most 1, and of a DIRK method at most 2. (See Hairer, Wanner IV.15)

C. Consider a system of ODEs $y' = L(y)$, where $y$ is a vector with $s$ components $v_i, i=1,..., s$. Assume the Euler forward discretization of the system $u^{n+1} = u^n + hL(u^n)$ has the property that for all $h \leq h_e$ the solution satisfies $TV(u^{n+1}) \leq TV(u^n)$, where $TV(v) = \sum_{1 \leq j \leq s} |v_{j+1} - v_j|$ denotes the total variation of the vector $v$. A one-step method is called strong stability preserving (SSP) if there exists a $C > 0$ such that the solution at consecutive time levels satisfies $TV(u^{n+1}) \leq TV(u^n)$ for all $h \leq C h_E$. Consider the following 2 second order Runge-Kutta methods, sometimes denoted Heun’s and Ralston’s methods, respectively.

\[
\begin{align*}
y_{1}^{n+1} &= y^{n} + \frac{h}{2} \left( L(y^{n}) + L(y^{n} + hL(y^{n})) \right) \\
y_{2}^{n+1} &= y^{n} + \frac{h}{4} \left( L(y^{n}) + 3L \left( y^{n} + \frac{2h}{3} L(y^{n}) \right) \right)
\end{align*}
\]

Are these methods SSP under some restriction $h \leq C h_E$?

Read about SSP methods in *Gottlieb, Shu, Tadmor*, Strong Stability-Preserving High-Order Time Discretization Methods, SIAM Rev., 43(1), 89–112