Testing finite state machines: State Identification and Verification

Automata Learning - reading group
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OREZ
Mealy Machines

- States: $s_1$, $s_2$, $s_3$
- Transitions:
  - $s_1 \xrightarrow{a/0} s_1$
  - $s_1 \xrightarrow{b/0} s_3$
  - $s_2 \xrightarrow{a/0} s_1$
  - $s_2 \xrightarrow{b/1} s_3$
  - $s_3 \xrightarrow{a/1} s_3$
Mealy Machines

\[(I, O, S, \delta, \lambda)\]
Mealy Machines

$$(I, O, S, \delta, \lambda)$$

Input

Diagram:

- States: $s^1$, $s^2$, $s^3$
- Transitions:
  - $s^1 \xrightarrow{a/0} s^1$
  - $s^1 \xrightarrow{b/0} s^2$
  - $s^2 \xrightarrow{b/1} s^3$
  - $s^2 \xrightarrow{b/1} s^2$
  - $s^3 \xrightarrow{b/1} s^3$
Mealy Machines

\((I, O, S, \delta, \lambda)\)

Output
Mealy Machines

\[ (I, O, S, \delta, \lambda) \]

States
Mealy Machines

\((I, O, S, \delta, \lambda)\)

\(\delta: S \times I \rightarrow S\)
Mealy Machines

\[(I, O, S, \delta, \lambda)\]

\[\delta : S \times I \to S\]
Mealy Machines

\((I, O, S, \delta, \lambda)\)

\(\delta: S \times I \rightarrow S\)
Mealy Machines

\[(I, O, S, \delta, \lambda)\]

\[\delta: S \times I \rightarrow S\]

\[\delta (s^2, \text{`a'}) = s^1\]
Mealy Machines

\[(I, O, S, \delta, \lambda)\]

\[\delta : S \times I \rightarrow S\]

\[\delta (s^2, 'a') = s^1\]

\[\delta (s^2, "aaab") = s^3\]
Mealy Machines

\((\mathcal{I},\mathcal{O},\mathcal{S},\delta,\lambda)\)

\(\delta: \mathcal{S} \times \mathcal{I} \rightarrow \mathcal{S}\)

\(\delta (s^2, \text{`a'} ) = s^1\)

\(\delta (s^2, \text{“aaab”} ) = s^3\)

\(\tilde{\delta} (\{s^1, s^3\}, \text{`b'} ) = \{s^3, s^2\}\)
Mealy Machines

\[(I, O, S, \delta, \lambda)\]

\[\lambda : S \times I \rightarrow O\]
Mealy Machines

\((I, O, S, \delta, \lambda)\)

\(\lambda: S \times I \rightarrow O\)

\(\lambda(s^2, 'a') = '0'\)
Mealy Machines

\((I, O, S, \delta, \lambda)\)

\(\lambda: S \times I \rightarrow O\)

\(\lambda(S, \text{"a"}) = \text{"0"}\)

\(\lambda(s^2, \text{"aaab"}) = \text{"0000"}\)
Mealy Machines

\[(I, O, S, \delta, \lambda)\]

\[\lambda: S \times I \rightarrow O\]

\[\lambda(s^2, 'a') = '0'\]

\[\lambda(s^2, "aaab") = "0000"\]

\[\lambda(\{s^1, s^3\}, 'b') = \{ '0', '1' \}\]
Lack of some informations

Goal: Deduce them.

Mean:
- Provide input Sequence
- Observe the output
Tests

- **Input Sequence / Observe output**
- **Preset Test:**
  - Fixed ahead of time.
- **Adaptive Test:**
  - Input symbol depends on the previous observed output symbol.
Testing: Problems

- State Identification
- State Verification
Testing: State Identification

- State Identification
  - State diagram given.
  - Initial state not identified.
  - Initial State Identification: Not always possible through testing

- Distinguishing Sequence

- State Verification
Testing: State Identification

State Identification

State Verification
Testing: Problems

- State Identification
- State Verification
Testing: Questions

- State Identification
- State Verification

- State diagram given.
- Initial state not identified.
- Machine is in its initial state.
- Goal: Check if it's in that state.
- Unique Input Output (UIO) Sequence
Testing: Questions

- **Existence** of the input sequence?
- How **long** is the sequence?
- **Complexity** to:
  - Check **existence** of the sequence,
  - **Construct** a sequence,
  - Construct a **short** one.
Testing: Problems

- State Identification
- State Verification
- Fault detection (black box), protocol testing

Link: Distinguishing Sequence
State Identification

Diagram of states:
- $s^1$: Transitions are $a/0$ and $b/0$.
- $s^2$: Transitions are $a/0$ and $b/1$.
- $s^3$: Transitions are $b/1$ and $a/1$. 
State Identification

- Preset Distinguishing sequence: Input $x$ such that the output produced by the machine is *different* for each state.

- The FSM needs to be *reduced*, to remove all the equivalent states.

- The machine *might not* have such a sequence.
Preset Distinguishing sequence

State Identification

Reduced

State \( s^1 \) transitions:
- \( a/0 \) to \( s^1 \)
- \( b/0 \) to \( s^3 \)
- \( a/1 \) to \( s^3 \)
- \( b/1 \) to \( s^2 \)

State \( s^2 \) transitions:
- \( b/1 \) to \( s^2 \)

State \( s^3 \) transitions:
- \( a/1 \) to \( s^3 \)
State Identification

Reduced

- 'b' distinguish $s^1$ from $s^2$ and $s^3$
State Identification

Reduced

- 'b' distinguish $s^1$ from $s^2$ and $s^{13}$
- 'a' distinguish $s^3$ from $s^2$
Preset Distinguishing sequence

State Identification

Reduced

But No (Unique) Sequence that distinguish (All states) at once
State Identification

Reduced

But No (Unique) Sequence that distinguish (All states) at once

- "b..." distinguish $s^1$ from $s^2$ and $s^3$, but "merges" $s^2$ and $s^3$

Preset Distinguishing sequence
State Identification

Reduced But No (Unique) Sequence that distinguish (All states) at once

- “b…” distinguish $s^1$ from $s^2$ and $s^3$, but “merges” $s^2$ and $s^3$
- “a…” distinguish $s^3$ from $s^1$ and $s^2$, but “merges” $s^1$ and $s^2
Preset Distinguishing sequence: Input $x$ such that the output produced by the machine is different for each state.

THEOREM: It is PSPACE-complete to test whether a given FSM has a preset distinguishing sequence.

Length of distinguishing sequence can be exponential

Did not get hold of the algorithm.
State Identification

- Preset Distinguishing sequence
- Adaptive Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence: Not a sequence but a decision tree: Input based on the last output.

Rooted tree, n leaves (states), Nodes: Input, Edges: Output.

Length of the sequence is the depth of the tree.
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

State Transition Diagram:

- \( s^1 \): a/1 [b/0]
- \( s^2 \): a/0 [b/0] \( \rightarrow \) a/1
- \( s^3 \): b/0 [a/0] \( \rightarrow \) b/0
- \( s^4 \): b/0 [a/0] \( \rightarrow \) b/0
- \( s^5 \): b/0 [a/0] \( \rightarrow \) a/1
- \( s^6 \): a/1 [b/0] \( \rightarrow \) b/0

Arrows indicate state transitions based on input symbols.
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

\[ a \rightarrow \{ s^1, s^2, s^3, s^4, s^5, s^6 \} \]
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_1, s_3, s_5\}

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_1, s_3, s_5\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

\[ s_1 \rightarrow s_2 \rightarrow a/1 \rightarrow s_3 \rightarrow b/0 \rightarrow s_4 \rightarrow b/0 \rightarrow s_5 \rightarrow a/0 \rightarrow s_6 \rightarrow b/0 \rightarrow s_1 \]
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

\[ \{s_1, s_2, s_3, s_4, s_5, s_6\} \]

\[ \{s_1, s_3, s_5\} \]  \rightarrow  \[ s_2, s_4, s_6\]
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing sequence

State Transition Diagram:

- **s^1**: Transition on **b/0** to **s^2**
- **s^2**: Transition on **b/0** to **s^3**
- **s^3**: Transition on **a/0** to **s^4**
- **s^4**: Transition on **b/0** to **s^5**
- **s^5**: Transition on **a/0** to **s^6**
- **s^6**: Transition on **b/0** to **s^1**

Nodes marked with **a/1** and **b/0** indicate specific transitions.
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_1, s_2, s_3, s_5\}

\{s^2, s^4, s^6\}

\{s^1, s^2, s^3, s^4, s^5, s^6\}
State Identification

- Adaptive Distinguishing Sequence
- Preset Distinguishing Sequence

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_1, s_3, s_5\} → \{s_2, s_4, s_6\} → \{s_1, s_2, s_3, s_4, s_5, s_6\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Initial

\{s^1, s^2, s^3, s^4, s^5, s^6\}

Current

\{s^2, s^4, s^6\} → \{s^1, s^3, s^5\} → \{s^2, s^4, s^6\}

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

0 \rightarrow 1

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_3, s_5, s_1\}

\{s_1, s_3, s_5\} \rightarrow \{s_2, s_4, s_6\}

\{s_1, s_2, s_3, s_4, s_5, s_6\} \rightarrow \{s_3, s_5, s_1\}
State Identification

- Adaptive Distinguishing Sequence
- Preset Distinguishing Sequence

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_3, s_5, s_1\}

0 \rightarrow 1

0 \rightarrow \{s_2, s_4, s_6\}

1 \rightarrow \{s_3, s_5, s_1\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

State Transition Diagram:

- $s_1$ transitions to $s_2$ on input $a$ and $s_5$ on input $b$.
- $s_2$ transitions to $s_3$ on input $a$ and $s_6$ on input $b$.
- $s_3$ transitions to $s_4$ on input $a$ and $s_5$ on input $b$.
- $s_4$ transitions to $s_1$ on input $b$.
- $s_5$ transitions to $s_2$ on input $a$ and $s_6$ on input $b$.
- $s_6$ transitions to $s_3$ on input $b$. 

Input Symbols:
- $a$: Transitioning states.
- $b$: Transitioning states.

States:
- $s_1$, $s_2$, $s_3$, $s_4$, $s_5$, $s_6$.
State Identification

Adaptive Distinguishing Sequence
Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial: $\{s^1, s^2, s^3, s^4, s^5, s^6\}$

Current: $\{s^3, s^5, s^1\}$

$\text{Initial} \rightarrow \text{Current}$

$\text{Initial} \rightarrow \{s^2, s^4, s^6\}$

$\text{Current} \rightarrow \{s^2, s^4, s^6\}$
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

- State $s_1$ transitions to $s_2$ with input $a/0$.
- State $s_2$ transitions to $s_3$ with input $a/1$.
- State $s_3$ transitions to $s_4$ with input $a/0$.
- State $s_4$ transitions to $s_5$ with input $b/0$.
- State $s_5$ transitions to $s_6$ with input $a/0$.
- State $s_6$ transitions to $s_1$ with input $b/0$.

- State $s_2$ transitions to $s_1$ with input $b/0$.
- State $s_3$ transitions to $s_2$ with input $b/0$.
- State $s_4$ transitions to $s_3$ with input $a/0$.
- State $s_5$ transitions to $s_4$ with input $b/0$.
- State $s_6$ transitions to $s_5$ with input $b/0$.

- State $s_1$ transitions to $s_6$ with input $a/1$.
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial: \(\{s_1, s_2, s_3, s_4, s_5, s_6\}\)

Current: \(\{s_3, s_5, s_1\}\)

Initial: \(\{s_2, s_4, s_6\}\)

Current: \(\{s_4, s_6, s_1\}\)

0 \rightarrow 1 \rightarrow 0 \rightarrow a \rightarrow b \rightarrow 0
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

States: $s^1, s^2, s^3, s^4, s^5, s^6$

Transitions:
- $s^1 \xrightarrow{a/0} s^2$
- $s^2 \xrightarrow{a/1} s^3$
- $s^3 \xrightarrow{a/0} s^4$
- $s^4 \xrightarrow{b/0} s^5$
- $s^5 \xrightarrow{b/0} s^6$
- $s^6 \xrightarrow{b/0} s^1$
- $s^1 \xrightarrow{b/0} s^1$ (self-loop)

Symbols: $a/0, a/1, b/0$
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_3, s_5, s_1\}

Transition diagram:
- From state 0, on input 1, the next state is b.
- From state b, on input 0, the next state is 0.

States: a, b, 0
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Diagram:
- States: $s_1, s_2, s_3, s_4, s_5, s_6$
- Transitions:
  - $s_1$ to $s_2$: $a/0$
  - $s_2$ to $s_3$: $a/1$
  - $s_3$ to $s_4$: $a/0$
  - $s_4$ to $s_5$: $b/0$
  - $s_5$ to $s_6$: $a/0$
  - $s_6$ to $s_1$: $b/0$
  - $s_1$ to $s_1$: $b/0$

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Invalid Input
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

- $s_1$: $a/0$, $b/0$
- $s_2$: $a/1$
- $s_3$: $b/0$
- $s_4$: $b/0$
- $s_5$: $a/0$
- $s_6$: $b/0$

Transition Labels:
- $a/0$
- $a/1$
- $b/0$
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

- Adaptive Distinguishing Sequence
- Preset Distinguishing Sequence

Diagram showing transitions between states:
- $s^1$ to $s^2$: a/0
- $s^2$ to $s^3$: a/1, b/0
- $s^3$ to $s^4$: a/0, b/0
- $s^4$ to $s^5$: b/0
- $s^5$ to $s^6$: a/0, b/0
- $s^6$ to $s^1$: b/0, a/1
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial

\{s^1, s^2, s^3, s^4, s^5, s^6\}
\{s^2, s^4, s^6\}
\{s^2, s^4, s^6\}

Current

\{s^3, s^5, s^1\}
\{s^3, s^5, s^1\}
\{s^2\}
\{s^5, s^1\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial

Current

\{s^1, s^2, s^3, s^4, s^5, s^6\}

\{s^2, s^4, s^6\}

\{s^2, s^4, s^6\}

\{s^6\}

\{s^5, s^1\}

\{s^2\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial: \{s_1, s_2, s_3, s_4, s_5, s_6\}

Current: \{s_3, s_5, s_1\}

Initial: \{s_2, s_4, s_6\}

Current: \{s_3, s_5, s_1\}

Initial: \{s_2, s_4, s_6\}

Current: \{s_4, s_6, s_1\}

Initial: \{s_2, s_4, s_6\}

Current: \{s_2\}

Initial: \{s_6\}

Current: \{s^5, s^1\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial
{s^1, s^2, s^3, s^4, s^5, s^6}
{s^2, s^4, s^6}
{s^6}

Current
{s^3, s^5, s^1}
{s^4, s^6, s^1}
{s^2}
{s^5, s^1}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial

Current

\{s^1, s^2, s^3, s^4, s^5, s^6\}

\{s^2, s^4, s^6\} \rightarrow \{s^3, s^5, s^1\}

\{s^2, s^4, s^6\} \rightarrow \{s^4, s^6, s^1\}

\{s^6\} \rightarrow \{s^2\}

\{s^2, s^4\} \rightarrow \{s^5, s^1\}
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence

Initial Current

\{s^1, s^2, s^3, s^4, s^5, s^6\}

\lambda(s^6, \text{"aba"}) = \text{"100"}

\delta(s^6, \text{"aba"}) = s^2

\{s^6\} \rightarrow \{s^2\} 

\{s^2, s^4\} \rightarrow \{s^5, s^1\}
State Identification

```
a
0
a

1
b

0

...

s_6

1

0

a

b
```
State Identification

- Preset Sequence is an Adaptive one
- Adaptive Sequence might be shorter
- FSM might have an Adaptive sequence, but no Preset one.
- Length is quadratic, if the seq. exists.
State Identification

Existence Algorithm: (Reason about the Initial Set)

Initialize State Partition $\Pi$ with one unique block

Iteration:

- Pick: Block $B$, Valid Input $a$.
- Refine the partition: Two states are assigned the same block in the new partition iff they produce the same output on $a$, and move to the same block (of the old partition).
State Identification

Existence Algorithm:

Initialize State Partition $\Pi$ with one unique block

Iteration:

- Pick: Block $B$, Valid Input $a$.
- Refine the partition...

Adaptive Seq. exists iff final partition is discrete
State Identification

Adaptive Distinguishing Sequence

Preset Distinguishing Sequence
State Identification

\[ \Pi = \{ s^1, s^2, s^3, s^4, s^5, s^6 \} \]
State Identification

\[ \Pi = \{s^1, s^3, s^5, s^6\} \]

\[ \Pi = \{s^1, s^3, s^5\}, \{s^2, s^4, s^6\} \]
State Identification

\[ \Pi = \{\{s^1, s^3, s^5\}, \{s^2, s^4, s^6\}\} \]
State Identification

\[ \Pi = \{s^1, s^3, s^5\}, \{s^2, s^4, s^6\} \]
State Identification

$\Pi = \{s_1, s_3, s_5\}, \{s_2, s_4, s_6\}\}

$\Pi = \{\{s_1\}, \{s_3, s_5\}, \{s_2, s_4, s_6\}\}$
State Identification

\[ \Pi = \{\{s^1\}, \{s^3, s^5\}, \{s^2, s^4, s^6\}\} \]
State Identification

\[ \Pi = \{\{s^1\}, \{s^3, s^5\}, \{s^2, s^4, s^6\}\} \]
State Identification

\[ \Pi = \{\{s^1\}, \{s^3, s^5\}, \{s^2, s^4, s^6\}\} \]

\[ \Pi = \{\{s^1\}, \{s^3, s^5\}, \{s^2, s^4\}, \{s^6\}\} \]
State Identification

\[ \Pi = \{\{s^1\}, \{s^3, s^5\}, \{s^2, s^4\}, \{s^6\}\} \]
State Identification

\[ \Pi = \{\{s^1\}, \{s^3, s^5\}, \{s^2, s^4\}, \{s^6\}\} \]
State Identification

\[ \Pi = \{\{s_1\},\{s_3, s_5\},\{s_2, s_4\},\{s_6\}\} \]

\[ \Pi = \{\{s_1\},\{s_3\},\{s_5\},\{s_2, s_4\},\{s_6\}\} \]
State Identification

\[ \Pi = \{\{s_1\},\{s_3\},\{s_5\},\{s_2, s_4\},\{s_6\}\} \]
State Identification

\[ \Pi = \{\{s^1\}, \{s^3\}, \{s^5\}, \{s^2, s^4\}, \{s^6\}\} \]
State Identification

\[ \Pi = \{\{s^1\}, \{s^3\}, \{s^5\}, \{s^2, s^4\}, \{s^6\}\} \]

\[ \Pi = \{\{s^1\}, \{s^3\}, \{s^5\}, \{s^2\}, \{s^4\}, \{s^6\}\} \]
State Identification

\[ \Pi = \{s^1, s^3, s^5, s^2, s^4, s^6\} \]
State Identification

Algorithm 3.2
and
3.3
State Verification

- State diagram given.
- **Initial state** not identified.
- Machine is in its initial state.
- Goal: Check if it’s in that state.
- Unique Input Output (UIO) Sequence
State Verification

Unique Input Output (UIO)
Sequence of a state $s^0$ is an input $x^0$ such that:

$$\lambda(s^0, x^0) \neq \lambda(s, x^0), \forall s$$
State Verification

◊ Unique Input Output (UIO)
  Sequence of a state $s^0$ is an input $x^0$ such that:

$$\lambda (s^0, x^0) \neq \lambda (s, x^0), \forall s$$

Preset/Adaptive Distinguishing Sequence

$\Rightarrow$ Existence of UIO
State Verification

All these problems are PSPACE-Complete:

- Does a given state $s$ of $M$ have a UIO
- Does all states of $M$ have a UIO
- Is there some states of $M$ with UIO
State Verification

- Length: There are machines whose states have UIO sequences but of exponential length.
Conclusion

- PSPACE-Completeness for Preset Seq. and UIO
- Polynomial time algorithm for checking existence and construction of the Adaptive Seq.
- Adaptive Seq. might be more useful
Just to remember ..
Assignment:

\[ s^1 \rightarrow a/0 \]
\[ s^2 \rightarrow b/1 \]
\[ s^3 \rightarrow b/0 \]

\[ s^1 \rightarrow b/1 \]
\[ s^2 \rightarrow b/1 \]
\[ s^3 \rightarrow a/0 \]
Assignment:

- Show Existence of Adaptive Distinguishing Sequence
- Draw the Adaptive Distinguishing Sequence