Pixels to Torques: Control using Deep Dynamical Models

Niklas Wahlström¹, John-Alexander M. Assael³, Thomas B. Schön², Marc P. Deisenroth³

¹Department of Electrical Engineering, Linköping University, Sweden

²Department of Information Technology, Uppsala University, Sweden

³Department of Computing, Imperial College London, UK
Short about me

  • 2007-2008: Exchange student, ETH Zürich, Switzerland

• 2010-2015: PhD student in Automatic Control, Linköping University
  • Spring 2014, Research visit, Imperial College, London, UK

• 2016-: Postdoc at Department of Information Technology, Uppsala University
My thesis

Three areas:

- Magnetic tracking
- Extended target tracking
- Deep dynamical models for control
Deep Learning: A recent example

First steps towards an autonomous system that learns by itself from raw pixel data.

- Deep autoencoder network + nonlinear dynamical model
- Model predictive control (MPC)
- Ref. value: $z_{\text{ref}} = f_d(y_{\text{ref}})$
- The model is automatically improved (in an iterative manner)

Deep Learning: A recent example
First steps towards an autonomous system that learns by itself from raw pixel data.

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Deep Learning: Another recent example

Automatically learn how to describe the contents of images.

Illustrates the modularity of the autoencoder, consisting of an encoder (vision deep CNN) and a decoder (language generating RNN).

A few examples where it failed

A large white **bird** standing in a forest.

A woman holding a **clock** in her hand.

A man wearing a hat and a **hat on a skateboard**.

A person is standing on a beach with a **surfboard**.

A woman is sitting at a table with a large **pizza**.

A man is talking on his cell **phone** while another man watches.
Deep learning: A very recent example

An AI defeated a human professional for the first time in the ancient game of Go

Outline

1. Introduction via three recent applications

2. What is a neural network (NN)?
   a) Concrete example for regression
   b) Learning and regularization

3. What is a deep neural network?

4. Learning deep neural networks
   a) Pre-training
   b) Defining and learning the autoencoder

5. Developing and learning a deep dynamical model
   a) Problem formulation
   b) Deep dynamical model

6. Some pointers, summary and the future
Constructing an NN for regression

A neural network (NN) is a nonlinear function $y = g_\theta(u)$ from an input variable $u$ to an output variable $y$ parameterized by $\theta$.

Linear regression
Constructing an NN for regression

A neural network (NN) is a nonlinear function \( y = g_\theta(u) \) from an input variable \( u \) to an output variable \( y \) parameterized by \( \theta \).

Linear regression models the relationship between a continuous target variable \( y \) and an input variable \( u \),

\[
y = \sum_{i=1}^{D} w_i u_i + b + \epsilon = \theta^T u + \epsilon,
\]

where \( \epsilon \) is noise and \( \theta \) is the parameters composed by the “weights” \( w_i \) and the offset (“bias”) term \( b \),

\[
\theta = \begin{pmatrix} b & w_1 & w_2 & \cdots & w_D \end{pmatrix}^T,
\]

\[
u = \begin{pmatrix} 1 & u_1 & u_2 & \cdots & u_D \end{pmatrix}^T.
\]
Generalized linear regression

We can generalize this by introducing nonlinear transformations of the predictor $\theta^T u$, 

$$y = f(\theta^T u).$$
Generalized linear regression

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$$y = f(\theta^T u).$$

Let us consider an example of a feed-forward NN, indicating that the information flows from the input to the output layer.
NN for regression – an example

1. Form $M$ linear combinations of the input $\mathbf{u} \in \mathbb{R}^D$

$$a_j^{(1)} = \sum_{i=1}^{D} w_{ji}^{(1)} u_i + b_j^{(1)}, \quad j = 1, \ldots, M.$$
NN for regression – an example

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$$a^{(1)}_j = \sum_{i=1}^{D} w^{(1)}_{ji} u_i + b^{(1)}_j, \quad j = 1, \ldots, M.$$  

2. Apply a nonlinear transformation

$$z_j = f \left( a^{(1)}_j \right), \quad j = 1, \ldots, M.$$
NN for regression – an example

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2. Apply a nonlinear transformation

$$z_j = f \left( a_j^{(1)} \right), \quad j = 1, \ldots, M.$$  

3. Form $M_y$ linear combinations of $z \in \mathbb{R}^M$

$$y_k = \sum_{j=1}^{M} w_{kj}^{(2)} z_j + b_k^{(2)}, \quad k = 1, \ldots, M_y.$$
NN for regression – an example

\[ \hat{y}_k(\theta) = \sum_{j=1}^{M} w_{kj}^{(2)} f \left( \sum_{i=1}^{D} w_{ji}^{(1)} u_i + b_j^{(1)} \right) + b_k^{(2)} \]
Multi-layer neural networks

We can think of the neural network as a sequential/recursive construction of several generalized linear regressions.

Each layer in a multi-layer NN is modelled as

$$z^{(l+1)} = f \left( W^{(l+1)} z^{(l)} + b^{(l+1)} \right),$$

starting with the input $z^{(0)} = u$. (The nonlinearity operates element-wise.)
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The scalar nonlinear function \( f(\cdot) \) is what makes the neural network nonlinear. Common functions are \( f(z) = 1/(1 + e^{-z}) \), \( f(z) = \tanh(z) \) and \( f(z) = \max(0, z) \).

The so-called **rectified linear unit (ReLU)** \( f(z) = \max(0, z) \) is heavily used for deep architectures.
Training a NN

The final layer $z^{(L)}$ of the network is used for making a prediction $\hat{y}(\theta) = z^{(L)}$ and we train the network by employing:

1. A set of training data.
2. A cost function $\mathcal{L}(\hat{y}(\theta), y)$.
3. An iterative scheme to optimize the cost function

$$J(\theta) = \sum_{n=1}^{N} \mathcal{L}(\hat{y}_n(\theta), y_n).$$
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Training a NN does involve a lot of engineering skill and is more of an art than a mathematically rigorous exercise.
Backpropagation

Recall our example network again:

\[ \hat{y}_k(\theta) = \sum_{j=1}^{M} w_{kj}^{(2)} f \left( \sum_{i=1}^{D} w_{ji}^{(1)} u_i + b_j^{(1)} \right) + b_k^{(2)} \]
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In solving the optimization problem

\[ \hat{\theta} = \arg\min_{\theta} J(\theta) \]

we typically employ gradient methods using \( \nabla J(\theta) \).
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Backpropagation amounts to computing the gradients via (recursive) use of the chain rule, combined with reuse of information that is needed for more than one gradient.
Tuning the model complexity

A neural network is a nonlinear parametric model that is built by recursively applying generalized linear regression,

\[ \hat{y} = f^{(L)} \circ \cdots \circ f^{(1)} \circ f^{(0)}(u). \]
## Tuning the model complexity

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**Problem:** As with any parametric method **overfitting** will occur if the number of free parameters is too large w.r.t. the training data. The model complexity typically needs to be **tuned**.
Tuning the model complexity

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**Weight decay:** Regularize using an Euclidean norm

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**Weight decay:** Regularize using an Euclidean norm

$$\tilde{J}(\theta) = J(\theta) + \lambda \|\theta\|^2.$$  

**Weight elimination:** Regularize using a zero-forcing term $h(\cdot)$

$$\tilde{J}(\theta) = J(\theta) + \lambda h(\theta).$$
Weight sharing is a constraint that forces certain connections in the network to have the same weights.
Networks with built-in constraints

**Weight sharing** is a constraint that forces certain connections in the network to have the same weights.

**Convolutional networks (ConvNets)** Makes use of the weight sharing idea. Nodes forms groups of 2D arrays.

Particularly successful in machine vision.

The convNet is a notable early successful deep architecture.
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Deep neural networks

Deep learning methods allow a machine to make use of raw data to automatically discover the representations (abstractions) that are necessary to solve a particular task.
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It is accomplished by using multiple levels of representation. Each level transforms the representation at the previous level into a new and more abstract representation,

\[ z^{(l+1)} = f \left( W^{(l+1)} z^{(l)} + b^{(l+1)} \right), \]

starting from the input (raw data) \( z^{(0)} = u \).
Deep neural networks

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starting from the input (raw data) \( z^{(0)} = u \).

**Key aspect:** The layers are **not** designed by human engineers, they are generated from (typically lots of) data using a learning procedure and lots of computations.
Hierarchy of features

Example: Image classification

The input layer represents an **image** and the output layer an **object identity**. Each hidden layer extracts increasingly abstract features.

Zeiler, M. D. and Fergus, R. Visualizing and understanding convolutional networks

Training deep neural networks

The main problem with a deep architecture is the training. The strategy sketched above will not work.

The breakthrough came 10 years ago:


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**Key idea:** Careful initialization by training each layer individually using an unsupervised algorithm. Referred to as **pre-training**.

Finally, a supervised algorithm (e.g. backpropagation) is used to fine-tune the parameters $\theta$ using the result from the pre-training as initial values.
Pre-training evolves sequentially from input to output. Here:

- 3 stages of unsupervised training
- 1 stage of supervised training
Pre-training – RBM

**Restricted Boltzmann machine (RBM):** an undir. graphical model with no connections among nodes of the same layer.

![Graphical representation of an RBM](image)
Pre-training – RBM

Restricted Boltzmann machine (RBM): an undir. graphical model with no connections among nodes of the same layer.

We have an observed input layer $u$ and an unobserved output layer $z$. 

\[
\hat{\theta} = \arg \max_{\theta} p_{\theta}(u),
\]

where $p_{\theta}(u)$ is found via marginalization,

\[
p_{\theta}(u) = \int p_{\theta}(u, z) \, dz.
\]
Pre-training – RBM

**Restricted Boltzmann machine (RBM):** an undir. graphical model with no connections among nodes of the same layer.

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Training strategy: Maximum likelihood

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The RBM is a **generative model**, which implies that we can simulate the output, which is then the input to the next layer.
Intuitive interpretation

Interpret the hidden layers as feature vectors and think of the deep architecture as a scheme for learning a hierarchy of features.
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Pre-training can in this way be thought of as a regularizer that forces the parameters to “good” regions, by exploiting extra information from the unsupervised learning stage.
Intuitive interpretation

Interpret the hidden layers as feature vectors and think of the deep architecture as a scheme for learning a **hierarchy of features**.

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There is still **no theoretical justification** as to why these deep networks exhibit such good generalization performance.... That is a good problem to solve.
Autoencoder

The autoencoder is an unsupervised learning procedure for **dimensionality reduction**.

It is a NN that learns compressed representations \( z \) of high-dimensional data \( u \), where \( \dim(u) \gg \dim(z) \).

**Encoder:** \( z = f_e(u) = f(W^T u + b) \).

**Decoder:** \( \hat{u} = f_d(z) = f(\bar{W}^T z + \bar{b}) \).
Training the autoencoder

The unknown parameters

$$\theta = \{ W, b, \bar{W}, \bar{b} \}$$

are estimated by minimizing the reconstruction error

$$e = u - \hat{u}(\theta),$$

using some cost function $J(\theta)$, for example LS

$$J(\theta) = \sum_{n=1}^{N} \| u_n - \hat{u}_n(\theta) \|^2.$$
Training the autoencoder

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J(\theta) = \sum_{n=1}^{N} \| u_n - \hat{u}_n(\theta) \|^2.
\]

After the training the encoder and the decoder will (by construction) be **approximate inverses** of each other,

\[ f_d(f_e(u)) \approx u. \]
Autoencoder

We can then easily transform either $u$ into $z$ or $z$ into $\hat{u}$ using either the encoder

$$z = f_e(W^T u + b),$$

or the decoder,

$$\hat{u} = f_d(\bar{W}^T z + \bar{b}).$$

The access to both of these two mappings is important for certain applications (such as the deep dynamical model).
Autoencoder

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\]

The access to both of these two mappings is important for certain applications (such as the deep dynamical model).

If \( f_e(\cdot) \) is chosen to be the identity (i.e. \( z = W^T u + b \)) and \( \dim u < \dim z \) then the autoencoder is equivalent to PCA. Hence, the autoencoder is a nonlinear generalization of PCA.
Deep autoencoder

The deep autoencoder is simply an autoencoder with several hidden layers.

Again, careful initialization is important for this to work, using the same pre-training as described before.

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   a) Problem formulation
   b) Deep dynamical model
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Motivation

- **Vision**: fully autonomous systems that learn by themselves from raw pixel data.
- **This work**: Modeling of high-dimensional pixel data
- **Strategy**: A deep dynamical model is proposed that contains a low-dimensional dynamical model.

N. Wahlström, T. B. Schön, M. P. Deisenroth Learning deep dynamical models from image pixels

*The 17th IFAC Symposium on System Identification (SYSID)*
Problem Formulation

Problem formulation: Modeling of high-dimensional pixel data

Example: Video stream of a pendulum

- **Input**: Torque of a pendulum
- **Output**: Pixel values of an $11 \times 11$ image
Problem Formulation

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The Autoencoder

Notation:

- $y_k$ - High-dim. observations
- $z_k$ - Low-dim. features
The Autoencoder

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- $y_k$ - High-dim. observations
- $z_k$ - Low-dim. features

Model components:
1. Encoder: $z_k = f_e(y_k; \theta_E)$
The Autoencoder

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The Autoencoder

Notation:
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Model components:
1. Encoder: \( z_k = f_e(y_k; \theta_E) \)
2. Decoder: \( \hat{y}_k^R = f_d(z_k; \theta_D) \)

Reconstruction error:
\[
V_R(\theta_E, \theta_D) = \sum_{k=1}^{N} \| y_k - \hat{y}_k^R(\theta_E, \theta_D) \|^2
\]
Deep Dynamical Model

Notation:

- $y_k$ - High-dim. observations
- $z_k$ - Low-dim. features
- $u_k$ - Inputs
Deep Dynamical Model

Notation:
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Deep Dynamical Model

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Model components:
1. Encoder: \( z_k = f_e(y_k; \theta_E) \)
2. Prediction model: \( \hat{z}_{k+1|k} = f(z_k, u_k, \ldots, z_{k-n+1}, u_{k-n+1}; \theta_P) \)
Deep Dynamical Model

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3. Decoder: \( \hat{y}_{k+1|k}^P = f_d(\hat{z}_{k+1|k}; \theta_D) \)
Deep Dynamical Model

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3. Decoder: $\hat{y}_{k+1|k}^P = f_d(\hat{z}_{k+1|k}; \theta_D)$

Prediction error:
$V_P(\theta_E, \theta_D, \theta_P) = \sum_{k=n}^{N-1} \| y_{k+1} - \hat{y}_{k+1|k}^P(\theta_E, \theta_D, \theta_P) \|^2$
Training

**Key ingredient:** The reconstruction error and the prediction error are minimized *simultaneously!*

\[
(\hat{\theta}_E, \hat{\theta}_D, \hat{\theta}_P) = \arg \min_{\theta_E, \theta_D, \theta_P} V_R(\theta_E, \theta_D) + V_P(\theta_E, \theta_D, \theta_P)
\]

\[
V_R(\theta_E, \theta_D) = \sum_{k=1}^{N} \| y_k - \hat{y}_k^R(\theta_E, \theta_D) \|^2,
\]

\[
V_P(\theta_E, \theta_D, \theta_P) = \sum_{k=n}^{N-1} \| y_{k+1} - \hat{y}_{k+1|k}^P(\theta_E, \theta_D, \theta_P) \|^2.
\]
Experiment: Pendulum

- Layers in encoder/decoder: 4
- Latent dim.: \( \dim(z) = 1 \)
- Order of prediction model: \( n = 4 \)
Experiment: Pendulum

- Layers in encoder/decoder: 4
- Latent dim.: \( \dim(z) = 1 \)
- Order of prediction model: \( n = 4 \)
Experiment: Agent in a Planar System

- **Input:** Offset in $x$-dir. ($u_1$) and $y$-dir. ($u_2$)
- **Output:** Pixel values of a $51 \times 51$ image
- **Latent dim.:** $\dim(z) = 2$
Experiment: Agent in a Planar System

- **Input**: Offset in $x$–dir. $(u_1)$ and $y$–dir. $(u_2)$
- **Output**: Pixel values of a $51 \times 51$ image
- **Latent dim.**: $\dim(z) = 2$
Experiment: Agent in a Planar System
Separate vs. Simultaneous Training

True frame

Simultaneous training

Separate training
Experiment: Agent in a Planar System

Simultaneous Training

Separate Training

Iteration: 0
Experiment: Agent in a Planar System

Simultaneous Training
Iteration: 0

Separate Training
Iteration: 0
Experiment: Agent in a Planar System

Simultaneous Training

Separate Training

Interaction: 450
Experiment: Agent in a Planar System

**Simultaneous Training**

Iteration: 450

**Separate Training**

Iteration: 450
The DDM is used to learn a closed-loop policy via nonlinear model predictive control (MPC). Future control signals are optimized by minimizing

\[ u_0^*, \ldots, u_{K-1}^* \in \arg\min_{u_0:K-1} \sum_{k=0}^{K-1} \| \hat{z}_k - z_{\text{ref}} \|^2 + \lambda \| u_k \|^2, \]

where \( z_{\text{ref}} = f_e(y_{\text{ref}}, \theta_e) \) is the feature of the reference image. When the control sequence \( u_0^*, \ldots, u_{K-1}^* \) is determined, the first control \( u_0^* \) is applied to the system. Hence, the MPC is only applied in the low-dimensional feature space!
Deep Dynamical Models for Control

Proposed algorithm

Follow a random control strategy and record data

**loop**

Update DDM with all data collected so far

**for** \( k = 0 \) to \( N - 1 \) **do**

- Get \( z_k, \ldots, z_{k-n+1} \) via encoder.
- \( u_k^* \leftarrow \epsilon\text{-greedy MPC policy using DDM prediction.} \)
- Apply \( u_k^* \) and record data.

**end for**

**end loop**

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N. Wahlström, T. B Schön, and M. P. Deisenroth

Experiment: Control of a Pendulum from Pixels Only

- Ref. image: Pendulum pointing upwards
- 100 images in each trial
- After 15 trials, a good controller was learned

![Graph showing pendulum control over time](image-url)
Application: Control of Two-Link Arm from Pixels Only

- Ref. image: Arm pointing upwards
- 1000 images in each trial
- After 8-9 trials a fairly good controller was learned.

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Some pointers

Key **publication channels** in machine learning, NIPS and ICML

From NIPS in December ([nips.cc/Conferences/2015](nips.cc/Conferences/2015)) three of the six tutorials deals with deep learning:

1. G. Hinton, Y. Bengio and Y. LeCun, Deep learning
2. B. Dally, High-performance hardware for Machine Learning
   [nips.cc/Conferences/2015/Schedule?event=4894](nips.cc/Conferences/2015/Schedule?event=4894)
3. J. Dean, Large-scale distributed systems for training NN
   [nips.cc/Conferences/2015/Schedule?event=4895](nips.cc/Conferences/2015/Schedule?event=4895)

A well written and timely introduction:


You will also find more material than you can possibly want here

[http://deeplearning.net/](http://deeplearning.net/)
Some pointers

Key **publication channels** in machine learning, NIPS and ICML

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Summary (I/II)

A neural network (NN) is a nonlinear function $y = g_\theta(u)$ from an input variable $u$ to an output variable $y$ parameterized by $\theta$. 
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The deep autoencoder makes use of a multi-layer “encoder” network to transform high-dimensional data into a low-dimensional code/feature and a similar “decoder” network is used to recover the data from the code.
Summary (II/II)

**Deep dynamical model:**

- Model for high-dimensional pixel data
- Simultaneous training is crucial
- Application: Control based on pixel data only
The future

The best predictive performance is obtained from highly flexible models (especially when large datasets are used). There are basically two ways of achieving flexibility:

1. Using models with a large number of parameters compared to the data set (e.g. deep NN).
2. Models using non-parametric components, e.g. Gaussian processes.
The future

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1. Using models with a large number of parameters compared to the data set (e.g. deep NN).
2. Models using non-parametric components, e.g. Gaussian processes.

Use the network also for “attention” and control. Use reinforcement learning to decide **where to look** for new data (resulting in new knowledge).

Deep reinforcement learning workshop at NIPS in december.