MAJORIZATION-MINIMIZATION TECHNIQUE FOR MULTI-STATIC RADAR CODE DESIGN

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This work was supported in part by the European Research Council (ERC) under Grant #228044, and the Swedish Research Council.

Problem Formulation: Data Modeling

We consider a multi-static pulsed-radar with one transmitter and N_r widely separated receive antennas. The baseband transmit signal can be formulated as

$$s(t) = \sum_{n=1}^{N} a_n \phi(t - [n-1]T_P) \tag{1}$$

where $\phi(t)$ is the basic unit-energy transmit pulse, T_P is the pulse repetition period, N is the number of transmitted pulses, and $\{a_n\}_{n=1}^{N}$ are the deterministic coefficients that are to be "optimally" determined. The vector $\mathbf{a} \triangleq [a_1 \quad a_2 \quad \ldots \quad a_N]^T$ is referred to as the code vector of the radar system.

Problem Formulation: Data Modeling

- Therefore, the discrete-time signal corresponding to a certain radar cell for the k^{th} receiver can be described as:

$$\mathbf{r}_k \triangleq \mathbf{s}_k + \mathbf{c}_k + \mathbf{w}_k = \alpha_k \mathbf{a} + \widetilde{\rho}_k \mathbf{a} + \mathbf{w}_k \tag{6}$$

binary hypothesis problem

$$\begin{cases} \mathrm{H}_0 : & \mathbf{r} = \mathbf{c} + \mathbf{w} \\ \mathrm{H}_1 : & \mathbf{r} = \mathbf{s} + \mathbf{c} + \mathbf{w} \end{cases} \tag{7}$$

$$\mathbf{D}_k \triangleq (\sigma_{\mathrm{c},k}^2 \mathbf{a}\mathbf{a}^H + \mathbf{M}_k)^{-\frac{1}{2}}$$

$$\mathbf{x}_k = \mathbf{D}_k \mathbf{r}_k$$

$$\theta_k \triangleq \mathbf{a}^H \mathbf{D}_k \mathbf{x}_k / \|\mathbf{a}^H \mathbf{D}_k\|_2$$

$$\lambda_k = \sigma_k^2 \mathbf{a}^H (\sigma_{\mathrm{c},k}^2 \mathbf{a}\mathbf{a}^H + \mathbf{M}_k)^{-1} \mathbf{a}$$

detection problem

$$T = \sum_{k=1}^{N_r} \frac{\lambda_k |\theta_k|^2}{1 + \lambda_k} \underset{\mathrm{H}_1}{\overset{\mathrm{H}_0}{\lessgtr}} \eta$$

From KL-Divergence to the Design Problem

$$\mathcal{D}\left(f(\mathbf{r}|\mathrm{H}_0)\|f(\mathbf{r}|\mathrm{H}_1)\right) = \sum_{k=1}^{N_r}\{\log(1+\lambda_k) - \lambda_k/(1+\lambda_k)\}$$

$$\begin{aligned} \max_{\mathbf{a},\lambda_k} \quad & \sum_{k=1}^{N_r}\{\log(1+\lambda_k) - \lambda_k/(1+\lambda_k)\} \\ \text{subject to} \quad & \lambda_k = \sigma_k^2 \mathbf{a}^H(\sigma_{\mathrm{c},k}^2 \mathbf{a}\mathbf{a}^H + \mathbf{M}_k)^{-1}\mathbf{a} \\ & \max_{n=1,\ldots,N}\{|a_n|^2\} \leq \zeta\,(e/N) \\ & \|\mathbf{a}\|_2^2 = e, \end{aligned}$$

total transmit energy

PAR-constrained code design

$$\mathrm{PAR}(\mathbf{a}) = \max_n \{|a_n|^2\}/(\tfrac{1}{N}\|\mathbf{a}\|_2^2) \leq \zeta$$

& The MaMi!

$$\min_{\mathbf{z}} \ \widetilde{f}(\mathbf{z}) \quad \text{subject to} \quad c(\mathbf{z}) \leq 0$$

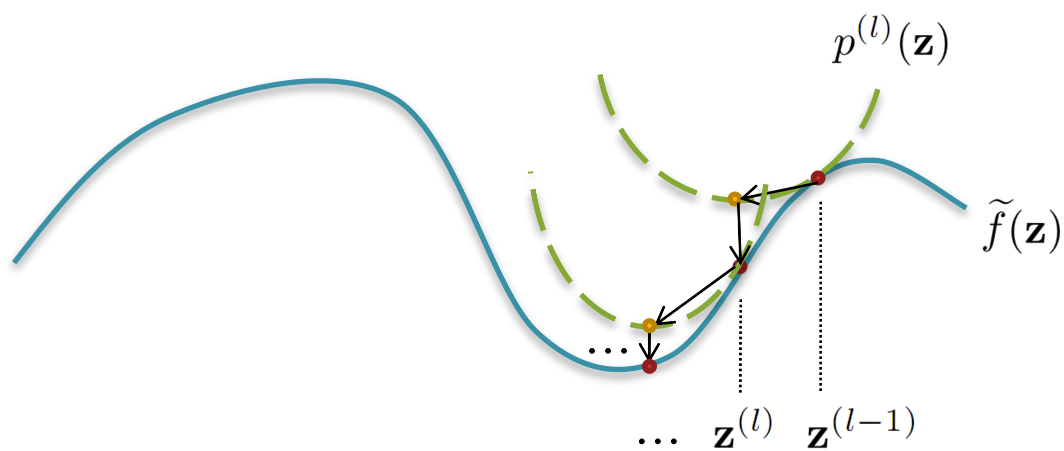
• Majorization Step: Finding $p^{(l)}(\mathbf{z})$ such that its minimization is simpler than that of $\widetilde{f}(\mathbf{z})$, and that $p^{(l)}(\mathbf{z})$ majorizes $\widetilde{f}(\mathbf{z})$, i.e.

$$p^{(l)}(\mathbf{z}) \geq \widetilde{f}(\mathbf{z}), \ \ \forall \mathbf{z} \ \text{ and } \ p^{(l)}(\mathbf{z}^{(l-1)}) = \widetilde{f}(\mathbf{z}^{(l-1)}) \tag{13}$$

with $\mathbf{z}^{(l-1)}$ being the value of $\mathbf{z}$ at the $(l-1)^{th}$ iteration.
• Minimization Step: Solving the optimization problem,

$$\min_{\mathbf{z}} \ p^{(l)}(\mathbf{z}) \quad \text{subject to} \quad c(\mathbf{z}) \leq 0 \tag{14}$$

to obtain $\mathbf{z}^{(l)}$.



The Final Form: QP@ Each Iteration.

$(l+1)^{th}$ iteration :

$$\begin{array}{lll} \min\limits_{\mathbf{a}} & \mathbf{a}^H \left(\sum_{k=1}^{N_r} \phi_k^{(l)} \mathbf{M}_k^{-1} \right) \mathbf{a} - \mathrm{Real} \left(\sum_{k=1}^{N_r} \mathbf{a}^H \mathbf{d}_k^{(l)} \right) & \\ \text{subject to} & \max\limits_{n=1,\ldots,N} \{|a_n|^2\} \leq \zeta\,(e/N) & (25) \\ & \|\mathbf{a}\|_2^2 = e & \end{array}$$

$$\mathbf{d}_k^{(l)} \triangleq (\psi_k^l / \sqrt{y_k^{(l)}}) \mathbf{M}_k^{-1} \mathbf{a}^{(l)}$$

$$\phi_k^{(l)} \triangleq \frac{\beta_k}{1+\beta_k y_k^{(l)}} + \beta_k(1+\gamma_k) + \frac{\gamma_k}{(1+\lambda_k^{(l)})^2} \left(\frac{\beta_k}{(1+\beta_k y_k^{(l)})^2} \right)$$

$$\psi_k^{(l)} \triangleq \sqrt{y_k^{(l)}} \left(\frac{2\beta_k(1+\gamma_k)}{1+\beta_k y_k^{(l)}(1+\gamma_k)} + 2\beta_k(1+\gamma_k) \right)$$

$$\gamma_k = \frac{\sigma_k^2}{\sigma_{\mathrm{c,k}}^2}$$

$$\beta_k = \sigma_{\mathrm{c,k}}^2$$

QP with PAR Constraint?

$$\begin{array}{ll} \max\limits_{\mathbf{a}} & \widehat{\mathbf{a}}^H \mathbf{K}\, \widehat{\mathbf{a}} \\ \text{subject to} & \max\limits_{n=1,\cdots,N} \{|a_n|^2\} \leq \zeta\,(e/N) \\ & \|\mathbf{a}\|_2^2 = e \end{array}$$

$$\widehat{\mathbf{a}} = [\mathbf{a} \;\; 1]^T, \mathbf{K} = \mu \mathbf{I}_{N+1} - \mathbf{J},$$

$$\mathbf{J} = \begin{bmatrix} \left(\sum_{k=1}^{N_r} \phi_k^{(l)} \mathbf{M}_k^{-1}\right) & -0.5\left(\sum_{k=1}^{N_r} \mathbf{d}_k^{(l)}\right) \\ -0.5\left(\sum_{k=1}^{N_r} \mathbf{d}_k^{(l)}\right)^H & 0 \end{bmatrix}$$

$\mu > \mu_{max} \longrightarrow$ maximum eigenvalue

Power Method-Like Iterations

The code vector $\mathbf{a}$ at the $(l+1)^{th}$ iteration of MaMi can be obtained from $\mathbf{a}^{(p)}$ (at convergence), using the power method-like iterations [11]:

$$\begin{aligned} \max_{\mathbf{a}^{(p+1)}} \quad & \|\mathbf{a}^{(p+1)} - \breve{\mathbf{a}}^{(p)}\| \\ \text{subject to} \quad & \max_{n=1,\cdots,N} \{|a_n^{(p+1)}|^2\} \leq \zeta(e/N) \\ & \|\mathbf{a}^{(p+1)}\|_2^2 = e \end{aligned} \tag{27}$$

where $\breve{\mathbf{a}}^{(p)}$ represents the vector containing the first N entries of $\mathbf{K}\ \widehat{\mathbf{a}}^{(p)}$.

Algorithm: Summary

Table 1. The MaMi Algorithm for maximizing the KL-divergence with a PAR constraint

Step 0: Initialize $\mathbf{a}$ with a random vector in $\mathbb{C}^N$ and set the iteration number l to 0.
Step 1: Solve the problem in (25) iteratively considering the nearest-vector problem in (27); set $l \leftarrow l + 1$.
Step 2: Compute $\phi_k^{(l)}$ and $\mathbf{d}_k^{(l)}$.
Step 3: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $\|\mathbf{a}^{(l+1)} - \mathbf{a}^{(l)}\|_2 \leq \xi$ for some $\xi > 0$.

Simulation Results

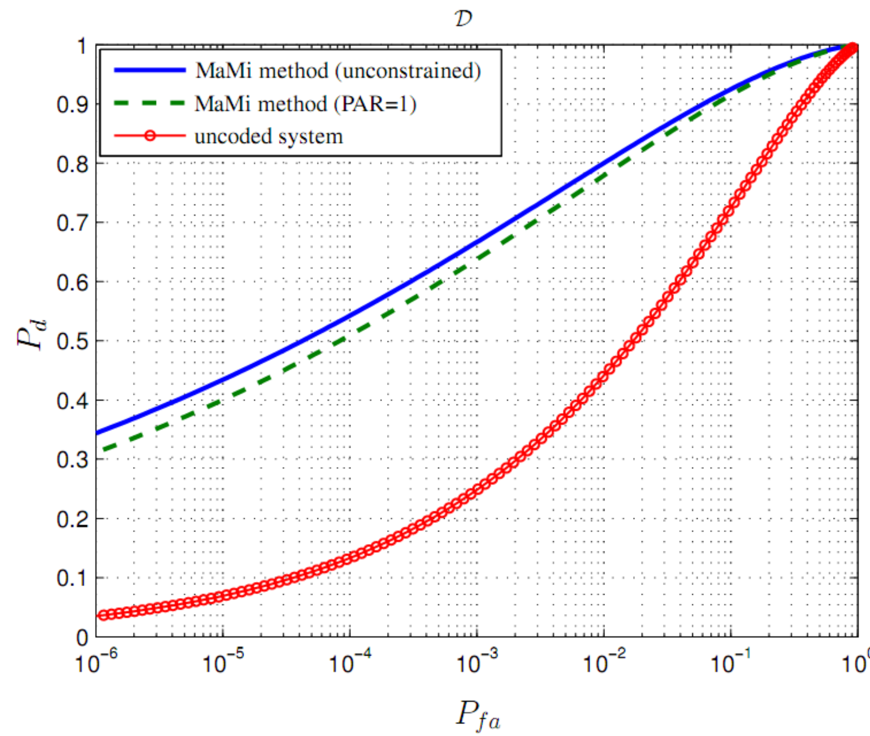


Fig. 1. ROCs of optimally coded and the uncoded systems.

Simulation Results

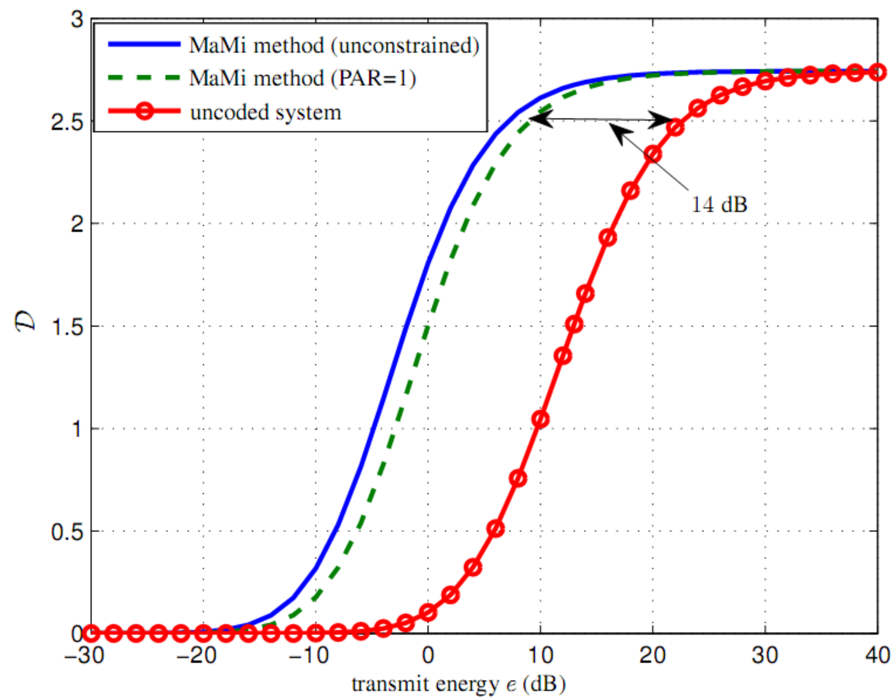


Fig. 2. Behavior of KL-divergence versus transmit energy e for the coded and uncoded systems.

Thank you for your kind attention! -Qs?