Is labeled-strong bisimulation decidable?

Process types

Definition: The set $\mathcal{T}$ of *process types* is inductively defined by the following grammar.

$$\alpha, \beta ::= \sum_{i \in I} l_i(\widetilde{\alpha}_i).\alpha_i \;\mid\; \sum_{i \in I} v.\alpha_i \;\mid\; (\alpha \parallel \beta) \;\mid\; t \;\mid\; \mu t.\alpha$$

Bisimilarity on process types

Definition:

1. A symmetric binary relation $R \subseteq \mathcal{T} \times \mathcal{T}$ is a *label-strong bisimulation*, if, whenever $\alpha \, R \, \beta$:

   (a) $\alpha \xrightarrow{l(\widetilde{\alpha})} \alpha'$ implies $\exists \beta', \widetilde{\beta}$ $(\beta \xrightarrow{l(\widetilde{\beta})} \beta'$ and $\alpha' \widetilde{\alpha} \, R \, \beta' \widetilde{\beta})$

   (b) $\alpha \xrightarrow{v} \alpha'$ implies $\exists \beta'$ $(\beta \Longrightarrow \beta'$ and $\alpha' \, R \, \beta')$.

2. Two types α and β are *label-strong bisimilar*, and we write $\alpha \approx \beta$, if there is a label-strong bisimulation R such that $\alpha \, R \, \beta$.

Theorem:

1. label-strong bisimilarity is an equivalence relation and the largest bisimulation.

2. label-strong bisimilarity is completely axiomatizable, for image-finite process types.

Given P and Q, is the problem '$P \approx Q$?' decidable?

State-of-the-art

1. Strong bisimilarity is decidable in BPP, a language that includes ours. The decision procedure is co-$\mathcal{NP}$-hard.

   BPP is equivalent to the communication-free nets, a subset of Petri-nets where every transition has exactly one input-place with arc-weigh one.

   In full Petri-nets strong bisimilarity is undecidable. Families of infinite state systems for which bisimilarity is known to be decidable are *finitely branching*.

2. The problem of whether two BPP processes are weak bisimilar is *not known* to be decidable (although it is *semi-decidable*).

   Weak bisimilarity is generally not finitely approximable, neither is the weak finiteness problem.

   If such procedures exist, they are $\mathcal{NP}$-hard (the lower bound is, in the polynomial hierarchy, Π_2^P).

3. Recently, Stirling proved decidability for a subset of processes for which *inequivalence need not to be finitely approximable.*

A. Ravara, P. Resende, and V. Vasconcelos. An algebra of behavioural types. Preprint, Section of Computer Science, Department of Mathematics, Instituto Superior Tcnico, 1049-001 Lisboa, Portugal, 2002. Submitted for publication.

Richard Mayr. On the Complexity of Bisimulation Problems for Basic Parallel Processes. ICALP 2000. Volume 1853 in LNCS. Springer Verlag. 2000.

Colin Stirling. Decidability of Weak Bisimilarity for a Subset of Basic Parallel Processes. FOSSACS 2001. Volume 2030 in LNCS. Springer Verlag. 2001.