Cognitive Radar Waveform Design for Spectral Coexistence in Signal-Dependent Interference

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***Abstract*—In this paper, we deal with cognitive design of the transmit signal and receive filter optimizing the radar detection performance without affecting spectral compatibility with some licensed overlaid electromagnetic radiators. We assume that the radar is embedded in a highly reverberating environment and exploit cognition provided by Radio Environmental Map (REM), to induce spectral constraints on the radar waveform, by a dynamic environmental database, to predict the actual scattering scenario, and by an Electronic Support Measurement (ESM) system, to acquire information about hostile active jammers. At the design stage, we develop an optimization procedure which sequentially improves the Signal to Interference plus Noise Ratio (SINR). Moreover, we enforce a spectral energy constraint and a similarity constraint between the transmitted signal and a known radar waveform. At the analysis stage, we assess the effectiveness of the proposed technique to optimizing SINR while providing spectral coexistence.**

I. Introduction

The ever growing demand of both high quality wireless services and accurate remote sensing capabilities is increasing the amount of required bandwidth, making the spectral coexistence among radar and licensed radiators a primary issue [1], [2]. Several papers in the open literature have considered the problem of synthesizing radar waveforms with a suitable frequency allocation [3], [4]. Most of them do not directly account for signal-dependent interference at the design stage [5], [6], [7].

In the present paper, we consider a radar operating in a highly reverberating environment, and propose a joint design of the transmit signal and receive filter, ensuring the coexistence with overlaid licensed radiators and optimizing radar detection performance. Precisely, we assume that the radar system exploits a Radio Environmental Map (REM) [8] to get spectrum cognition about the licensed radiators, and a dynamic environmental database [9] (including for instance a geographical information system, digital terrain maps, previous radar experiences, and spectral clutter models) to predict the actual scattering scenario [10]; moreover, we suppose that the information concerning hostile jammers (unlicensed radiators) are acquired by means of an Electronic Support Measurement (ESM) system. Hence, making use of the above information, we devise a suitable radar code and receive filter for a point-like stationary target embedded in signal-dependent clutter (modeled as a collection of incoherent scatterers) and signal-independent disturbance (due to jammer, Electromagnetic Interference (EMI) and noise), considering the Signal to Interference plus Noise Ratio (SINR) as the figure of merit, and constraining the amount of interference energy produced on reserved frequency bands. We also enforce an energy constraint and a similarity constraint with a prescribed waveform, so as to control some relevant characteristics of the waveform, such as range-Doppler resolution and variations in the signal modulus.

The devised constrained optimization procedure sequentially improves the SINR. Each iteration of the algorithm requires the solution of two hidden-convex optimization problems. The resulting computational complexity is linear with the number of iterations and polynomial with the receive filter length. At the analysis stage, we assess the performance of the proposed algorithm assuming a homogeneous clutter environment, and considering a signal-independent disturbance produced by interference sources from licensed telecommunication systems and unlicensed active jammers. The results show that considerable SINR improvements (with respect to the not optimized code) can be obtained by jointly optimizing the transmitter and the receiver, while ensuring spectral coexistence with the overlaid licensed radiators.

The paper is organized as follows. In Section II, we describe the system model. In Section III, we formulate the constrained optimization problem for the design of the radar code and the receive filter, and propose a sequential optimization procedure to solve it. In Section IV, we assess the performance of the proposed algorithm. Finally, in Section V, we draw conclusions and discuss possible future research tracks.

A. Notation

We adopt the notation of using boldface for vectors $\boldsymbol{a}$ (lower case), and matrices $\boldsymbol{A}$ (upper case). The i-th element of $\boldsymbol{a}$ and the (m,l)-th entry of $\boldsymbol{A}$ are respectively denoted by $a(i)$ and $\boldsymbol{A}(m,l)$. The transpose, the conjugate, and the conjugate transpose operators are denoted by the symbols $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^\dagger$ respectively. $\mathrm{tr}(\cdot)$ and $\mathrm{rank}(\cdot)$ are the trace and the rank of the square matrix argument. $\boldsymbol{I}$ and $\boldsymbol{0}$ denote respectively the identity matrix and the matrix with zero entries (their size is determined from the context). $\mathbb{C}^N$ and $\mathbb{H}^N$ are respectively the sets of the N-dimensional vectors of complex numbers and $N \times N$ Hermitian matrices. For any $\boldsymbol{A} \in \mathbb{H}^N$, $\boldsymbol{A} \succeq \boldsymbol{0}$ means that $\boldsymbol{A}$ is a positive semidefinite matrix ($\boldsymbol{A} \succ \boldsymbol{0}$ for positive definiteness). The Euclidean norm of the vector $\boldsymbol{x}$ is denoted by $\|\boldsymbol{x}\|$. The letter j represents the imaginary unit (i.e. $j = \sqrt{-1}$). For any complex number x, $|x|$ and $\arg(x)$ represent the modulus and the argument of x. $\mathbb{E}[\cdot]$ denotes the statistical expectation. Finally, for any optimization problem $\mathcal{P}$, $v(\mathcal{P})$ represents its optimal value.

II. System Model

We consider a monostatic radar system which transmits a coherent burst of N sub-pulses. The N-dimensional column vector $\boldsymbol{v} \in \mathbb{C}^N$ of the fast-time observations, from the range-azimuth cell under test, can be expressed as $\boldsymbol{v} = \alpha_T \boldsymbol{s} + \boldsymbol{c} + \boldsymbol{n}$, with $\boldsymbol{s} \in \mathbb{C}^N$ the fast-time radar code, α_T a complex parameter accounting for the response of the target within the range-azimuth bin of interest, $\boldsymbol{c} \in \mathbb{C}^N$ the vector of clutter samples, and $\boldsymbol{n} \in \mathbb{C}^N$ the vector of the signal-independent interference samples. In particular, $\boldsymbol{n}$ accounts for both white internal thermal noise and interfering (licensed and unlicensed) radiators, and it is modeled as a complex, zero-mean, circularly symmetric Gaussian random vector, with covariance matrix $\mathbb{E}[\boldsymbol{n}\boldsymbol{n}^\dagger] = \boldsymbol{R}_{\text{ind}}$. As to $\boldsymbol{c}$, it is the superposition of the returns from the range cells adjacent that under test, namely [11]:

$$\boldsymbol{c} = \sum_{\substack{k=-N+1 \\ k \neq 0}}^{N-1} \alpha_k \boldsymbol{J}_k \boldsymbol{s}\,, \tag{1}$$

where $\boldsymbol{J}_k$, $k = \pm 1, \ldots, \pm(N-1)$ denotes the shift matrix $\boldsymbol{J}_k(l,m) = 1$ if $l - m = k$, $\boldsymbol{J}_k(l,m) = 0$ elsewhere, with $(l,m) \in \{1, \ldots, N\}^2$, and $\{\alpha_k\}_{k \neq 0}$ represent the scattering coefficients of the range cells adjacent that under test, modeled as independent complex, zero-mean, circularly symmetric, Gaussian random variables with $\mathbb{E}[|\alpha_k|^2] = \beta_k$.

As to the licensed radiators coexisting with the radar of interest, we suppose that each of them is working over a frequency band $\Omega_k = \left[f_1^k, f_2^k\right]$, $k = 1, \ldots, K$, where f_1^k and f_2^k denote the lower and upper normalized frequencies for the k-th system, respectively. To guarantee spectral compatibility with the overlayed telecommunication services, the radar has to control the amount of interfering energy produced on the shared frequency bands. From an analytical point of view, this is tantamount to forcing the following constraint [4]

$$\boldsymbol{s}^\dagger \boldsymbol{R}_I \boldsymbol{s} \leq E_I, \tag{2}$$

where E_I denotes the maximum allowed interference which can be tolerated by the coexisting telecommunication networks,

$$\boldsymbol{R}_I = \sum_{k=1}^{K} \widetilde{w}_k \boldsymbol{R}_I^k \tag{3}$$

and

$$\boldsymbol{R}_I^k(m,l) = \begin{cases} f_2^k - f_1^k & \text{if } m = l\,, \\ \dfrac{e^{j2\pi f_2^k(m-l)} - e^{j2\pi f_1^k(m-l)}}{j2\pi(m-l)} & \text{if } m \neq l, (m,l) \\ & \in \{1, \ldots, N\}^2\,. \end{cases} \tag{4}$$

Notice that, by suitably choosing the coefficients $\widetilde{w}_k \geq 0$, $k = 1, \ldots, K$, different weights can be given to the coexisting wireless networks, for instance based on their distance from the radar and their tactical importance. It is worth pointing out that radar systems can exploit a REM to get cognizance of the licensed radiators (e.g. their spatial location and bandwidth occupation), a dynamical environmental map to predict the actual reverberating scenario (i.e. β_k, $k \neq 0$) and an ESM system to obtain awareness of the hostile unlicensed radiators.

III. Code Design

In this section, we deal with the joint design of the radar transmit waveform and receive filter in order to maximize the SINR providing a control both on the interfering energy produced in the licensed bands and desirable features of the transmitted signal. Specifically, we assume that the vector of the observations $\boldsymbol{v} \in \mathbb{C}^N$ is filtered through $\boldsymbol{w} \in \mathbb{C}^N$, so that the SINR at the output of the filter can be written as:

$$\text{SINR} = \frac{|\alpha_T|^2 \left|\boldsymbol{w}^\dagger \boldsymbol{s}\right|^2}{\boldsymbol{w}^\dagger \left[\boldsymbol{\Sigma}_{\boldsymbol{c}}(\boldsymbol{s}) + \boldsymbol{R}_{\text{ind}}\right] \boldsymbol{w}}\,, \tag{5}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{c}}(\boldsymbol{s}) = \mathbb{E}\left[\boldsymbol{c}\boldsymbol{c}^\dagger\right] = \sum_{k=-N+1,\, k\neq 0}^{N-1} \beta_k \boldsymbol{J}_k \boldsymbol{s}\boldsymbol{s}^\dagger \boldsymbol{J}_k^T$.

Furthermore, to ensure coexistence among the radar and wireless telecommunication infrastructures sharing the same spectrum, we enforce the transmitted waveform to be compliant with the constraint (2). Besides, other than an energy constraint $\|\boldsymbol{s}\|^2 = 1$, we impose a similarity constraint with a unit energy reference code $\boldsymbol{s}_0$, namely $\|\boldsymbol{s} - \boldsymbol{s}_0\|^2 \leq \epsilon$ (where $\epsilon \geq 0$ rules the size of the similarity region), so as to indirectly control some desirable features of the sought radar waveform (such as its range resolution). Thus, the joint design of the radar code and the receive filter can be formulated in terms of the following constrained optimization problem:

$$\mathcal{P} \begin{cases} \max\limits_{\boldsymbol{s},\boldsymbol{w}} & \dfrac{|\alpha_T|^2 \left|\boldsymbol{w}^\dagger \boldsymbol{s}\right|^2}{\boldsymbol{w}^\dagger \left[\boldsymbol{\Sigma}_{\boldsymbol{c}}(\boldsymbol{s}) + \boldsymbol{R}_{\text{ind}}\right] \boldsymbol{w}} \\ \text{s.t.} & \|\boldsymbol{s}\|^2 = 1 \\ & \boldsymbol{s}^\dagger \boldsymbol{R}_I \boldsymbol{s} \leq E_I \\ & \|\boldsymbol{s} - \boldsymbol{s}_0\|^2 \leq \epsilon \end{cases}. \tag{6}$$

As highlighted in [4], not all the pairs (E_I, ϵ) produce a feasible problem $\mathcal{P}$. Nevertheless, the set of pairs (E_I, ϵ) associated with feasible problems, in the following referred to as I/S achievable region, can be determined with arbitrary precision (see [4] for further details).

Notice that problem $\mathcal{P}$ is a non-convex optimization problem, since the objective function is a non-convex function and the constraint $\|\boldsymbol{s}\|^2 = 1$ defines a non-convex set. Therefore, following the guidelines in [5], we aim to derive optimized solutions to $\mathcal{P}$ via a sequential maximization procedure. The idea is to iteratively improve the SINR, controlling, at the same time, the total amount of energy injected in the licensed bandwidth, as well as radar waveform features. Specifically, given $\boldsymbol{w}^{(n-1)}$, we search for an admissible radar code $\boldsymbol{s}^{(n)}$ at step n improving the SINR corresponding to the receive filter $\boldsymbol{w}^{(n-1)}$ and the transmitted signal $\boldsymbol{s}^{(n-1)}$. Whenever $\boldsymbol{s}^{(n)}$ is found, we fix it and search for the filter $\boldsymbol{w}^{(n)}$ which improves the SINR corresponding to the radar code $\boldsymbol{s}^{(n)}$ and the receive filter $\boldsymbol{w}^{(n-1)}$, and so on. Otherwise stated, $\boldsymbol{w}^{(n)}$ is used as starting point at step $n+1$. To trigger the procedure, the optimal receive filter $\boldsymbol{w}^{(0)}$, for an admissible code $\boldsymbol{s}^{(0)}$, is considered. Notice that the proposed optimization procedure requires a condition to stop the iterations; to this end, an iteration gain constraint can be forced, namely

$$|\mathrm{SINR}^{(n)} - \mathrm{SINR}^{(n-1)}| \leq \zeta \,,$$

where $\zeta \geq 0$ is the desired precision.
From an analytical point of view, $\boldsymbol{w}^{(n)}$ can be computed solving the optimization problem

$$\mathcal{P}_{\boldsymbol{w}}{}^{(n)} \left\{ \begin{array}{ll} \max\limits_{\boldsymbol{w}} & \dfrac{|\alpha_T|^2 \left|\boldsymbol{w}^\dagger \boldsymbol{s}^{(n)}\right|^2}{\boldsymbol{w}^\dagger \left[\boldsymbol{\Sigma_c}\left(\boldsymbol{s}^{(n)}\right) + \boldsymbol{R}_{\text{ind}}\right] \boldsymbol{w}} \end{array} \right. , \tag{7}$$

whose optimal solution, for any fixed $\boldsymbol{s}^{(n)}$, is given by

$$\boldsymbol{w}^{(n)} = \frac{\left[\boldsymbol{\Sigma_c}\left(\boldsymbol{s}^{(n)}\right) + \boldsymbol{R}_{\text{ind}}\right]^{-1} \boldsymbol{s}^{(n)}}{\boldsymbol{s}^{(n)\dagger}\left[\boldsymbol{\Sigma_c}\left(\boldsymbol{s}^{(n)}\right) + \boldsymbol{R}_{\text{ind}}\right]^{-1} \boldsymbol{s}^{(n)}} \,. \tag{8}$$

On the other hand, $\boldsymbol{s}^{(n)}$ is an optimal solution to the following non-convex optimization problem $\mathcal{P}_{\boldsymbol{s}}{}^{(n)}$:

$$\mathcal{P}_{\boldsymbol{s}}{}^{(n)} \left\{ \begin{array}{ll} \max\limits_{\boldsymbol{s}} & \dfrac{|\alpha_T|^2 |\boldsymbol{w}^{(n-1)\dagger}\boldsymbol{s}|^2}{\boldsymbol{w}^{(n-1)\dagger}\left[\boldsymbol{\Sigma_c}(\boldsymbol{s}) + \boldsymbol{R}_{\text{ind}}\right]\boldsymbol{w}^{(n-1)}} \\ \text{s.t.} & \|\boldsymbol{s}\|^2 = 1 \\ & \boldsymbol{s}^\dagger \boldsymbol{R}_I \boldsymbol{s} \leq E_I \\ & \|\boldsymbol{s} - \boldsymbol{s}_0\|^2 \leq \epsilon \end{array} \right. . \tag{9}$$

It is possible to show that problem $\mathcal{P}_{\boldsymbol{s}}{}^{(n)}$ is a hidden-convex optimization problem. Precisely, its optimal solution can be computed in polynomial time (resorting to the rank-one matrix decomposition theorem [12, Theorem 2.3]), starting from an optimal solution to the semidefinite programming (SDP) problem

$$\mathcal{P}_1 \left\{ \begin{array}{ll} \max\limits_{\boldsymbol{S},t} & \operatorname{tr}(\boldsymbol{Q}\boldsymbol{S}) \\ \text{s.t.} & \operatorname{tr}(\boldsymbol{M}\boldsymbol{S}) = 1 \\ & \operatorname{tr}(\boldsymbol{S}) = t \\ & \operatorname{tr}(\boldsymbol{R}_I\boldsymbol{S}) \leq tE_I \\ & \operatorname{tr}(\boldsymbol{S}_0\boldsymbol{S}) \geq t\delta_\epsilon \\ & t \geq 0\,,\ \boldsymbol{S} \succeq \boldsymbol{0} \end{array} \right. , \tag{10}$$

with t an auxiliary variable, $\boldsymbol{S}_0 = \boldsymbol{s}_0\boldsymbol{s}_0^\dagger$, $\boldsymbol{S} \in \mathbb{H}^N$, $\boldsymbol{Q} = \boldsymbol{w}^{(n-1)}\boldsymbol{w}^{(n-1)\dagger}$, $\boldsymbol{M} = \sum_{k=-N+1,\,k\neq 0}^{N-1} \beta_k\ \boldsymbol{J}_k^T\ \boldsymbol{w}^{(n-1)}\boldsymbol{w}^{(n-1)\dagger}\ \boldsymbol{J}_k + \boldsymbol{w}^{(n-1)\dagger}\boldsymbol{R}_{\text{ind}}\boldsymbol{w}^{(n-1)}\boldsymbol{I}$, and $\delta_\epsilon = (1-\epsilon/2)^2$. **Algorithm 1** summarizes the procedure leading to an optimal solution $\boldsymbol{s}^{(n)}$ of $\mathcal{P}_{\boldsymbol{s}}{}^{(n)}$.

Algorithm 1 : Algorithm for Radar Code Optimization

Input: $\boldsymbol{M}$, $\boldsymbol{Q}$, $\boldsymbol{R}_I$, $\boldsymbol{s}_0$, δ_ϵ, E_I.
Output: An optimal solution $\boldsymbol{s}^{(n)}$ to $\mathcal{P}_{\boldsymbol{s}}{}^{(n)}$.

solve SDP $\mathcal{P}_1$ finding an optimal solution $(\boldsymbol{S}^\star, t^\star)$ and the optimal value $v^\star$;
let $\boldsymbol{S}^\star := \boldsymbol{S}^\star / t^\star$;
if rank$(\boldsymbol{S}^\star) = 1$ **then**
  perform an eigen-decomposition $\boldsymbol{S}^\star = \boldsymbol{s}^\star(\boldsymbol{s}^\star)^\dagger$ and get $\boldsymbol{s}^\star$.
else
  apply the rank-one decomposition theorem [12, Theorem 2.3] to the set of matrices $(\boldsymbol{S}^\star, \boldsymbol{Q} - v^\star\boldsymbol{M}, \boldsymbol{s}_0\boldsymbol{s}_0^\dagger, \boldsymbol{I}, \boldsymbol{R}_I)$ and get $\boldsymbol{s}^\star$;
end
output $\boldsymbol{s}^{(n)} := \boldsymbol{s}^\star e^{j\arg(\boldsymbol{s}^{\star\dagger}\boldsymbol{s}_0)}$ and terminate.

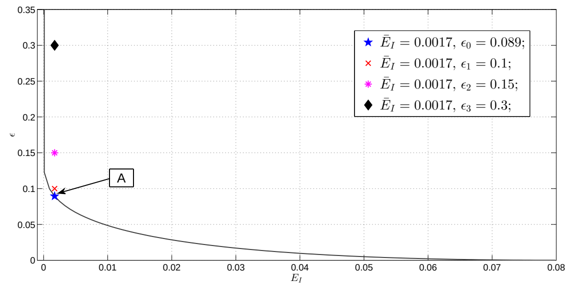

Figure 1: I/S achievable region.

As to the computational complexity of the iterative procedure, it is linear with respect to the number of iterations $\bar{N}$, whereas, in each iteration, it includes the computation of the inverse of $\boldsymbol{\Sigma_c}\left(\boldsymbol{s}^{(n)}\right) + \boldsymbol{R}_{\text{ind}}$ and the complexity effort of **Algorithm 1**. The former is in the order of $\mathcal{O}(N^3)$. The latter is connected with the complexity of SDP solution, i.e. $\mathcal{O}(N^{3.5})$.

IV. Performance Analysis

In this section, we assess the performance of the proposed waveform design technique in terms of achievable SINR, waveform spectral shape, and disturbance rejection features. We consider a radar whose baseband equivalent transmitted signal has a two-sided bandwidth of 810 KHz and it is sampled at $f_s = 810$ KHz. As to the signal-dependent interference, we assume a uniform clutter environment with $\beta_k = 8$ dB, $k = \pm 1, \ldots, \pm(N-1)$. Moreover, we model the covariance matrix of the signal-independent interference as

$$\boldsymbol{R}_{\text{ind}} = \sigma_0\boldsymbol{I} + \sum_{k=1}^{K} \frac{\sigma_{I,k}}{\Delta f_k}\boldsymbol{R}_I^k + \sum_{k=1}^{K_J} \sigma_{J,k}\boldsymbol{R}_{J,k} \,,$$

where: 1) $\sigma_0 = 0$ dB is the thermal noise level; 2) $K = 2$ is the number of licensed cooperative radiators; 3) $\sigma_{I,k}$ accounts for the energy of the k-th coexisting telecommunication network operating on the normalized frequency band $[f_2^k, f_1^k]$ ($\sigma_{I,k} = 10$ dB, $k = 1, \ldots, K$); 4) $\Delta f_k = f_2^k - f_1^k$ is the bandwidth associated with the k-th licensed radiator, for $k = 1, \ldots, K$; 5) $K_J = 2$ is the number of active and unlicensed narrowband jammers; 6) $\sigma_{J,k}$, $k = 1, \ldots, K_J$, accounts for the energy of the k-th active jammer ($\sigma_{J,1} = 25$ dB, $\sigma_{J,2} = 30$ dB); 7) $\boldsymbol{R}_{J,k} = \boldsymbol{r}_{J,k}\boldsymbol{r}_{J,k}^{\dagger}$, $k = 1, \ldots, K_J$, is the normalized disturbance covariance matrix of the k-th active unlicensed jammer, with $r_{J,k}(i) = e^{j2\pi f_{J,k} i/f_s}$, $i = 0, \ldots, N-1$, where $f_{J,k}$ denotes the frequency shift of the k-th jammer ($f_{J,1}/f_s = 0.7$, and $f_{J,2}/f_s = 0.75$). As to the overlaid and foreseen telecommunication systems, which spectrally coexist with the radar of interest, we consider the baseband equivalent radar stop-bands $\Omega_1 = [0.05, 0.08]$ and $\Omega_2 = [0.4, 0.435]$, and suppose that they have the same relevance, namely $\widetilde{w}_k = 1$, for $k = 1, 2$. Furthermore, we model the reference waveform $\boldsymbol{s}_0$ as a unit norm Linear Frequency Modulated (LFM) pulse, namely $s_0(i) = 1/\sqrt{N} e^{j2\pi K_s (i/f_s)^2}$, $i = 0, \ldots, N-1$, with a duration of 148 μsec and a chirp rate $K_s = (750 \times 10^3)/(148 \times 10^{-6})$ Hz/sec (which results in $N = 120$ samples due to the considered sampling frequency), and assume a Signal to Noise Ratio (SNR) $|\alpha_T|^2/|\sigma_0| = 10$ dB. In the exit condition, we set $\zeta = 10^{-5}$.

In **Figure 1**, we represent the I/S achievable region for the considered scenario, and highlight some possible choices for the pairs (E_I, ϵ) (referred to in the sequel as operative points). Notice that the radar designer can properly select the operative points to suitably control spectral coexistence, desirable radar features and achievable SINR of the system.

In **Figure 2a**, we plot the SINR behavior versus the number of iterations, for the operative points $(\bar{E}_I, \epsilon_p)$, $p = 1, 2, 3$, with $\bar{E}_I = 0.0017$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.3$[1]. As expected, increasing ϵ is tantamount to improving the optimal value of the SINR, since the feasible set of the optimization problem becomes larger and larger (with performance gains up to approximatively 1.6 dB). In **Figure 2b**, with reference to the same operative points of **Figure 2a**, we report the Energy Spectral Density (ESD) of the synthesized signals versus the normalized frequency, together with that of the reference code $\boldsymbol{s}_0$. The stop-bands in which the licensed systems are located, are shaded in light gray. The curves show that the proposed technique is capable to suitably control the amount of energy produced over the shared frequency bands. Additionally, increasing the similarity parameter ϵ, namely increasing the available degrees of freedom, smarter and smarter distributions of the useful energy are achieved. In fact, there is a progressive reduction of the radar emission in correspondence of the shared frequencies, as well as an enhancement of the jammer rejection capabilities. Finally, in **Table 1**, we provide the Integrated Sidelobe Level (ISL) and the Peak Sidelobe Level (PSL) for the Cross-Correlation Functions (CCFs) of the radar codes and receive filters, corresponding to the operative point $(\bar{E}_I, \epsilon_3) = (0.017, 0.3)$, for different values of the iteration number ($n = 1, 10, 30, 50$). Interestingly, the values in the table reflect the capability of the proposed joint transmit-receive optimization procedure to sequentially achieve better and better signal-dependent disturbance suppression levels.

[1]We set the starting sequence $\boldsymbol{s}^{(0)}$ as that corresponding to the operative point **A** of **Figure 1** (namely, $\bar{E}_I = 0.0017$, $\epsilon_0 = 0.089$).

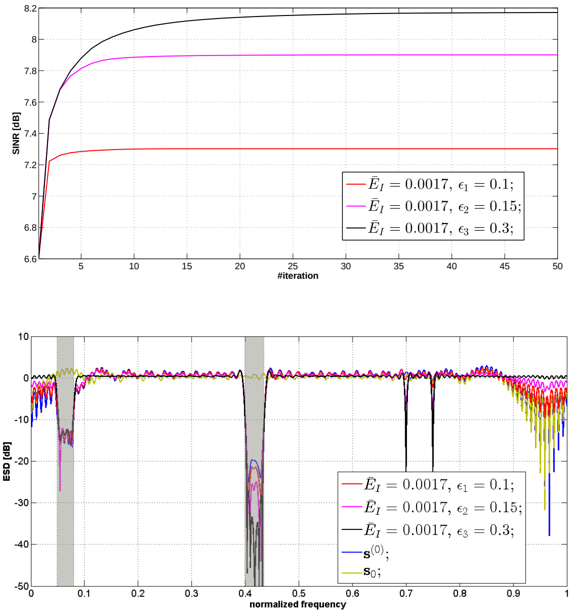

Figure 2: a) SINR; b) ESD (stop-bands shaded in light gray); brown curve: reference code $\boldsymbol{s}_0$; blue curve: starting sequence $\boldsymbol{s}^{(0)}$; red curve: $\boldsymbol{s}^{\star}$ for $\bar{E}_I = 0.0017$, $\epsilon_1 = 0.1$; magenta curve: $\boldsymbol{s}^{\star}$ for $\bar{E}_I = 0.0017$, $\epsilon_2 = 0.15$; black curve: $\boldsymbol{s}^{\star}$ for $\bar{E}_I = 0.0017$, $\epsilon_3 = 0.3$.

Table 1: ISL and PSL of the CCFs for the transmit waveform $\boldsymbol{s}^{(n)}$ and the receive filter $\boldsymbol{w}^{(n)}$, for iteration number $n = 1, 10, 30, 50$ and $(\bar{E}_I, \epsilon_3) = (0.0017, 0.3)$.

n	1	10	30	50
ISL [dB]	-7.91	-10.84	-11.24	-11.28
PSL [dB]	-19.03	-21.53	-22.02	-22.05

V. Conclusions

In this paper, focusing on a radar which operates in a highly reverberating environment, we considered the cognitive design of the transmit signal and the receive filter in a spectrally crowded environment, where some frequency bands are shared among the radar and other telecommunications systems. We exploited the cognition provided by a REM to get spectral awareness of the licensed radiators, a dynamic environmental database to predict the actual scattering scenario, and an ESM system to obtain information about active jammers. Then, we proposed an iterative procedure which sequentially improves the SINR, while controlling the interference energy produced in the licensed bands. At each step, the proposed procedure requires the solution of two hidden-convex optimization problems, with a computational complexity which is linear with the number of iterations and polynomial with the receive filter length. At the analysis stage, we assessed the performance of the devised waveform and filter in terms of achievable

SINR and spectral compatibility; furthermore, we showed the capability of the joint transmit-receive optimization procedure to achieve better and better signal-dependent interference suppression levels at each iteration step.

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