A Practical Sparse Channel Estimation for Current OFDM Standards

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***Abstract*—Wireless channels especially for OFDM transmissions can be precisely approximated by a time varying filter with sparse taps (in the time domain). Sparsity of the channel is a criterion which can highly improve the channel estimation task in mobile applications. In sparse signal processing, many efficient algorithms have been developed for finding the sparsest solution to linear equations (Basis Pursuit, Matching Pursuit) in the presence of noise. In current OFDM standards, a number of the ending subcarriers at both positive and negative frequencies are left unoccupied (for ease of analog filtering at the receiver) which results in an ill-conditioned frequency to time transformation matrix. This means that the initial estimate for the impulse response of the channel (in time) easily varies as the noise vector changes. Thus in this case we cannot use most of the proposed algorithms in sparse signal processing. In this paper, we propose iteration with adaptive thresholding and MMSE methods to overcome this difficulty. Simulation results indicate that the proposed method is almost perfect for stationary channels and only minor performance degradation is observed with increase of Doppler frequency.**

I. Introduction

Owing to the fact that OFDM transmission is robust against multipath fading, most of the current digital wireless video services such as DVB (-T, -H, -T2) [1], ISDB-T [2], T-DMB [3] and etc. are based on this modulation. Insertion of the data at orthogonal subcarriers, in addition to a long enough cyclic prefix, enables the receiver to decode different parts of the transmitted data (different subcarriers) almost independently. Therefore, unwanted frequency notches caused by the channel discards only a fraction of the data. On the other hand, accurate decoding of the data requires compensation for the channel distortion which in turn requires an approximate of the Channel Frequency Response (CFR).

The basis of all OFDM channel estimators are the noisy samples of the CFR at *pilot* tones. *Pilot* tones are a number of subcariers which are known a priory at the receiver; in other words, the transmitter sends a known pattern at a number of predefined subcarriers in order to help the channel estimation block of the receiver. For mobile purposes where the channel may vary quickly, pilot tones are distributed in all OFDM symbols to catch up with rapid changes of the channel. Usually the location of the pilots are not exactly the same in consecutive OFDM symbols which offers a wider range of channel frequency samples (but at different time instants).

The samples obtained at pilots are further processed to estimate the channel response at non-pilot subcarriers and then the data are equalized using these estimates. The traditional method for estimation of the channel at non-pilot subcarriers is interpolation between the samples at pilot locations. According to the continuous and lowpass nature of the channel frequency response, a wide range of interpolating methods starting from the simple Linear Interpolation (LI) up to more complicated interpolants such as splines are introduced [4]. Since the time spread of the channel is assumed to be less than the cyclic length, further denoising of the interpolated estimate is achieved by discarding the nonzero time samples outside the cyclic interval. A more general approach is to consider the time varying channel as a 2-D object (Time-Frequency) which resembles a 2-D lowpass signal under limited time spread and Doppler frequency conditions [5]. The channel estimation is thus equivalent to the reconstruction of a 2-D lowpass signal from its irregular samples (values obtained at pilots at different OFDM symbol).

After recent results in sparse signal processing including Sparse Component Analysis (SCA) [6] and Compressed Sensing [7], there has been a trend to exploit the sparsity of the Channel Impulse Response (CIR) in its estimation [8], [9], [10]. In [8] reduction of the number of required pilots for an estimation method based on one of the l_1 minimization algorithms (Basis Pursuit summarized in [11]) is devised while in [10] another method based on the Matching Pursuit algorithm is introduced.

A point which is often ignored in these techniques is that, according to current OFDM standards, the bandwidth is not fully occupied; i.e., a number of the subcarriers at both edges of the bandwidth are set to zero to increase the allowable transition band of the analog bandpass filter at the receiver. Although the ending zero subcarriers reduce the complexity of the receiver, they eliminate the possibility to have samples of the CFR at these locations. Therefore, interpolation at these parts are impossible and the frequency response can not be stably translated to the sparse time taps (the coefficient matrix relating the time domain signal to the pilot sub-carriers is ill-conditioned due to the zero subcarriers). The drawback of the previous sparsity-based methods is that they require an almost accurate initial state for convergence which can not be

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guaranteed with the mentioned zero padding at the edges (ill-conditioned frequency to time transformation matrix). In this paper we propose a sparsity-based estimation method which works for OFDM transmissions even with zero padding. Simulation results confirm the accuracy of the proposed method for time invariant channels. Moreover, the performance of the proposed method shows little degradation with increase of the Doppler frequency.

II. Problem Statement

So as to clearly describe the previously mentioned challenge in channel estimation, we explain the issue in three subsections. In II-A the main OFDM components related to the channel estimation are briefly described. In II-B, OFDM channel estimation is restated as a sparse signal processing problem and finally in II-C we will point out the difficulty of using time domain sparsity in conjunction with partial use of the bandwidth.

A. OFDM System Components

In OFDM transmission, coded binary data, after interleaving are grouped and mapped to constellation points based on a specific modulation scheme. In the next step, these complex symbols are divided into blocks and for each block a number of pilots are inserted among the data at predefined locations. The resulting blocks which have less samples than the required block size is zero padded to form the final blocks (OFDM symbols) which we denote in vector form by $\mathbf{X}^{(n)}$ (n is the block number). These blocks are converted into the time domain ($\mathbf{x}^{(n)}$) by means of IFFT (Inverse Fast Fourier Transform) and each block (in time) is extended by a Cyclic Prefix (CP) containing a copy of the last samples; these extended blocks are serially transmitted. The role of CP is to avoid Inter Symbol Interference (ISI) between the adjacent OFDM symbols (after passing through the multipath channel).

The channel which affects the transmitted signal is often modeled as a linear time-varying multi-path channel with Additive White Gaussian Noise (AWGN). The channel (h) at time t for an impulse sent to the channel τ seconds ago and its 2D Fourier transform (H) can be expressed as:

$$\begin{aligned} h(t,\tau) &= \sum_{l=0}^{L-1} \alpha_l(t)\delta(t-\tau_l) \\ H(t,f) &= \int_{-\infty}^{+\infty} h(t,\tau)e^{-j2\pi f\tau}d\tau \end{aligned} \tag{1}$$

where L is the number of paths, α_l is the lth complex path gain, and τ_l is the corresponding path delay.

In the case of proper cyclic prefixing and perfect timing, it can be shown that the digital channel affecting the data in the frequency domain is given by:

$$H[n,k] \triangleq H(nT_f, k\Delta f) = \sum_{l=0}^{L-1} h[n,L]e^{-\frac{j2\pi kl}{K}} \tag{2}$$

where T_f is the symbol length which also includes CP, Δf is the subcarrier spacing, $T_s = \frac{1}{\Delta f}$ is the sample interval and K is the length of the OFDM symbols. At the receiver, after removing the guard interval in the time domain and demultiplexing the time samples ($y(n)$) into parallel OFDM symbols, the data are converted into the frequency domain ($Y(n)$) by means of FFT. If we denote the sampled vector of the AWGN noise in the frequency domain by W and we assume no interference, the equation relating the n^{th} transmitted and received OFDM symbols, respectively $X(n)$ and $Y(n)$ is:

$$Y(n) = X(n) \odot H(n) + W(n) \tag{3}$$

where $\odot$ represents element by element multiplication of the two vectors. As a result, the time varying fading channel is similar to a 2-D discrete signal defined at the integer lattice of the time-frequency plane. Due to the existence of the pilots, certain noisy samples of this lattice are known and the goal of the channel estimation is to estimate the rest via interpolation. The estimated channel is used in the equalization block to obtain an approximation of the transmitted OFDM symbol, $X(n)$. This block is usually followed by demapping, deinterleaving and decoding of the data.

B. Restating OFDM Channel Estimation as a Sparse Problem

As stated earlier, the goal of the channel estimation process is to obtain the channel coefficients in the frequency domain using noisy values of the channel at pilot positions. Due to the sparse distribution of the scattering objects, the equivalent discrete OFDM channel is sparse in the time domain. Taking sparsity into consideration, we look for a time domain sparse channel (h) whose frequency samples are known:

$$H_{k_p} = F_{k_p} \cdot h + W_{k_p} \tag{4}$$

where k_p is the index vector representing the pilot positions in the channel frequency spectrum, H_{k_p} is a vector containing the value of the channel frequency spectrum at these pilot subcarriers and F_{k_p} denotes the reduced DFT matrix by keeping the rows pertaining to the pilot positions. Also $W(k_p)$ denotes the additional noise on the pilots in the frequency domain. As mentioned earlier, in a well designed OFDM system, the length of the cyclic prefix is larger than the channel time spread; thus, the only non-zero values of the vector h reside in the first N_{CP} samples where N_{CP} is the length of the cyclic prefix. Using this fact, the equation can be simplified to:

$$H_{k_p} = F_{k_p,\, N_{CP}} \cdot h_{CP} + W_{k_p} \tag{5}$$

where $F_{k_p,N_{CP}}$ is obtained from the matrix F_{k_p} by keeping the first N_{CP} columns and the sparse vector h_{CP} is the first N_{CP} points of h. So in this sense the channel estimation problem is equivalent to finding the sparse vector h_{CP} from the above equation. In the next subsection we present a major difficulty in using time-domain sparsity, the partial use of the bandwidth, an issue that most of the authors neglected.

C. Sparsity with partial Use of Bandwidth

Conventional channel estimation methods are not able to exploit the inherent sparsity of the channel in the time domain. Very recently the idea of using time-domain sparsity in OFDM channel estimation has been proposed in [8]. Exploiting the sparsity has many advantages; it increases the system performance, and can be used to decrease the number of pilots, thus decreasing overhead and increasing bandwidth efficiency. In [8] the authors proposed the idea of using compressive sensing methods in OFDM channel estimation and proved that the OFDM channel estimation problem as stated, satisfies the uniform uncertainty principle described in [7], and thus linear programming-based algorithms similar to the ones used in [11] can be employed for channel estimation. Simulation results show that this method works effectively even in quickly changing channels. However, the authors of [8] did not consider zero padding at the endpoints of the bandwidth in their scenario, which is an essential part of the current standards based on OFDM transmission. This assumption, which we hereafter name partial use of bandwidth, causes the matrix $F_{k_p,N_{CP}}$ defined in subsection II-B to contradict the RIP (Restricted Isometry Property) defined in [7] (Since the FFT matrix and its submatrices are Vandermond it is easy to evaluate their determinant; the submatrices which we are dealing with are close to singularity). This makes the use of compressive sensing algorithms described in [8] impractical. The problem that zero padding poses is that the $F_{k_p,N_{CP}}$ matrix is ill-conditioned in the presence of the zero padding block. Also due to this block we do not have any pilots in the zero padded parts, complicating the use of time-domain techniques. We propose a method that exploits this inherent sparsity and also solves the zero padding problem, which is briefly described in the next section.

III. Proposed Channel Estimation Method

As mentioned in section II, the time domain OFDM channel (CIR) is sparse. In this section we will propose a new channel estimation scheme that exploits this inherent sparsity.

Similar to other sparsity-based estimators, our method is based on an iterative technique which improves the estimates in each step starting at an initial value. However, the sensitivity of our algorithm to this initial state is not restricting; better initial values result in faster convergence. Since the samples of the channel are in the frequency domain and the sparsity criterion is valid in the time domain, we shall switch between the two domains to benefit from both sets of information.

To save the computational capacity for the iterations, we use a simple initial state; i.e., we begin by the spectrum of the estimated channel at the previous OFDM symbol as the initial value. At the start of the reception when there is no previous estimate, we begin by the linear interpolated version (linear interpolation between the samples taken at pilot subcarriers). As stated in section II-C, the linear interpolation is not possible at the zero-padded end points; thus, we leave these parts as zero, which means we have rejected the highpass coefficients of CIR.

Up to this point we have not employed the samples obtained at pilot locations which are considered as the most confident set of available data. The simple way to use them is to replace the estimated values at pilot locations with the obtained samples.

Now it is turn to consider the time sparsity criterion. For this aim, we should convert the estimated spectrum into the time samples by means of the IFFT operation. If we had the exact spectrum, the signal after the IFFT would have been sparse; however, rejection of the highpass coefficients spreads each original nonzero sample over a range of the neighboring samples and existence of the additive noise changes the original zero samples into arbitrary nonzero values. Therefore, the initial sparsity criterion is no longer valid. With use of an adaptive thresholding method on the current time samples which we call *MAT* (Modified Adaptive Thresholding) we will find the most likely combination of the nonzero locations which form a sparse signal (will be described in the following subsection). It should be emphasized that the output of MAT is only the location of the nonzero samples, not their values.

The last step in each iteration is to estimate the value of the locations reported by MAT. In fact, Since the values of the time samples before MAT are not reliable, we only employ them for finding the nonzero locations and we again estimate their values using the MMSE method. The employed MMSE which is based on the matrix relating the pilot values to the time sample values of the MAT-reported locations will be discussed in III-B. After finding the values, the time samples are again converted to the frequency domain (FFT) and the next iteration will be started; i.e., the samples at pilot values are replaced with their respective estimated values and so on. The stepwise algorithm in each iteration is briefly shown below:

1) Replace the estimated values (results of the previous iteration or the initial estimate) at pilot locations by the samples obtained from the received data and convert the spectrum into the time domain (IFFT).
2) Locate channel tap positions using MAT based on the into-time-transformed estimated spectrum.
3) Estimate the corresponding tap values using MMSE method.

For the stopping condition of the iterations we use the Power Ratio (PR) measure defined as the power of the estimated channel divided by the power of the actual channel:

$$PR = \frac{Estimated\ \ Channel\ \ Power}{Actual\ \ Channel\ \ Power} \tag{6}$$

The receiver is aware of the actual channel power by calculating the power of the to-be-equalized data (power of the equalized data is the same as the average power of the constellation). The iterations should continue until PR exceeds $1-\epsilon$ or a predefined maximum iteration number is reached; ϵ is a small positive real determining the trade-off between the accuracy and complexity.

The block diagram of the method is depicted in Fig. 1.

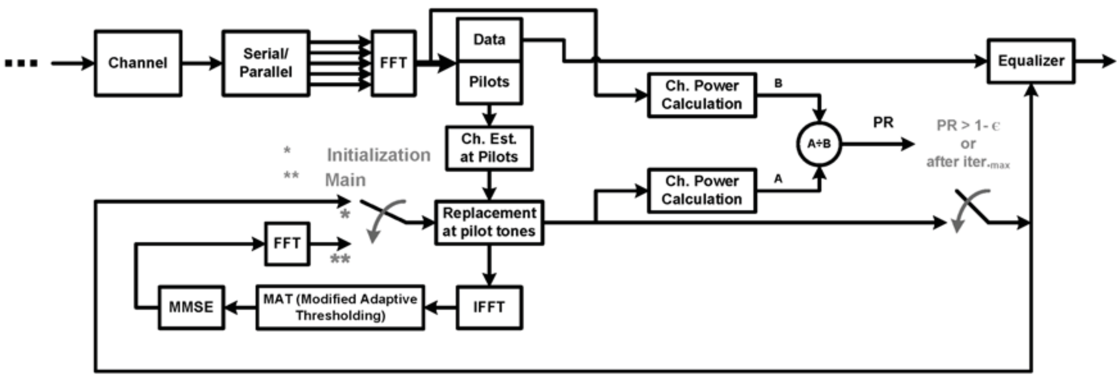

Fig. 1. Overall block diagram of the proposed estimation method

A. Channel Tap Detection Using MAT

A sparse channel is characterized by the number, position and value of its taps. MAT is a method that by means of thresholding, decides for the number and the position of the taps. It is composed of three successive thresholds (all thresholds in this paper will be shown by η with proper indices) with the main based on the CML-CFAR (Censored Mean Level Constant False Alarm Rate). In [12] a CFAR-based method for impulse noise cancellation is proposed which we have modified it here to suit the estimation problem (that explains the word "Modified" in the name MAT).

In the employed CFAR algorithm, amplitude of each sample is compared (hard decision) to an average of the neighboring amplitudes (which ideally represents the standard deviation of the noise) in order to decide whether it is a tap or just a noisy sample. Since the neighboring samples may include a tap, among the considered l samples, only the least m amplitudes are averaged to avoid including the tap amplitudes (practical choices are $l = 6$ and $m = 3$):

$$\eta_{CFAR}[i] = \frac{\big|h[i_1]\big| + \cdots + \big|h[i_m]\big|}{m} \tag{7}$$

where

$$\big\{h[i_1] \leq \cdots \leq h[i_l]\big\} = \big\{h[i+k]\big\}_{k=-\frac{l}{2}}^{k=\frac{l}{2}} - \big\{h[i]\big\} \tag{8}$$

Since for each sample we require a CFAR threshold and each threshold is formed by an average, it is reasonable to exclude unlikely samples prior to CFAR so as to both decrease the computational complexity and increase the detection probability. Therefore, we discard the samples with amplitudes less than the following threshold prior to CFAR:

$$\eta_{initial} = \beta e^{-\alpha i} \tag{9}$$

where i represents the iteration number; i.e., the threshold exponentially decreases as the iterations proceed. α and β are constants that depend on the number of taps and initial powers of noise and channel taps; since these parameters are not initially known, a rough estimate shall be used. The above choice of the threshold will be justified in section IV.

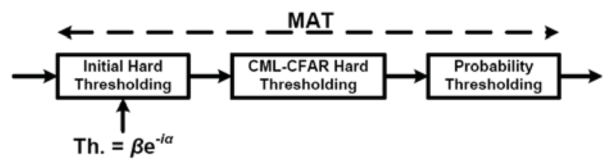

Fig. 2. The block diagram of MAT

In the third and the final thresholding (the first two are the above initial and the followed CFAR thresholds), it is calculated that in what fraction of the previous iterations a location is detected as a tap after CFAR. The locations with the detection probability less than 0.3 are again discarded; 0.3 is a number found through simulations. The importance of this probability thresholding is to prevent the possible oscillations in the decision upon a location. The locations with probability greater than 0.3 are reported to MMSE estimator as the detected tap locations in this iteration.

The block diagram of the MAT showing the three thresholding schemes is depicted in Fig. 2

B. Value Estimation using MMSE

In the problem at hand we wish to solve a linear equation in the presence of noise. That is, we wish to obtain the value of the CIR vector $\mathbf{h}$ at set of tap positions T reported by MAT, from the equation

$$\widetilde{\mathbf{H}}_{k_p} = \mathbf{F}_{k_p,T}\mathbf{h}_T + \mathbf{W}_{k_p} \tag{10}$$

where the vector $\widetilde{\mathbf{H}}_{k_p}$ is the measured CFR vector at pilot positions, $\mathbf{F}_{k_p,T}$ is obtained from the DFT matrix by selecting the rows that pertain to pilot positions (k_p) and the columns that pertain to actual channel taps T and $\mathbf{W}_{k_p}$ is the noise vector at pilot positions. The MMSE estimator tries to estimate the vector $\mathbf{h}_T$ by minimizing $E\{\|\mathbf{h}_T - \hat{\mathbf{h}}_T\|_2\}$, hence the name Minimum Mean Squared Error. This estimate vector is given by:

$$\hat{\mathbf{h}}_T = \mathbf{R}_{\mathbf{h}_T}\mathbf{F}_{k_p,T}^H\big(\mathbf{F}_{k_p,T}\mathbf{R}_{\mathbf{h}_T}\mathbf{F}_{k_p,T}^H + \mathbf{R}_{\mathbf{W}}\big)^{-1}\widetilde{\mathbf{H}}_{k_p} \tag{11}$$

where $.^H$ denotes the Hermitian operation and $\mathbf{R}_{\mathbf{h}_T}$ and $\mathbf{R}_{\mathbf{W}}$ are the auto-covariance matrices of $\mathbf{h}_T$ and $\mathbf{W}_{k_p}$ respectively.

When the noise vector $\mathbf{W}_{k_p}$ is a complex zero-mean random white Gaussian process, $\mathbf{R_W}$ is equal to $2\sigma_W^2\mathbf{I}$, where the variance of both real and imaginary parts are assumed to be $\sigma_{\mathbf{W}}^2$. Also $\mathbf{R}_{\mathbf{h}_T}$ can be estimated with $P_{\mathbf{h}}\mathbf{I}$ where $P_{\mathbf{h}}$ is the average power of the channel (it can be obtained as mentioned earlier in this section). Thus the estimate vector can be written as:

$$\hat{\mathbf{h}}_T = \mathbf{F}_{k_p,T}^H(\mathbf{F}_{k_p,T}\mathbf{F}_{k_p,T}^H + \frac{2\sigma_W^2}{P_{\mathbf{h}}}\mathbf{I})^{-1}\widetilde{\mathbf{H}}_{k_p} \tag{12}$$

IV. Mathematical Analysis

For simplicity of the analysis, let us assume that both channel taps and the additive noise in the time domain at the i^{th} iteration are zero-mean normal complex random variables with variance of σ_{tap}^2 and $\sigma_{n,i}^2$ (for each of the real and imaginary parts), respectively. Hence, their amplitudes which are of main concern in MAT follow the Rayleigh distribution. Moreover, we assume that the probability of a time sample to be a channel tap is p_{tap}; i.e., if N_{CP} represents the length of the cyclic prefix, we expect to have a channel with $p_{tap}\cdot N_{CP}$ taps. As shown in [12], the best hard threshold (η_{opt}) with respect to the error probability which distinguishes the taps and the solely noise samples is given by:

$$\eta_{opt} = \sigma_{n,i}\cdot\sqrt{2\frac{1+SNR_i}{SNR_i}\ln\big(\frac{1-p_{tap}}{p_{tap}}(1+SNR_i)\big)} \tag{13}$$

where $SNR_i = \frac{\sigma_{tap}^2}{\sigma_{n,i}^2}$. If we define $\bar{\eta}_{opt,i}^2 = \frac{\eta_{opt}^2}{2\sigma_{n,i}^2}$, the power of the noise samples detected as tap ($\Sigma_{tap|n}$) and the power of the undetected taps ($\Sigma_{n|tap}$) in the i^{th} iteration will be:

$$\begin{cases} \Sigma_{tap|n} = \sigma_{n,i}^2\big(1+\bar{\eta}_{opt,i}^2\big)e^{-\bar{\eta}_{opt,i}^2} \\ \Sigma_{n|tap} = \sigma_{tap}^2\Bigg(1-\big(1+\frac{\bar{\eta}_{opt,i}^2}{1+SNR_i}\big)e^{-\frac{\bar{\eta}_{opt,i}^2}{1+SNR_i}}\Bigg) \end{cases} \tag{14}$$

For good initial states and converging conditions, SNR_i is large enough to use the following approximations:

$$\begin{cases} \bar{\eta}_{opt,i}^2 = \frac{\eta_{opt}^2}{2\sigma_{n,i}^2} \approx \ln\big(\frac{1-p_{tap}}{p_{tap}}\big) + \ln(SNR_i) \\ \Sigma_{tap|n} \approx \sigma_{n,i}^2\big(1+\ln\big(\frac{1-p_{tap}}{p_{tap}}SNR_i\big)\big)\frac{p_{tap}}{(1-p_{tap})\cdot SNR_i} \\ \Sigma_{n|tap} \approx \sigma_{tap}^2\frac{\ln^2\big(\frac{1-p_{tap}}{p_{tap}}SNR_i\big)}{2SNR_i^2} \end{cases} \tag{15}$$

After thresholding, some of the taps are not detected and some of the noise samples are detected as tap. Although we have assumed i.i.d. distribution for the time samples, due to rejection of the highpass coefficients, neighboring samples are correlated; i.e., if the amplitude of a sample stays above the threshold, the amplitudes of the neighboring samples are likely to do so. However, CFAR thresholding keeps almost one of these neighboring samples; hence, after CFAR the i.i.d. condition is almost satisfied. We model the first two thresholding blocks (initial and CFAR) as a simple thresholding for an i.i.d. input. Thus, the noise variance of the next iteration can be approximated by:

$$\sigma_{n,i+1}^2 = p_{tap}\cdot\Sigma_{n|tap} + (1-p_{tap})\cdot\Sigma_{tap|n} \tag{16}$$

TABLE I
SIMULATION PARAMETERS

Parameter	Specifications
DVB-H mode	2K
Number of carriers	1705
OFDM symbol duration	$224\mu s$
Gaurd Interval	$1/8$ (256)
Signal Constellation	16QAM
Channel Model	Brazil Channel D
$iter_{max}$	8
(ϵ , α , β)	(0.05 , 0.1 , 0.5)

TABLE II
MULTI-PATH PROFILE (BRAZIL CHANNEL D)

Delay (μs)	Amplitude (dB)
0.0	-0.1
$+0.48$	-3.9
$+2.07$	-2.6
$+2.90$	-1.3
$+5.71$	0.0
$+5.78$	-2.8

therefore:

$$\begin{aligned} \frac{\sigma_{n,i+1}^2}{\sigma_{n,i}^2} &\approx \frac{p_{tap}}{SNR_i}\Bigg(1+\ln\big(\frac{1-p_{tap}}{p_{tap}}SNR_i\big) \\ &\qquad + 0.5\ln^2\big(\frac{1-p_{tap}}{p_{tap}}SNR_i\big)\Bigg) \\ &\lessapprox \frac{p_{tap}}{SNR_i}e^{\ln\big(\frac{1-p_{tap}}{p_{tap}}SNR_i\big)} = 1-p_{tap} \end{aligned} \tag{17}$$

which means:

$$\begin{cases} \sigma_{n,i}^2 \approx \sigma_{n,0}^2\cdot(1-p_{tap})^i \\ SNR_i \approx SNR_0\cdot(1-p_{tap})^{-i} \end{cases} \tag{18}$$

The last approximation, reveals the linear change of $\bar{\eta}_{opt,i}^2$ with respect to the iteration number:

$$\bar{\eta}_{opt,i}^2 \approx \ln\big(\frac{1-p_{tap}}{p_{tap}}\big) + \ln(SNR_0) - i\ln(1-p_{tap}) \tag{19}$$

As shown in (18), $\sigma_{n,i}$ exponentially decreases with respect to the iteration number, while (19) shows linear increase of $\bar{\eta}_{opt,i}^2$; consequently, the overall effect on η_{opt} would be the exponential decrease with respect to the iteration number (i) which justifies the choice of $\eta_{initial}$ in (9).

V. Simulation Results

We performed computer simulations using MATLAB to verify the performance of the proposed method applied to the DVB-H standard. Parameters of the system used in the simulations are shown in table I. Since in this paper we only focus on the channel estimation, we have ignored other issues such as synchronization or frequency offset. As shown in table I, the length of the considered cyclic prefix exceeds the time spread of the applied channel ("Brazil D"channel with the profile as shown in table II) which means we have no ISI in the received symbols.

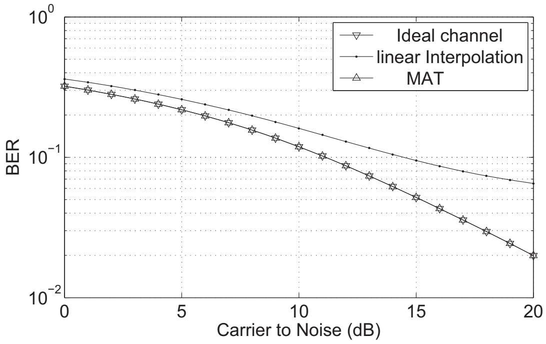

Fig. 3. Performance of the ideal, linear-interpolated and the proposed channel estimates under time-invariant Brazil D channel.

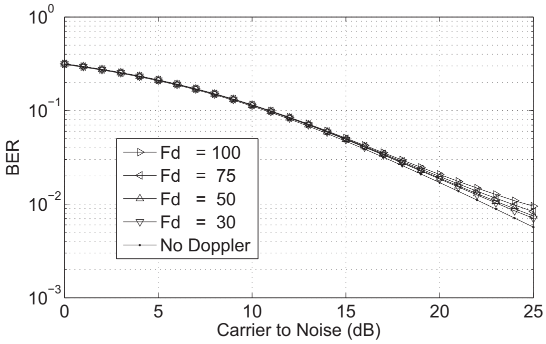

Fig. 4. Performance of the proposed method at different Doppler frequencies for Brazil D channel.

The performance of the proposed channel estimation measured as Bit Error Rate (BER) is shown in Fig. 3 for the time-invariant Brazil D channel. For the purpose of comparison, we have also used the ideal channel (which is just a hypothetical upperbound for the estimation in simulations) for MMSE equalization of the data. As can be seen in Fig. 3, the BER obtained using MAT coincides with that of the ideal channel; we should mention that an AR time averaging on the estimated channel is also applied (excluding this time averaging we expect deviation from the ideal channel at low SNRs). That is if we used the ideal channel for equalization rather than the proposed scheme we would get the same BER. In this sense the proposed channel estimation is perfect. Furthermore, the BER of the linear interpolation method is shown in Fig. 3 as a conventional channel estimation method for comparison.

To show the efficacy of the proposed method in time varying channels we tested the proposed method under different Doppler frequencies. As can be seen in Fig. 4 the proposed method shows little performance degradation with increase of the Doppler frequency.

Not only does the MAT method work perfectly under severe multipath channels such as Brazil D, but it also shows similar trends under simple channels such as AWGN (Fig. 5). As can be seen, the channel estimation performance obtained using the proposed method is close to the one obtained using the ideal channel. This again shows the channel estimation to be perfect in terms of BER.

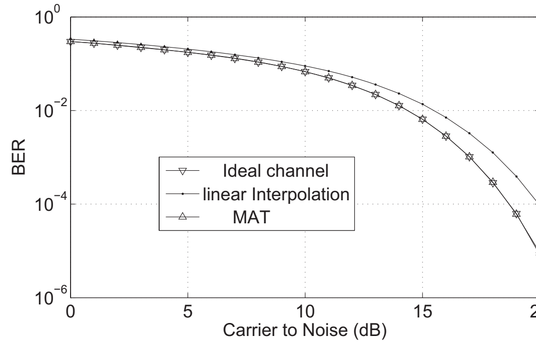

Fig. 5. Performance of the ideal, linear-interpolated and the proposed channel estimates under AWGN channel.

VI. Conclusion

In this paper, we have considered the OFDM channel estimation problem with use of the sparsity of the equivalent discrete channel in time. The main difficulty in exploiting this sparsity is that due to the partial use of the bandwidth in current OFDM standards, the reduced frequency to time transformation is no longer stable which sometimes leads to divergence of the previously introduced sparsity-based estimators. We have proposed a thresholding method which solves this problem. The simulation results show that the performance of this method after data equalization is almost the same as if we used the exact channel.

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