# RADAR CODE OPTIMIZATION FOR MOVING TARGET DETECTION

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The discrete-time received signal $\mathbf{r}$ for the range-cell corresponding to the time delay $\tau$ can be written as

$$\mathbf{r} = \alpha\,\mathbf{a} \odot \mathbf{p} + \mathbf{a} \odot \mathbf{c} + \mathbf{w}$$

code vector (to be designed) — clutter component — signal-independent interferences

$$\text{detection problem}: \begin{cases} H_0: & \mathbf{r} = \mathbf{a} \odot \mathbf{c} + \mathbf{w} \\ H_1: & \mathbf{r} = \alpha\mathbf{a} \odot \mathbf{p} + \mathbf{a} \odot \mathbf{c} + \mathbf{w} \end{cases}$$

$$\text{optimal detector: } \left|\mathbf{r}^H \left(\mathbf{M} + \mathbf{ACA}^H\right)^{-1} (\mathbf{a} \odot \mathbf{p})\right|^2 \underset{H_1}{\overset{H_0}{\lessgtr}} \eta$$

In cases where target Doppler shift is unknown, we consider the following *average* metric:

$$\operatorname{tr}\left\{\mathbf{A}^H(\mathbf{M} + \mathbf{ACA}^H)^{-1}\mathbf{AW}\right\}$$
$$\text{where } \mathbf{W} = \mathrm{E}\{\mathbf{pp}^H\}.$$

code optimization problem:

$$\max_{\mathbf{A}} \quad \operatorname{tr}\left\{\left(\mathbf{A}^{-1}\mathbf{M}\mathbf{A}^{-H} + \mathbf{C}\right)^{-1}\mathbf{W}\right\}$$
$$\text{subject to} \quad \operatorname{tr}\left\{\mathbf{AA}^H\right\} \leq e$$

$$\mathbf{W} = \mathbf{VV}^H \qquad \mathbf{V} \in \mathbb{C}^{N\times\delta} \text{ full column-rank}$$

$$\mathbf{R} \triangleq \begin{bmatrix} \theta\mathbf{I} & \mathbf{V}^H\mathbf{A}^H \\ \mathbf{AV} & \mathbf{M} + \mathbf{ACA}^H \end{bmatrix} \qquad (18)$$

$$\mathbf{U} \triangleq [\mathbf{I}_\delta \ \ \mathbf{0}_{N\times\delta}]^T$$

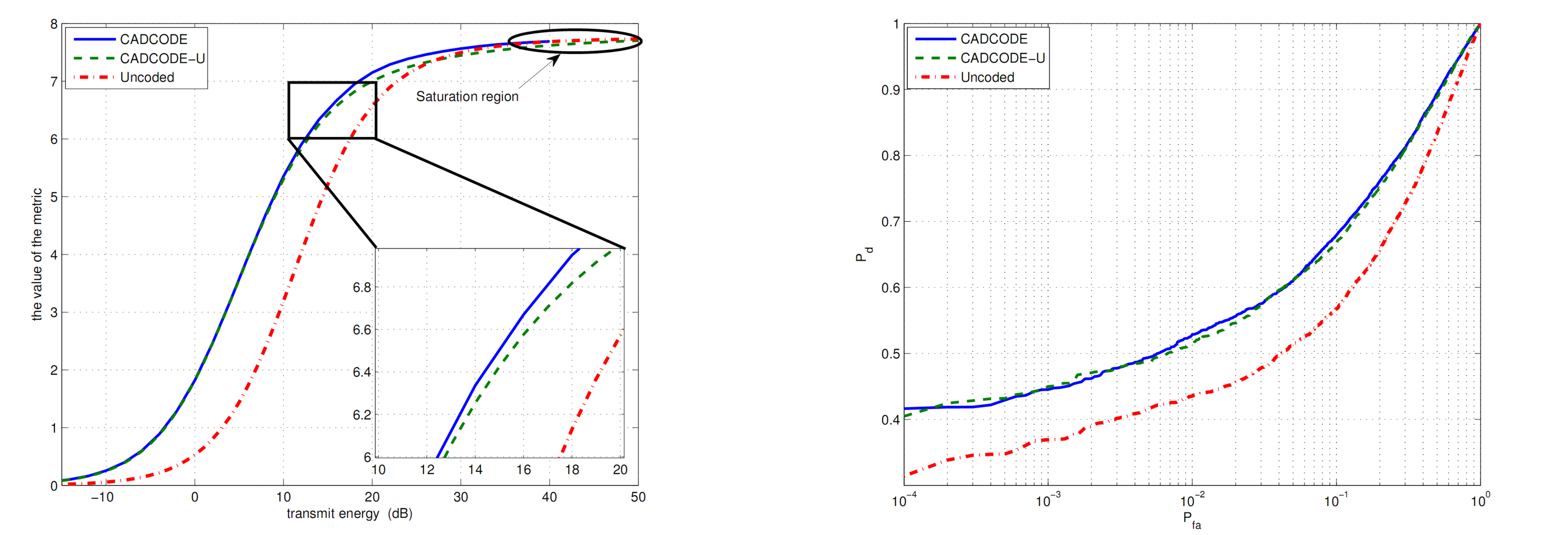
$$\min_{\mathbf{a}} \quad \mathbf{a}^H\left((\mathbf{Y}_2\mathbf{Y}_2^H) \odot \mathbf{C}^T\right)\mathbf{a} + 2\Re(\mathbf{d}^H\mathbf{a}) \quad (22)$$
$$\text{subject to} \quad \mathbf{a}^H\mathbf{a} \leq e$$

$$\mathbf{Y} \triangleq [\mathbf{Y}_{1\ \delta\times\delta} \ \ \mathbf{Y}_{2\ N\times\delta}]^T \text{ and } \mathbf{d} \triangleq \operatorname{diag}(\mathbf{V}^*\mathbf{Y}_1^*\mathbf{Y}_2^T)$$

## CADCODE framework
*converges to a stationary point*

**Table 1**. CADCODE for Optimal Radar Code Design

**Step 0**: Initialize the code vector $\mathbf{a}$ using a random vector in $\mathbb{C}^N$, and form $\mathbf{R}$ as defined in (18).
**Step 1**: Compute $\mathbf{Y} = \mathbf{R}^{-1}\mathbf{U}(\mathbf{U}^H\mathbf{R}^{-1}\mathbf{U})^{-1}$.
**Step 2**: Solve the optimization problem (22) to obtain the code vector $\mathbf{a}$.
**Step 3**: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $\|\mathbf{a}^{(k+1)} - \mathbf{a}^{(k)}\| \leq \epsilon$ for some $\epsilon > 0$.




(a) (b)

**Fig. 1**. The design of optimal codes of length $N = 16$. (a) depicts the values of the metric for CADCODE and CADCODE-U methods as well as the uncoded system vs. the transmit energy. (b) plots the ROC of the optimal detector (unknown $\omega$) for $\sigma^2 = 3$ and transmit energy= 10.

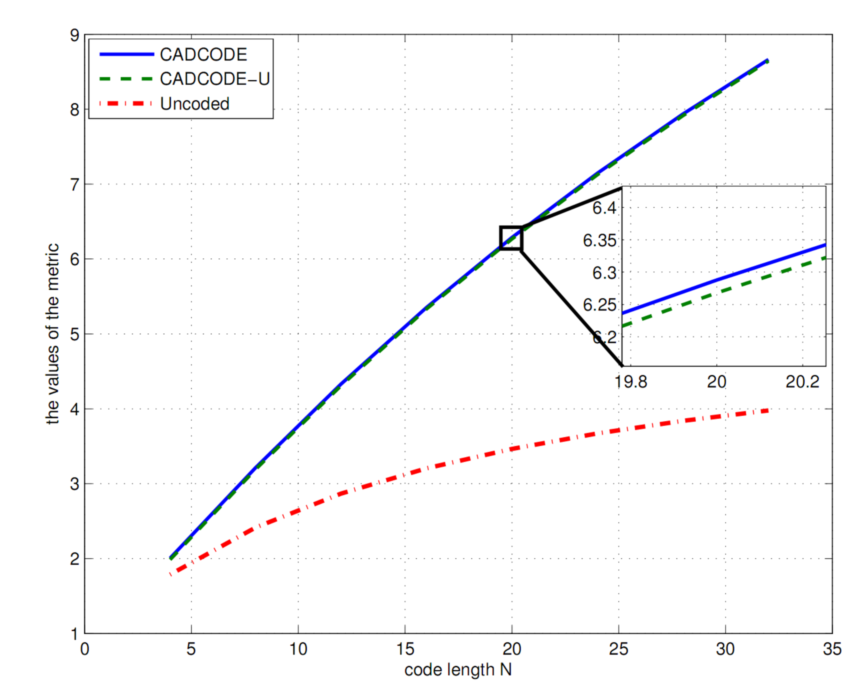



**Fig. 2**. The values of the metric associated with CADCODE, CADCODE-U, and the uncoded system vs. the code length $N$.

## NUMERICAL EXAMPLES

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