A MAX-MIN DESIGN OF TRANSMIT SEQUENCE AND RECEIVE FILTER

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ABSTRACT

In this paper, we study the joint design of Doppler robust transmit sequence and receive filter to improve the performance of an active sensing system dealing with signal-dependent interference. The signal-to-interference-plus-noise ratio (SINR) of the filter output is considered as the performance measure of the system. The design problem is cast as a max-min optimization problem to robustify the system SINR with respect to the unknown Doppler shifts of the targets. To tackle the design problem, we devise a novel method to obtain optimized pairs of transmit sequence and receive filter sharing the desired robustness property.

Keywords: Doppler shift, max-min, receive filter, robust design, transmit sequence

1. INTRODUCTION

The performance of an active sensing system can be significantly improved by judiciously designing its transmit sequence and receive filter. Such a design usually encompasses several challenges including the fact that Doppler shifts of moving targets are often unknown at the transmit side, the existence of signal-dependent interference (clutter) as well as signal-independent interference at the receive side, and practical constraints such as similarity to a given code.

Joint design of the transmit sequence and the receive filter has been considered in a large number of studies during the last decades. Most of the works have been concerned with either stationary targets or targets with known Doppler shifts (see e.g. [1–5]). In [6], considering a stationary target, a frequency domain approach has been employed to obtain an optimal receive filter and corresponding optimal energy spectral density of the transmit signal; then a synthesis procedure has been used to approximately provide the time domain signal. The work of [7] considers a related problem to that of [6] under a peak-to-average power ratio (PAR) constraint. The reference [8] deals with joint design of transmit sequence and receive filter under a similarity constraint in cases where the Doppler shift of the target is known (see also [9]). In [10], the unknown target Doppler shift has been dealt with via employing an average approach.

Several researches consider signal-independent clutter scenarios (see e.g. [11]). The unknown Doppler shift of the target has been taken into account in [11].

In this paper, we devise a novel method for Doppler robust joint design of transmit sequence and receive filter of a radar system in the presence of (signal-dependent) clutter. We consider the SINR at the output of the receive filter as the performance measure of the system. Besides an energy constraint, a similarity constraint is imposed on the transmit sequence to control certain characteristics of the transmit waveform. The design problem is cast as a max-min optimization to robustify the system performance. We devise a cyclic maximization to tackle a relaxed version of the design problem as well as a synthesis stage to obtain an optimized solution to the problem.

The rest of this paper is organized as follows. The problem formulation is presented in Section 2. Section 3 contains the steps for the derivation of the proposed method.Numerical results are provided in Section 4. Finally, conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

We consider a radar system with transmit sequence $\mathbf{x} \in \mathbb{C}^N$ and receive filter $\mathbf{w} \in \mathbb{C}^N$. The discrete-time received signal backscattered from a moving target corresponding to the range-azimuth cell under the test can be modeled as (see, e.g. [8]):

$$\mathbf{r} = \alpha_T \mathbf{x} \odot \mathbf{p}(\nu) + \mathbf{c} + \mathbf{n}, \tag{1}$$

where α_T is a complex parameter associated with backscattering effects of the target as well as propagation effects, $\mathbf{p}(\nu) = [1, e^{j\nu}, \dots, e^{j(N-1)\nu}]^T$ with ν being the normalized target Doppler shift, $\mathbf{c}$ is the N-dimensional column vector containing clutter samples, and $\mathbf{n}$ is the N-dimensional column vector of interference samples. The SINR at the output

This work was supported in part by the European Research Council (ERC) under Grant #228044 and the Swedish Research Council. *Please address all the correspondence to Mohammad Mahdi Naghsh, Phone: +983113912450; Fax: +983113912451; Email: mm_naghsh@ec.iut.ac.ir

of the receive filter can be formulated as

$$SINR(\nu) = \frac{|\alpha_T|^2 \left|\mathbf{w}^H \left(\mathbf{x} \odot \mathbf{p}(\nu)\right)\right|^2}{\mathbf{w}^H \mathbf{\Sigma_c}\left(\mathbf{x}\right)\mathbf{w} + \mathbf{w}^H \mathbf{M}\mathbf{w}} \tag{2}$$

where $\mathbf{M} \triangleq \mathrm{E}\{\mathbf{n}\mathbf{n}^H\}$ and $\mathbf{\Sigma_c}\left(\mathbf{x}\right)$ is the covariance matrix of $\mathbf{c}$.

We consider the SINR in (2) as the performance measure of the system and aim to find a robust design of the transmit sequence and the receive filter with respect to the unknown Doppler shift of the target. In addition to an energy constraint, a similarity constraint is imposed on the transmit sequence and hence the design problem can be cast as:

$$\mathcal{P}\begin{cases} \max\limits_{\mathbf{x},\mathbf{w}} \min\limits_{\nu\in\Omega} & \dfrac{\left|\mathbf{w}^H \left(\mathbf{x} \odot \mathbf{p}(\nu)\right)\right|^2}{\mathbf{w}^H \mathbf{\Sigma_c}\left(\mathbf{x}\right)\mathbf{w} + \mathbf{w}^H \mathbf{M}\mathbf{w}} \\ \text{subject to} & \|\mathbf{x}\|^2 = e \\ & \|\mathbf{x} - \mathbf{x}_0\|^2 \leq \delta \end{cases} \tag{3}$$

where $\mathbf{x}_0$ is the given code associated with the similarity constraint, $\Omega = [\nu_l, \nu_u] \subseteq [-\pi, \pi]$ denotes a given interval of the target Doppler shift ν and e denotes the maximum available transmit energy. Let $\mathbf{X} = \mathbf{x}\mathbf{x}^H$ and $\mathbf{W} = \mathbf{w}\mathbf{w}^H$. Using standard properties of the Hadamard product and Lemma 3.1 in [8], $SINR(\nu)$ can be alternatively expressed as follows (the proof is omitted due to the lack of space):

$$SINR(\nu) = \frac{|\alpha_T|^2 \mathbf{p}(\nu)^H (\mathbf{W}\odot\mathbf{X}^*)\mathbf{p}(\nu)}{\mathrm{tr}\{(\mathbf{\Sigma_c}(\mathbf{X})+\mathbf{M})\mathbf{W}\}} \tag{4}$$

$$= \frac{|\alpha_T|^2 \mathbf{p}(\nu)^H (\mathbf{W}\odot\mathbf{X}^*)\mathbf{p}(\nu)}{\mathrm{tr}\left\{\left(\mathbf{\Theta_c}(\mathbf{W})+(\frac{\beta}{e})\mathbf{I}\right)\mathbf{X}\right\}} \tag{5}$$

where $\beta = \mathrm{tr}\{\mathbf{M}\mathbf{W}\}$, and

$$\mathbf{\Sigma_c}\left(\mathbf{X}\right) = \sum_{k=0}^{N_c-1}\sum_{i=0}^{L-1} \sigma^2_{(k,i)} \mathbf{J}_k \left(\mathbf{X} \odot \mathbf{\Phi}_{\epsilon_{(k,i)}}^{\bar{\nu}_{d_{(k,i)}}}\right)\mathbf{J}_k^T,$$

$$\mathbf{\Theta_c}\left(\mathbf{W}\right) = \sum_{k=0}^{N_c-1}\sum_{i=0}^{L-1} \sigma^2_{(k,i)} \left(\mathbf{J}_k^T \mathbf{W}\mathbf{J}_k\right) \odot \left(\mathbf{\Phi}_{\epsilon_{(k,i)}}^{\bar{\nu}_{d_{(k,i)}}}\right)^*$$

with $N_c \leq N$ being the number of range rings that interfere with the range-azimuth bin of interest $(0,0)$, L is the number of discrete azimuth sectors, $\sigma^2_{(k,i)}$ is the mean interfering power associated with the clutter patch located at the $(k,i)^{th}$ range-azimuth bin whose Doppler shift is supposed to be uniformly distributed in the interval $\Omega_c = \left(\bar{\nu}_{d_{(k,i)}} - \frac{\epsilon_{(k,i)}}{2}, \bar{\nu}_{d_{(k,i)}} + \frac{\epsilon_{(k,i)}}{2}\right)$. Herein we have

$$\mathbf{\Phi}_{\epsilon_{(k,i)}}^{\bar{\nu}_{d_{(k,i)}}}(l,m) = \begin{cases} 1, & l = m \\ e^{\left(j(l-m)\bar{\nu}_{d_{(k,i)}}\right)} \frac{\sin[0.5(l-m)\epsilon_{(k,i)}]}{[0.5(l-m)\epsilon_{(k,i)}]}, & l \neq m \end{cases}$$

and $\mathbf{J}_k$ denotes the aperiodic shift matrix [8] for $0 \leq k \leq N_c - 1$.

Using (4), we cast the design problem w.r.t $(\mathbf{X}, \mathbf{W})$ and relax the rank-one constraints on these matrices. Then, we consider the following problem:

$$\mathcal{P}_1\begin{cases} \max\limits_{\mathbf{X},\mathbf{W}} \min\limits_{\nu\in\Omega} & \dfrac{\mathbf{p}(\nu)^H \left(\mathbf{W}\odot\mathbf{X}^*\right)\mathbf{p}(\nu)}{\mathrm{tr}\left\{\left(\mathbf{\Sigma_c}\left(\mathbf{X}\right)+\mathbf{M}\right)\mathbf{W}\right\}} \\ \text{subject to} & \mathrm{tr}\{\mathbf{X}\} = e \\ & \mathrm{tr}\{\mathbf{X}\mathbf{X}_0\} \geq \epsilon_\delta \\ & \mathbf{X} \succeq \mathbf{0}, \mathbf{W} \succeq \mathbf{0} \end{cases} \tag{6}$$

where $\mathbf{X}_0 = \mathbf{x}_0\mathbf{x}_0^H$ and $\epsilon_\delta = ((2e-\delta)/2)^2$. Note that optimized solutions to the design problem $\mathcal{P}$ can be obtained from optimal solutions to the above problem, see below.

3. THE PROPOSED METHOD TO TACKLE THE DESIGN PROBLEM $\mathcal{P}$

In this section, we devise a novel method to tackle the non-convex optimization problem $\mathcal{P}$. The method includes solving the relaxed problem $\mathcal{P}_1$ via a cyclic maximization approach followed by a synthesis stage.

• **Optimal X for fixed W**: Let $\widetilde{t} \in \mathbb{R}$ denote a slack variable. For fixed $\mathbf{W}$, the optimization in (6) is equivalent to the following maximization problem:

$$\mathcal{P}_X\begin{cases} \max\limits_{\mathbf{X},\widetilde{t}} & \dfrac{\widetilde{t}}{\mathrm{tr}\left\{\left(\mathbf{\Theta_c}(\mathbf{W}) + (\frac{\beta}{e})\mathbf{I}\right)\mathbf{X}\right\}} \\ \text{subject to} & \mathbf{p}(\nu)^H \left(\mathbf{W}\odot\mathbf{X}^*\right)\mathbf{p}(\nu) \geq \widetilde{t}, \ \ \forall\, \nu\in\Omega \\ & \mathrm{tr}\{\mathbf{X}\} = e \\ & \mathrm{tr}\{\mathbf{X}\mathbf{X}_0\} \geq \epsilon_\delta \\ & \mathbf{X} \succeq \mathbf{0}. \end{cases} \tag{7}$$

Note that problem $\mathcal{P}_X$ is a linear-fractional maximization problem with infinitely many constraints (see the first constraint). Inspired by Charnes-Cooper transform [13], we let $\mathbf{Y} = s\mathbf{X}$, $t = s\,\widetilde{t}$ for an auxiliary variable $s \geq 0$; it can be shown that the following problem is equivalent to $\mathcal{P}_X$:

$$\mathcal{P}'_X\begin{cases} \max\limits_{\mathbf{Y},t,s} & t \\ \text{subject to} & \mathrm{tr}\left\{\left(\mathbf{\Theta_c}(\mathbf{W}) + (\frac{\beta}{e})\mathbf{I}\right)\mathbf{Y}\right\} = 1 \\ & \mathbf{p}(\nu)^H \left(\mathbf{W}\odot\mathbf{Y}^*\right)\mathbf{p}(\nu) \geq t, \ \ \forall\, \nu\in\Omega \\ & \mathrm{tr}\{\mathbf{Y}\} = e\, s \\ & \mathrm{tr}\{\mathbf{Y}\mathbf{X}_0\} \geq \epsilon_\delta\, s \\ & \mathbf{Y} \succeq \mathbf{0}, s \geq 0. \end{cases} \tag{8}$$

To deal with the constraint set, we note that the constraint $\mathbf{p}(\nu)^H \left(\mathbf{W}\odot\mathbf{Y}^*\right)\mathbf{p}(\nu) \geq t, \ \forall\nu\in\Omega$ implies the non-negativity of a trigonometric polynomial of ν over the interval Ω. More specifically, let $z_k \triangleq \sum_{i=1}^{N-k} Z_{i+k,i}$ for $0 \leq k \leq N-1$ with $\mathbf{Z} = \mathbf{W}\odot\mathbf{Y}^*$. It is straightforward to verify that for any $\nu\in\Omega$, the aforementioned constraint is equivalent to $h(\nu) \triangleq z_0 - t + 2\Re\sum_{k=1}^{N-1} z_k e^{-jk\nu} \geq 0$.

We employ a semidefinite representation of the above constraint using Theorem 3.4 in [14] (see also [15]). In particular, let $\mathbf{z} = [z_0, z_1, \ldots, z_{N-1}]^T$; there should exist an $N\times N$ Hermitian matrix $\mathbf{Z}_1 \succeq \mathbf{0}$ and an $(N-1)\times(N-1)$ Hermitian matrix $\mathbf{Z}_2 \succeq \mathbf{0}$ such that

$$\mathbf{z} = t\mathbf{e}_1 + \mathbf{F}_1^H\left(\mathrm{diag}(\mathbf{F}_1\mathbf{Z}_1\mathbf{F}_1^H) + \mathbf{q}\odot\mathrm{diag}(\mathbf{F}_2\mathbf{Z}_2\mathbf{F}_2^H)\right)$$

where $\mathbf{q} = [q_0, q_1, \ldots, q_{n-1}]^T$ with $q_k = \cos(2\pi k/n - (\nu_l + \nu_u)/2) - \cos((\nu_u+\nu_l)/2)$, $\mathbf{F}_1 = [\mathbf{f}_0, \ldots, \mathbf{f}_{N-1}]$ and $\mathbf{F}_2 = [\mathbf{f}_0, \ldots, \mathbf{f}_{N-2}]$ in which $\mathbf{f}_k = [1, e^{-jk\theta}, \ldots, e^{-j(n-1)k\theta}]^T$

with $\theta = 2\pi/n$, and $n = 2N - 1$. Consequently, $\mathcal{P}'_X$ is equivalent to the following semidefinite program (SDP):

$$\begin{cases} \max\limits_{\mathbf{Y},\mathbf{Z}_1,\mathbf{Z}_2,t,s} \quad t \\ \text{subject to} \quad \mathrm{tr}\left\{\left(\boldsymbol{\Theta}_{\mathbf{c}}(\mathbf{W}) + (\frac{\beta}{e})\mathbf{I}\right)\mathbf{Y}\right\} = 1 \\ \mathbf{z} = t\mathbf{e}_1 + \mathbf{F}_1^H\left(\mathrm{diag}(\mathbf{F}_1\mathbf{Z}_1\mathbf{F}_1^H) + \mathbf{q}\odot\mathrm{diag}(\mathbf{F}_2\mathbf{Z}_2\mathbf{F}_2^H)\right) \\ \mathrm{tr}\{\mathbf{Y}\} = e\, s \\ \mathrm{tr}\{\mathbf{Y}\mathbf{X}_0\} \geq \epsilon_\delta\, s \\ \mathbf{Y} \succeq \mathbf{0}, \mathbf{Z}_1 \succeq \mathbf{0}.\mathbf{Z}_2 \succeq \mathbf{0}, s \geq 0 \end{cases} \tag{9}$$

Let $(\mathbf{Y}, \mathbf{Z}_1, \mathbf{Z}_2, t, s)$ denote an optimal solution to the above SDP. The corresponding optimal $\mathbf{X}$ (i.e., an optimal solution to $\mathcal{P}_X$) for fixed $\mathbf{W}$ is given by $\mathbf{Y}/s$.

• **Optimal W for fixed X**: Using (5) and techniques similar to those for fixed $\mathbf{W}$, we obtain the following SDP for fixed $\mathbf{X}$:

$$\begin{cases} \max\limits_{\mathbf{W},\mathbf{Z}'_1,\mathbf{Z}'_2,\breve{t}} \quad \breve{t} \\ \text{subject to} \quad \mathrm{tr}\left\{\left(\boldsymbol{\Sigma}_{\mathbf{c}}(\mathbf{X}) + \mathbf{M}\right)\mathbf{W}\right\} = 1 \\ \mathbf{z}' = \breve{t}\mathbf{e}_1 + \mathbf{F}_1^H\left(\mathrm{diag}(\mathbf{F}_1\mathbf{Z}'_1\mathbf{F}_1^H) + \mathbf{q}\odot\mathrm{diag}(\mathbf{F}_2\mathbf{Z}'_2\mathbf{F}_2^H)\right) \\ \mathbf{W} \succeq \mathbf{0}, \mathbf{Z}'_1 \succeq \mathbf{0}, \mathbf{Z}'_2 \succeq \mathbf{0} \end{cases} \tag{10}$$

where $\mathbf{z}'$ includes $z'_k = \sum_{i=1}^{N-k} Z'_{i+k,i}$ for $\mathbf{Z}' = \mathbf{W}\odot\mathbf{X}^*$.

• **The synthesis stage:** A judicious synthesis of the optimized transmit sequence $\mathbf{x}_\star$ and receive filter $\mathbf{w}_\star$ from the obtained $(\mathbf{W}_\star, \mathbf{X}_\star)$ (via the above cyclic algorithm) is required to maintain the Doppler robustness. If $\mathbf{W}_\star$ is rank-one, $\mathbf{w}_\star$ is available via considering $\mathbf{W}_\star = \mathbf{w}_\star\mathbf{w}_\star^H$; whereas if $\mathbf{X}_\star = \mathbf{x}\mathbf{x}^H$, for $\mathbf{x}_\star$ we have $\mathbf{x}_\star = \mathbf{x}e^{j\,\arg(\mathbf{x}^H\mathbf{x}_0)}$ [16]. In cases where the rank of either $\mathbf{W}_\star$ or $\mathbf{X}_\star$ is larger than one, the synthesis of $\mathbf{w}_\star$ or $\mathbf{x}_\star$ is more complicated. To tackle the synthesis problem, we exploit the rank-one decomposition method [17, Theorem 2.3]; precisely, a rank-one matrix $\mathbf{x}\mathbf{x}^H$ can be constructed such that

$$\mathbf{x}^H\mathbf{A}_i\mathbf{x} = \mathrm{tr}\left\{\mathbf{X}\mathbf{A}_i\right\}, \quad i = 1,2,3,4.$$

where $\mathbf{X}$ denotes a given Hermitian positive semidefinite matrix and $\{\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4\}$ are Hermitian matrices (satisfying some mild conditions).

Let $(\mathbf{W}_\star, \mathbf{X}_\star)$ denote an optimal solution to $\mathcal{P}_1$, and let

$$\nu_\star \;=\; \mathrm{argmin}_{\nu\in\Omega}\; \mathbf{p}(\nu)^H(\mathbf{W}_\star\odot\mathbf{X}_\star^*)\mathbf{p}(\nu). \tag{11}$$

Considering the above Theorem and the problem $\mathcal{P}_X$, a suitable rank-one matrix $\mathbf{x}_\star\mathbf{x}_\star^H$ can be found such that

$$\begin{cases} \mathrm{tr}\{\underbrace{(\boldsymbol{\Theta}_c(\mathbf{W}_\star) + (\beta/e)\mathbf{I})}_{\mathbf{R}_1}\mathbf{X}_\star\} = \mathbf{x}_\star^H\mathbf{R}_1\mathbf{x}_\star \\ \mathrm{tr}\{\underbrace{(\mathbf{W}_\star\odot\left((\mathbf{p}(\nu_\star)\mathbf{p}(\nu_\star)^H)^*\right)}_{\mathbf{R}_2}\mathbf{X}_\star\} = \mathbf{x}_\star^H\mathbf{R}_2\mathbf{x}_\star \\ \mathrm{tr}\{\mathbf{X}_0\mathbf{X}_\star\} = \mathbf{x}_\star^H\mathbf{X}_0\mathbf{x}_\star \\ \mathrm{tr}\{\mathbf{X}_\star\} = \mathbf{x}_\star^H\mathbf{x}_\star \end{cases} \tag{12}$$

We denote the vector obtained via above Theorem by $\mathbf{x}_\star = \mathcal{D}(\mathbf{X}_\star, \mathbf{R}_1, \mathbf{R}_2, \mathbf{X}_0, \mathbf{I})$. Similarly, an optimized receive filter $\mathbf{w}_\star$ is available via using $\mathbf{w}_\star = \mathcal{D}(\mathbf{W}_\star, \mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4)$ where

$$\begin{cases} \mathbf{Q}_1 \triangleq \boldsymbol{\Sigma}_c(\mathbf{X}_\star) + \mathbf{M} \\ \mathbf{Q}_2 \triangleq \mathbf{X}_\star\odot(\mathbf{p}(\nu_\star)\mathbf{p}(\nu_\star)^H) \\ \mathbf{Q}_3 \triangleq \mathbf{X}_\star\odot(\mathbf{p}(\nu')\mathbf{p}(\nu')^H) \\ \mathbf{Q}_4 \triangleq \mathbf{X}_\star\odot(\mathbf{p}(\nu'')\mathbf{p}(\nu'')^H) \end{cases} \tag{13}$$

Herein ν' and ν'' are two arbitrary Doppler shifts in Ω. Employing these points leads to considering the behavior of the SINR associated with the optimal solution w.r.t ν in three points and so a better synthesis (as compared to considering just $\nu_\star$).

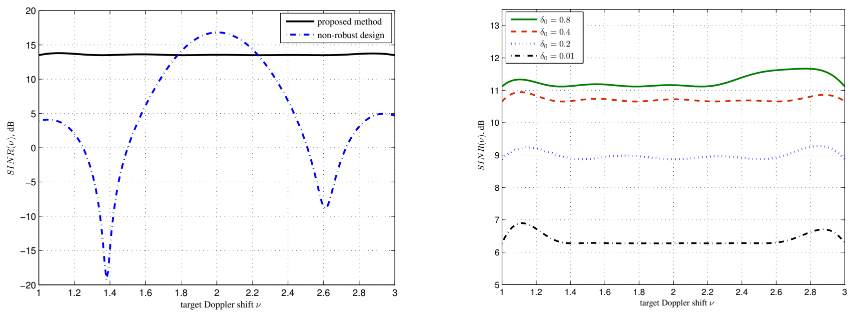
Table 1 summarizes the steps of the proposed method for max-min design of transmit sequence and receive filter. Steps 1 and 2 are related to the devised cyclic algorithm and are handled via solving the two SDPs stated in (9) and (10). Steps 4 and 5 aim to synthesize optimized pair of transmit code and receive filter using Theorem 2.3 in [17]. Herein we remark on the fact that the ranks of $\mathbf{W}_\star$ and $\mathbf{X}_\star$ depend on the employed starting point in addition to the parameters of the design problem. Therefore, for a fixed design problem, it is possible to try several random initiations and choose the best result (considering the ranks of the solutions).

Table 1. Proposed method for max-min design of transmit sequence and receive filter

Step 0: Initialize $\mathbf{X}$ with $\mathbf{x}\mathbf{x}^H$ where $\mathbf{x}$ is a random vector in $\mathbb{C}^N$.
Step 1: Solve SDP in (10) to obtain $\mathbf{W}$.
Step 2: Solve SDP in (9) to obtain $\mathbf{X}$.
Step 3: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $\|\mathbf{X}^{(\kappa+1)} - \mathbf{X}^{(\kappa)}\|_F \leq \mu$ for a given $\mu > 0$ where κ denotes the iteration number.
Step 4 (Receive filter synthesis): If $\mathbf{W}_\star$ is rank-one, perform an eigen-decomposition $\mathbf{W}_\star = \mathbf{w}_\star\mathbf{w}_\star^H$ to obtain $\mathbf{w}_\star$. Otherwise, define $\mathbf{w}_\star = \mathcal{D}(\mathbf{W}_\star, \mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4)$.
Step 5 (Transmit sequence synthesis): If $\mathbf{X}_\star$ is rank-one, perform an eigen-decomposition $\mathbf{X}_\star = \mathbf{x}\mathbf{x}^H$ to obtain $\mathbf{x}_\star = \mathbf{x}e^{j\,\arg\,(\mathbf{x}^H\mathbf{x}_0)}$. Otherwise, define $\mathbf{x}_\star = \mathcal{D}(\mathbf{X}_\star, \mathbf{R}_1, \mathbf{R}_2, \mathbf{X}_0, \mathbf{I})$.

4. NUMERICAL EXAMPLES

In this section we provide several numerical examples to examine the effectiveness of the proposed method. Throughout the simulations, we consider a sequence length $N = 20$, number of interfering range rings $N_c = 2$, and number azimuth sectors $L = 100$. The interfering signals backscattered from various azimuth sectors are weighted according to the azimuth beam-pattern characteristic of a typical linear array (see [8] for details). A uniformly distributed clutter is assumed with $\sigma^2_{(k,i)} = \sigma^2 = 100$ for all (k,i). As to the target, we set $\alpha_T = 1$. Concerning the covariance matrix



(a) (b)

Fig. 1. (a) comparison of the robust design and non-robust design for a typical example, (b) the effect of the similarity constraint.

$\mathbf{M}$ of the signal-independent interference, it is assumed that $\mathbf{M}_{m,n} = \rho^{|m-n|}$ with parameter $\rho = 0.5$. The generalized Barker code is used for sequence $\mathbf{x}_0$ [18]. The size of the similarity region is controlled by $\delta_0 = \delta/e$. The total transmit energy e is supposed to be equal to N. The convex optimization problems are solved via the CVX toolbox [19].

We investigate the robustness of the system SINR w.r.t target Doppler shift assuming $\Omega = [\nu_l, \nu_u] = [1, 3]$ and $\Omega_c = [\bar{\nu}_d - \frac{\epsilon}{2}, \bar{\nu}_d + \frac{\epsilon}{2}] = [-0.1, 0.1]$ [20]; also the result of non-robust design [8] with given Doppler shift $\nu_{given} = 0.5(\nu_l + \nu_u)$ is considered as the benchmark for the comparison. The results are plotted in Fig. 1 (a) for $\delta_0 = 0.5$. It is observed that using the proposed method leads to a robustness for the system performance; furthermore, a significant increase in the minimum value of $SINR(\nu)$ is observed as compared with that of non-robust design. Note that in this example, the ranks of the optimal $\mathbf{W}_\star$ and $\mathbf{X}_\star$ were equal to one. Examples of design with various sizes of similarity region are provided in Fig. 1 (b). In this figure, the behavior of the system SINR is shown w.r.t target Doppler shift for δ_0 in $\{0.01, 0.2, 0.4, 0.8\}$. The robustness property with respect to the target Doppler shift ν is observed in all examples. As expected, the larger the δ_0, the larger the worst value of the $SINR(\nu)$. This is due to a larger feasibility set for the optimization problem associated with obtaining $\mathbf{X}$ and the fact that the optimal $\mathbf{W}_\star$ and $\mathbf{X}_\star$ are rank-one.

Next we consider an example in which the rank of optimal $\mathbf{W}_\star$ and $\mathbf{X}_\star$ are larger than one. As discussed earlier, the rank of the solutions to the relaxed problem $\mathcal{P}_1$ depend on the employed starting point in addition to the design parameters. Note that it was numerically observed that the rank of $\mathbf{X}_\star$ is equal to one as long as $\Omega \cap \Omega_c = \varnothing$. As to the rank of $\mathbf{W}_\star$, a similar observation was made for most of the employed random starting points. Nevertheless, by setting $\Omega = [1, 2]$, $\Omega_c = [-0.25, 0.25]$, $\delta_0 = 0.1$, we find a case for which we have $\mathrm{rank}(\mathbf{W}_\star) = 2$ and $\mathrm{rank}(\mathbf{X}_\star) = 1$. The SINR associated with the optimal solutions and the employed rank-one decomposition method are illustrated in Fig. 2. For the current example, we have $\nu_\star = 1.71$ and the best result for the decomposition method is obtained with ν'=1.3 and $\nu'' = 1.5$ (via trying several pairs of (ν', ν'')). It is observed that the synthesized pair of transmit sequence and receive filter follow well the general behavior of the SINR corresponding to the optimal solutions. A minor degradation for the minimum value of the synthesized $SINR(\nu)$ is also observed due to synthesis loss.

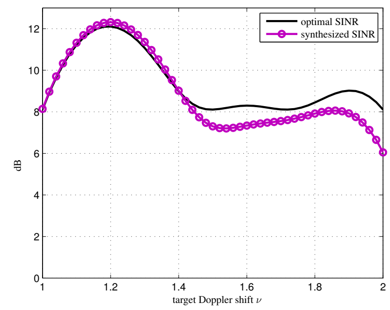


Fig. 2. An example of using the rank-one decomposition method for a case in which $\mathrm{rank}(\mathbf{W}_\star) = 2$, $\mathrm{rank}(\mathbf{X}_\star) = 1$.

5. CONCLUDING REMARKS

A joint max-min design of the transmit sequence and receive filter was considered for cases where the Doppler shift of the target is unknown. A novel method was proposed to tackle the design problem under the similarity constraint. The proposed method consists of a cyclic algorithm (to tackle a relaxed version of the design problem) along with a synthesis stage. We considered a reformulation of $SINR(\nu)$ by using $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ and $\mathbf{X} = \mathbf{x}\mathbf{x}^H$, relaxation of the rank-one constraints on the aforementioned matrices, cyclic maximization of the relaxed problem, and a synthesis stage (based on a new rank-one decomposition method). Simulation results showed that employing the proposed method leads to a considerable robustness of the system performance w.r.t the target Doppler shift.

6. REFERENCES

[1] W. D. Rummler, “A technique for improving the clutter performance of coherent pulse trains,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 3, pp. 898–906, Nov. 1967.

[2] D. F. Delong and E. M. Hofstteter, “On the design of optimum radar waveforms for clutter rejection,” *IEEE Trans. Inf. Theory*, vol. 13, pp. 454–463, Jul. 1967.

[3] B. Friedlander, “Waveform design for MIMO radars,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, pp. 1227–1238, Jul. 2007.

[4] P. Stoica, J. Li, and Y. Xie, “On probing signal design for MIMO radar,” *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4151–4161, 2007.

[5] M. M. Naghsh, M. Modarres-Hashemi, S. Shahbaz-Panahi, M. Soltanalian, and P. Stoica, “Unified optimization framework for multi-static radar code design using information-theoretic criteria,” *IEEE Trans. Signal Process.*, vol. 61, no. 21, pp. 5401–5416, 2013.

[6] S. M. Kay, “Optimal signal design for detection of Gaussian point targets in stationary Gaussian clutter/reverberation,” *IEEE J. Sel. Topics Signal Process.*, vol. 1, pp. 31–41, Jun. 2007.

[7] P. Stoica, H. He, and J. Li, “Optimization of the receive filter and transmit sequence for active sensing,” *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1730 –1740, Apr. 2012.

[8] A. Aubry, A. De Maio, A. Farina, and M. Wicks, “Knowledge-aided (potentially cognitive) transmit signal and receive filter design in signal-dependent clutter,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49, pp. 93–117, Jan. 2013.

[9] A. Aubry, A. De Maio, M. Piezzo, A. Farina, and M. Wicks, “Cognitive design of the receive filter and transmitted phase code in reverberating environment,” *Radar, Sonar Navigation, IET*, vol. 6, no. 9, pp. 822–833, 2012.

[10] M. M. Naghsh, M. Soltanalian, P. Stoica, and M. Modarres-Hashemi, “Radar code optimization for moving target detection,” in *EUSIPCO*, Marrakech, Morocco, 2013.

[11] A. De Maio, Y. Huang, M. Piezzo, S. Zhang, and A. Farina, “Design of optimized radar codes with a peak to average power ratio constraint,” *IEEE Trans. Signal Process.*, vol. 59, pp. 2683–2697, Jun. 2011.

[12] P. Stoica and Y. Selen, “Cyclic minimizers, majorization techniques, and the expectation-maximization algorithm: a refresher,” *IEEE Signal Process. Mag.*, vol. 21, no. 1, pp. 112 – 114, Jan. 2004.

[13] A. Charnes and W. Cooper, “Programming with linear fractional functionals,” *Naval Research logistics quarterly*, vol. 9, no. 3-4, pp. 181–186, 1962.

[14] T. Roh and L. Vandenberghe, “Discrete transforms, semidefinite programming, and sum-of-squares representation of nonnegative polynomials,” *SIAM Journal on Optimization*, vol. 16, no. 4, pp. 939–964, 2006.

[15] P. Stoica, T. McKelvey, and J. Mari, “MA estimation in polynomial time,” *IEEE Trans. Signal Process.*, vol. 48, pp. 1999–2012, 2000.

[16] A. De Maio, S. De Nicola, Y. Huang, S. Zhang, and A. Farina, “Code design to optimize radar detection performance under accuracy and similarity constraints,” *IEEE Trans. Signal Process.*, vol. 56, pp. 5618–5629, Nov. 2008.

[17] W. Ai, Y. Huang, and S. Zhang, “New results on Hermitian matrix rank-one decomposition,” *Mathematical programming*, vol. 128, no. 1, pp. 253–283, 2011.

[18] M. Friese, “Polyphase Barker sequences up to length 36,” *IEEE Trans. Inf. Theory*, vol. 42, no. 4, pp. 1248–1250, 1996.

[19] M. Grant and S. Boyd. (2012, February) CVX package. [Online]. Available: http://www.cvxr.com/cvx

[20] M. Skolnik, *Radar Handbook*, 3rd ed. New York: McGraw-Hill, 2008.