Calibrating Inertial Sensors using a Dynamic Model and Maximum Likelihood Estimation

Fredrik Olsson*, Manon Kok**, Kjartan Halvorsen*, Thomas B. Schön*

*Department of Information Techonology, Uppsala University, **Department of Electrical Engineering, Linköping University

UPPSALA UNIVERSITET

LINKÖPINGS UNIVERSITET

Background and contribution

Accelerometers and gyroscopes (inertial sensors) are used to measure linear acceleration and angular velocity, respectively. These sensors have many different applications in for example navigation, orientation estimation and motion capture. Inertial sensors are widely available, for instance in smartphones. To obtain accurate position and orientation estimates, it is important that the sensors are properly calibrated. We present ongoing work where a practical calibration algorithm for inertial sensors is developed.

Formulation of the calibration problem

Similarly to Kok et al. (2014)[1], we formulate the calibration problem as an orientation estimation problem in the presence of unknown model parameters θ. A nonlinear state space model is used

$$\mathbf{x}_{t+1} = \mathbf{f}_t(\mathbf{x}_t, \omega_t, \mathbf{v}_t, \theta), \quad (1)$$

$$\mathbf{y}_t = \mathbf{h}_t(\mathbf{x}_t, \theta) + \mathbf{e}_t, \quad (2)$$

where the state $\mathbf{x}_t, t \in \{1, \dots, N\}$, represents the sensor orientation. The nonlinear functions $\mathbf{f}_t(\cdot)$ and $\mathbf{h}_t(\cdot)$ represent the dynamic model and the measurement model, respectively. The negative log-likelihood is approximated and used to formulate the cost function

$$V(\theta) = \frac{1}{2}\sum_{t=1}^{N} \|\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}(\theta)\|^2_{\mathbf{S}_t^{-1}(\theta)} + \log\det \mathbf{S}_t(\theta), \quad (3)$$

where $\|\mathbf{y}_t - \hat{\mathbf{y}}_{t|t-1}(\theta)\|_{\mathbf{S}_t^{-1}(\theta)}$ is the residual norm with covariance matrix $\mathbf{S}_t(\theta)$ obtained from an Extended Kalman Filter (EKF). A Maximum Likelihood estimator for θ is then formulated as

$$\hat{\theta} = \arg\min_{\theta} V(\theta). \quad (4)$$

[1] M. Kok and T. B. Schön, **Maximum likelihood calibration of a magnetometer using inertial sensors.** *Proceedings of the 19th World Congress of the International Federation of Automatic Control (IFAC), pp. 92-97, Cape Town, South Africa*, August 2014.

Dynamic model with gyroscope input

The dynamic model uses the gyroscope measurements as input to predict the orientation of the sensor as

$$\mathbf{x}_{t+1} = \mathbf{x}_t \odot \boldsymbol{exp}\frac{T}{2}\omega_t, \quad (5)$$

where the state variables $\mathbf{x}_t$ are unit quaternions, $\odot$ denotes the quaternion multiplication and $\boldsymbol{exp}$ the vector exponential. The input to the dynamic model is the angular velocity ω_t, sampled with interval T. The gyroscope measurements, which are used to estimate ω_t, are modelled as

$$\mathbf{y}_{\omega,t} = \omega_t + \mathbf{b}_\omega + \mathbf{v}_t, \quad (6)$$

where $\mathbf{b}_\omega \in \mathbb{R}^{3\times 1}$ is the gyroscope bias and $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_\omega)$ is Gaussian measurement noise.

Accelerometer measurement model

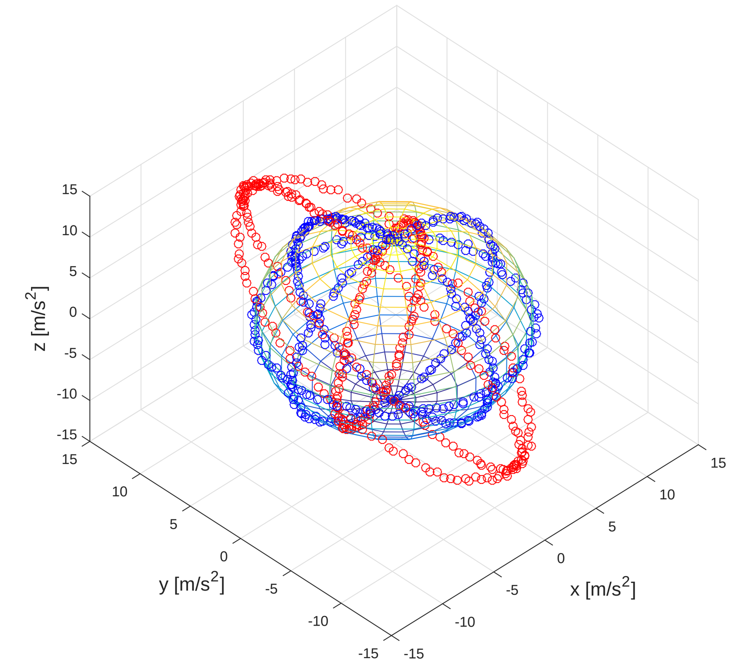
The accelerometer measurements are modelled as

$$\mathbf{y}_{a,t} = \mathbf{D}\mathbf{R}(\mathbf{x}_t)\mathbf{g} + \mathbf{b}_a + \mathbf{e}_{a,t}, \quad (7)$$

using the assumption that the sensor only measures the gravitational acceleration $\mathbf{g}$, which is true for stationary sensors. The rotation matrix $\mathbf{R}(\mathbf{x}_t)$ describes the orientation of the sensor. Sensor errors are modelled by the matrix $\mathbf{D} \in \mathbb{R}^{3\times 3}$, which include non-orthogonality and gain of the sensitive axes. Bias is modelled by the vector $\mathbf{b}_a \in \mathbb{R}^{3\times 1}$ and $\mathbf{e}_{a,t} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_a)$ is Gaussian measurement noise. The calibrated accelerometer measurements are then calculated as

$$\mathbf{y}^{\text{cal}}_{a,t} = \mathbf{D}^{-1}(\mathbf{y}_{a,t} - \mathbf{b}_a). \quad (8)$$

The accelerometer measurements can be visualized in 3D. This figure shows an example with simulated accelerometer measurements. The calibrated measurements (blue) are centered on the sphere with radius $\|\mathbf{g}\|_2$ and its center in origo, while the uncalibrated measurements (red) form an ellipsoid centered around the sensor bias.

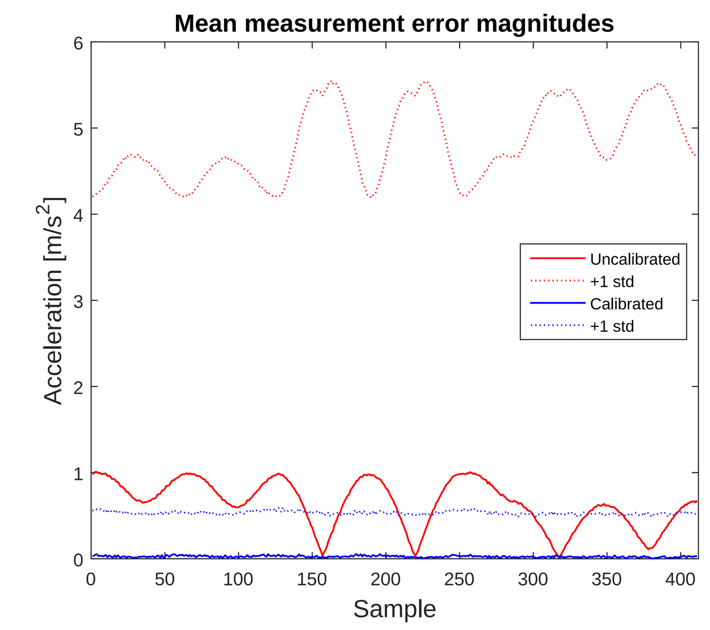


Calibration algorithm

1. Estimate accelerometer and gyroscope covariance, gyroscope bias and the initial orientation of the sensor, $\hat{\boldsymbol{\Sigma}}_a, \hat{\boldsymbol{\Sigma}}_\omega, \hat{\mathbf{b}}_\omega, \hat{\mathbf{x}}_0$ from a stationary portion of the sampled measurements.
2. Set $i = 0$ and repeat:
3. Run the EKF using the current estimates $\hat{\theta}_i = \{\hat{\mathbf{D}}_i, \hat{\mathbf{b}}_{a,i}\}$ and initial orientation $\hat{\mathbf{x}}_0$.
4. Calculate $\hat{\theta}_{i+1}$ by solving (4) as an unconstrained optimization problem using a Gauss-Newton method, the numerical gradient and approximate Hessian of the cost function (3), and a line search algorithm.
5. Set $i = i + 1$ and repeat from step 3 until convergence.

Simulations

1000 Monte Carlo simulations were performed with randomly generated parameters θ. In the simulations the sensors were rotated once around each axis.



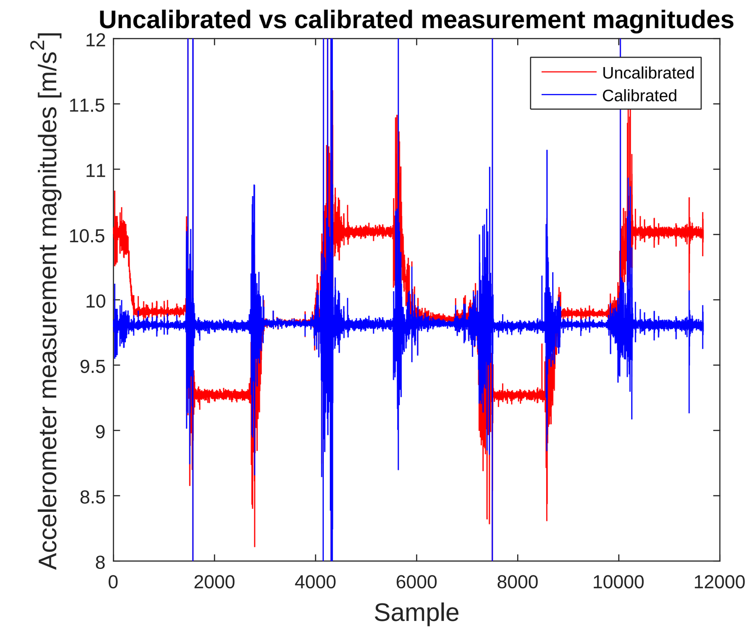
Root mean square errors (RMSE)

Param.	$\mathbf{D}$	$\mathbf{b}_a[m/s^2]$	$\mathbf{b}_\omega[rad/s]$
RMSE	**0.02304**	**0.0858**	**0.01149**
Param.	$\boldsymbol{\Sigma}_a$	$\boldsymbol{\Sigma}_\omega$	
RMSE	**0.01206**	**0.001325**	

Real sensor data

Data was collected from the inertial sensors in a Samsung Galaxy S5 smartphone. The phone was rotated by hand, which violates the assumption that the accelerometer only measures the gravitational acceleration since the sensors will not always be stationary. The magnitude of the accelerometer measurements varies for different orientations before the calibration and becomes centered around the local gravity magnitude after calibration. The bursts in the measurements come from rotating the phone by hand.

Even in the presence of these disturbances the algorithm is able to calibrate the accelerometer.



Future work

- Algorithm robustness against increasing noise levels and outliers.
- Find a method to detect outliers and exclude them from the measurements.
- Deriving a lower bound for the estimates.
- More experiments on real data, for example to see how long a sensor calibration remains valid.

fredrik.olsson@it.uu.se, manon.kok@liu.se, kjartan.halvorsen@it.uu.se, thomas.schon@it.uu.se