Extended target tracking using Gaussian processes

Niklas Wahlström, Emre Özkan

Division of Automatic Control
Linköping University
Linköping, Sweden

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Extended target tracking

Many sensors generates more than one measurement per target

Extended target (definition)

Targets that potentially give rise to multiple measurements at each time step

Goal: We want to estimate target position, target orientation and target extent jointly.
Related work

- Elliptical targets using inverse Wishart prior
  - Multiple ellipses per target (Lan and Li, Fusion 2012).
  - Encoding orientation (Granström and Orguner, Trans. A erospace 2014)

- Parametrized objects, rectangles, ellipses etc. (Granström, Fusion 2011)

- Random hyper-surface model (Baum and Hanebeck, Fusion 2011)

Modeling using polar coordinates

- $x$-coordinate
- $y$-coordinate
- Radial distance $r$
- Angle $\theta$

$r = f(\theta)$
We model $f(\theta)$ using a Gaussian process.

$$f(\theta) \sim \mathcal{GP}(0, k(\theta, \theta')),$$

$$\mathbb{E}[f(\theta)f(\theta')] = k(\theta, \theta')$$
We use a periodic covariance function to model the periodicity.

We use an additional constant covariance to model a constant (but unknown) mean, corresponding to the mean radius of the target.

\[ k(\theta, \theta') = \sigma_f^2 e^{-\frac{2 \sin^2 \left( \frac{|\theta - \theta'|}{2} \right)}{l^2}} + \sigma_r^2 \]
Recursive GP regression

**Idea:** Consider function values $f^1, f^2, \ldots, f^{N_f}$ to be the state components

$$x^f = \begin{bmatrix} f^1 \\ f^2 \\ \vdots \\ f^{N_f} \end{bmatrix}$$

This can be cast into a state space model

$$x^f_{k+1} = x^f_k$$
$$y_k = H x^f_k + e_k$$
$$x^f_0 \sim \mathcal{N}(0, P^f_0)$$
Idea: Consider function values \( f^1, f^2, \ldots, f^{N_f} \) to be the state components.

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\]

Advantages

- Recursive update with KF
Recursive GP regression

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This can be cast into a state space model

$$x^f_{k+1} = F x^f_k + w_k$$
$$y_k = H x^f_k + e_k$$
$$x^f_0 \sim \mathcal{N}(0, P^f_0)$$

**Advantages**
- Recursive update with KF
- Add process noise
Recursive GP regression

**Idea:** Consider function values \( f_1, f_2, \ldots, f^f_N \) to be the state components.

\[
x^f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f^f_N \end{bmatrix}
\]

This can be cast into a state space model:

\[
x^f_{k+1} = F x^f_k + w_k \\
y_k = H x^f_k + e_k \\
x^f_0 \sim \mathcal{N}(0, P^f_0)
\]

**Advantages**

- Recursive update with KF
- Add process noise
- \( x^f_k \) can be augmented with target position \( x^c_k \) and orientation \( \psi_k \).
Measurement is the sum of target position and offset due to target extent

\[ y_{k,l} = x^c_k + \begin{bmatrix} \cos(\theta^G_{k,l}) \\ \sin(\theta^G_{k,l}) \end{bmatrix} f(\theta^L_{k,l}) + e_{k,l}, \quad \theta^G_{k,l} = \theta^G_{k,l}(x^c_k) \]

\[ \theta^L_{k,l} = \theta^L_{k,l}(x^c_k, \psi_k) \]
Measurement is the sum of target position and offset due to target extent

\[ \mathbf{y}_{k,l} = \mathbf{x}_k^c + \begin{bmatrix} \cos(\theta_{k,l}^G) \\ \sin(\theta_{k,l}^G) \end{bmatrix} \mathbf{H}(\theta_{k,l}^L) \mathbf{x}_k^f + \mathbf{e}_{k,l}, \quad \theta_{k,l}^G = \theta_{k,l}^G(\mathbf{x}_k^c) \]

\[ \theta_{k,l}^L = \theta_{k,l}^L(\mathbf{x}_k^c, \psi_k) \]
This can be summarized into a non-linear sensor model

\[ y_{k,l} = x^c_k + H(x^c_k, \psi_k)x^f_k + e_{k,l} \]
Real data experiment

- Laser range data
- Multi-target scenario (cars, bicycles, humans)
- Almost no clutter

We used a simple logic-based multi-target tracker:
- Gating based likelihood
- Associate a measurement with the most likely target
- Cluster all ungated measurements and form new targets
Real data experiment - result

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[Graph showing data points with axes labeled from -20 to 20 on the x-axis and from 0 to 30 on the y-axis.]
Real data experiment - comparison

Green: RHM (Baum and Hanebeck). Black: Elliptical target (Koch...), Blue: proposed model
If we assume that \( f(\theta) \) has a period of \( \pi \) instead of \( 2\pi \), we can encode symmetry assumptions.
If the measurements originate from the target interior, we can add a random scalar to compensate for that

\[ y_{k,l} = x^c_k + s_{k,l} H(x^c_k, \psi_k) x^f_k + e_{k,l}, \quad s_{k,l} \in [0, 1] \]
Conclusions and Future work

Conclusions

- Model the target extent with a Gaussian process
- Estimate target extent and kinematic state jointly
- Fully recursive update provided

Future work

- Use Rao-Blackwellized PF - target extent state can be marginalized
- Exploit target symmetry properties even further
- A more sophisticated multi-target tracker