1. INTRODUCTION

1.1. Background, Problem, and Purpose

For the time being there are three basic characteristics that must be fulfilled to economically defend development of knowledge system applications. The knowledge should be expensive, distributed among experts and have a straightforward representation. Indisputably, legal knowledge fulfills the two first characteristics. However, the impression it may give of also fulfilling the third is misleading [14,34]. This may be a disappointment from an economical point of view. On the other hand legal knowledge offers interesting challenges for research in knowledge representation. The purpose of this thesis is to identify some of the basic difficulties and discuss how these could be approached.

Background. Before the purpose can be described in more detail we need a background concerning the problem domain.

When the layman hears the word ‘law’ he probably equates it with acts and the like. The legal term for this source of law is statute law. Besides statute law we have however several important legal sources e.g., case-law, equity, legal doctrine etc. Some of these legal sources have been the object of study for knowledge representation in well-known research projects. We have, for example, Sergot’s project on statute law [43] (the British Nationality Act), Gardner’s on case-law (American contract law) [14], and Ashley’s and Rissland’s on case-law [1]; conflicts between case-law and literal statutory requirements have been studied by McCarty [35].

This study is demarcated to statute law and legal sources being systematized in a way akin to statute law, e.g., regulations, ordinances, decrees, conventions, treaties etc. A certain property of these has probably been a dominating source for optimism concerning knowledge representation of law. They are drafted in the form of provisions, e.g.,

(1) “The employer’s dismissal must have fair grounds”

Although these provisions are often, as here, written as un-conditional statements, they are, with one exception, possible to reformulate to ‘if...then...’ statements, e.g.,

(2) if fair ground for dismissal exists then the employee may be dismissed.

where the circumstances in the antecedent (here ‘fair ground for dismissal’) lead to the legal consequence in the consequent (here ‘the employee may be dismissed’).

The exception is when the provision is a so-called ‘legislative manifest.’ These are expressions of political rhetoric and moral conceptions. For example

(3) “Man and wife are under the obligation of mutual faithfulness and assistance.”

Legislative manifests are rare—in fact manifest 3 for a long period of time was quite unique in the Swedish legislation. According to legal doctrine these exceptions have little affect on the legal usage; they cannot be reformulated in conformity with the ‘if...then...’ scheme above and are therefore lege imperfectae, i.e., law without sanction ([44], pp. 242 ff.). Therefore, it is not far-fetched to presume that

(4) only provisions that, without loss of legal adequacy, can be reformulated to ‘if...then...’ statements are of interest to capture in a representation.

Presumption 4 is not absolutely true. Legislative manifests have a role. They affect the interpretation of provisions ([44], p. 243). It may be true that the influence legislative manifests have in that respect is small. It is however important to realize the significance of the things that exercise influence over the interpretation of legal rules.
The Main Problem. The impression that the ‘if . . . then . . .’ statements inherent in provisions\(^1\) give the essence of the legislation is appealing with regard to knowledge representation. However, the impression is incorrect. The statutes that belong to some branch of law give only an incomplete picture of the legal knowledge of that field. J. C. Gray once expressed this as: “Statutes are sources of law . . . not part of the Law itself” ([17], p. 276). Gray’s point is that statutory rules are not rules to be applied directly by the courts—they have no independent interpretation: “their meaning is declared by the courts, and it is with the meaning declared by the courts and no other meaning that they are imposed upon the community as Law.” ([18], p. 170) (emphasis in original). It cannot be denied that the application of provisions occasionally is rather mechanical ([22], p. 12). Also in these cases, however, the court’s interpretation of the provisions exercises an influence that cannot simply be ignored. And in many cases only a fraction of the fact situations to which the rules apply (or may possibly apply) would be captured if direct depictions of the provisions were the sole content of the representation. A knowledge system with such a basis only would be very limited and in most cases not practical. Instead, what needs to be captured is a more abstract kind of knowledge which we may term legal norms.

The notion of a legal norm is important but both complex and disputed in legal philosophy. In a study of this kind it cannot be investigated at any depth. To give a hint of the problem a rough and incomplete description suffices however. There exists no general agreement concerning the norm notion. Different attitudes are embraced by natural law and legal positivism which are the two main branches of legal philosophy. Natural law assigns to the norm notion a wider extension than legal positivism. Few, if any, contemporary adherents of legal positivism would however equate the norm with the provision. Norms have been described variously. The following writings are arbitrarily selected from legal doctrine. According to Kelsen ‘the general norm created by the legislative body are called “statutes” . . .’ but “the function of ascertaining the existence of the general norm to be applied by the court implies the important function of interpreting this norm, of determining its meaning” ([29], pp. 257, 143). In a somewhat different context he describes norms as “the expression of the idea that something ought to occur” ([29], p. 36). Other writers have described norms as the meaning of the tokens physically expressing the norm ([38], p. 9), as rule complexes and conceptions which are only incompletely reflected by the provisions ([44], p. 178), as the essence as contrasted to the existence (i.e., statutory rules and the like) of law [28], etc.

Suffice it to say here that legal reasoning is the application of norms, not of provisions. Reaching the norm behind a provision requires that it be interpreted ‘legally.’ Several things affect this interpretation. To mention a few: the legal branch (penal law, contract law, etc.) to which the provision belongs; legislative manifests such as manifest 3 above; other provisions in the same regulation, also perhaps in other regulations, that ought to be juxtaposed and considered together with the rule at hand; contemporary ethical conceptions; etc.

The legal interpretation is thus complex. There exist, however, general principles for how to perform legal reasoning and these exercise control over the interpretation. Since legal hermeneutics is an important branch of jurisprudence some of these principles are fairly well understood and documented in legal doctrine. Most principles are however in want of further analyses.

Basing a representation of legal knowledge on provisions is appealing because such a representation would be transparent. But as we may infer from above this together with a uniform logical interpretation is insufficient. If this approach is chosen the provisions ought as far as possible be interpreted as what they are, i.e., depictions of legal norms. Providing such an interpretation involves complicated considerations. The first problem we arrive at is

\(^1\) Below the term ‘provision’ is used interchangeably for the provision as such and the ‘if . . . then . . .’ form it may be reformulated to. What meaning is intended will be clear from context.
to what extent may principles for how to interpret provisions be formalized and represented?

We must acquire an understanding of the principles before we approach the representation problem. The questions involved in problem 5 are connected to a recurrent theme in jurisprudence: the question ‘What is Law?’ In modern legal theory a two level model is, among others, advanced as the explanation. The model includes inter alia how legal rules are interpreted. Its originator, Hart, stresses the important role of what he terms secondary rules (metarules) and the following quotations from his quite influential work ‘The Concept of Law’ ([22], pp. 92, 79) are elucidative:

‘... they may all be said to be on a different level from the primary rules, for they are all about such rules; in the sense that while primary rules are concerned with the actions that individuals must or must not do, these secondary rules are all concerned with the primary rules themselves. They specify the ways in which the primary rules may be conclusively ascertained, introduced, eliminated, varied, and the fact of their violation conclusively determined”

and

‘... introduce new rules of the primary type, extinguish or modify old ones, or in various ways determine their incidence or control their operations.’

Hart argues that this model of explanation is nothing less than the very essence of jurisprudence

‘... we shall make a general claim that in the combination of these two types of rule there lies what Austin wrongly claimed to have found in the notion of coercive orders, namely, “the key to the science of jurisprudence.”’

Principles for how to interpret provisions have an important role. Deductive interpretation suffices for trivial cases only. Hart enumerates various ways in which secondary rules affect the interpretation. Provisions may be changed into covering more or fewer cases than the meaning of their premises originally suggested, they may be labeled obsolete, new rules may be introduced, etc.

Secondary rules control, e.g., how existing provisions may be interpreted by analogy. This means that a provision is modified in a certain way to make inferable conclusions that originally were not deducible. The extent according to which provisions may be modified depends on the current branch of law. For example, in penal law legal analogy is subjected to severe constraints but in laws of contracts there are less restrictions, on condition that the parties are equal. Knowledge concerning the more detailed application of analogy is thus demarcated to branches of law. The content and form of secondary rules for the inference depend on this and on the case in issue. In spite of these differences it is nevertheless possible to give a schematic description of legal analogical reasoning ([44], p. 71). This schema describes the common characteristics for secondary rules for analogy in all branches of law. It is impossible to settle once and for all what are the content and form of a certain field specific secondary rule. This holds for analogy as well as for other secondary rules. Therefore, schemata are particularly important. They constitute the most firm knowledge about the rules at these levels. We propose that both schemata of this kind and branch specific knowledge for their interpretation must be captured in an adequate representation of legal knowledge. Furthermore, that this calls for rules at a ‘tertiary’ level (meta-metalevel) and even higher levels with respect to provisions. Extending Hart’s terminology we term such rules tertiary, quaternary, etc. Below we refer to rules above the primary level as ‘higher-level rules.’

Our objective is not to capture legal rules as these look in their written form. Instead we focus on how legal experts conceive legal rules; i.e., the conceptions we here term legal...
norms. We must keep this perspective before our sight when we turn to next topic of the study, that of semantics: How do legal experts conceive legal rules and what semantics agrees best with this? The questions involved here are of fundamental import for the representation of legal knowledge. Also, they are important for understanding the role of the higher-level rules.

A look at provision 1 reveals several semantic problems. First, the rule contains a deontic expression and seems to be an imperative statement rather than a sentence in the indicative mood. This forces us to ask whether

(6) premises of provisions are capable of truth or if other semantic notions apply instead?

If the answer is that they are capable of truth there exists still a problem. Rule 1 has the premise ‘fair ground.’ This is a vague legal concept and the question is

(7) how do truth values get assigned to the legal concepts in a provision?

The existence of vague legal concepts entails that not even the truth notion itself can be taken for granted without analysis. We have the truth theoretical problem

(8) how shall we understand the notion of a premise being true in the legal domain?

This study is demarcated to the semantic problems 6, 7, and 8 and to the problem 5 about higher-level rules. It will be shown that in order to establish when higher-level rules should be used we must first acquire some understanding of the semantic problems. And inversely, a clarification of how higher-level rules function will give an important key for how to handle these semantic problems. The four problems 5, 6, 7, and 8 are mutually dependent and we therefore study them together.

Some problems above may raise metaphysical question e.g., the semantics of vague legal concepts. Since we focus on how legal experts conceive provisions we can disregard from such questions however. Our objective is to acquire an understanding of and possibly represent legal knowledge. Even if lawyers comprehend the reality in a way that seems incorrect we still aim to represent their knowledge. Therefore, we do not have to bother about the ontology of their knowledge. The important thing is that we really capture how lawyers and jurists comprehend law. We should be able to motivate the view of legal knowledge we advance e.g., by pointing to legal philosophy and/or empirical experiences from interviews with lawyers, etc.

**Summary.** Reformulated provisions are appealing as a basis for the representation of legal knowledge. In this case however, the representation must support an interpretation of the provisions as norms. Providing an interpretation of this kind includes a lot of problems. The specific purpose of this study is to investigate two categories of these problems, namely

(9) the semantic problems raised by premises and consequents of provisions,

and

(10) the problems of understanding higher-level rules for legal reasoning and the problems of integrating these in a representation.

It will be shown that categories 9 and 10 are interdependent. Before we approach how higher-level rules look we attempt to establish the circumstances under which they apply. We begin therefore with the semantic problems in category 9. Next, concerning category 10 the emphasis will be put first on how the higher-level rules informally should be understood. Here the main object of study will be the use of analogy in legal reasoning. The integration of higher-level rules in a formal representation on the other hand raises several hard questions.
We elaborate an attempt along an approach being close at hand [20]. This shows that there exist prospects for capturing higher-level rules in a runnable formalism. It is too early to settle what is the best solution however. Both philosophical and technical difficulties exist and we point out open questions in need of further analyses.

The rest of this thesis is structured as follows: chapter 2 examines the semantics of legal concepts and settles an appropriate approximation of legal knowledge. It ends up with an identification of the case when higher-level rules should be used. Chapter 3 shows the affinity between the higher-level rules used to handle vague legal concepts and those used for legal analogy. Chapter 4 shows, with an example from legal analogy, how rules at various levels function together. Chapter 5 chooses a formal representation for the lowest level. Chapter 6 discusses the problems of incorporating the higher-level rules in a formal representation. Chapter 7 concludes the study and directions for further research are pointed out.
2. THE SEMANTICS OF LEGAL CONCEPTS

2.1. The Problem Stated
Conceiving legal reasoning as a process of deductive application of provisions is inadequate. Deductive application is only possible when it has first been established whether or not the legal concepts in a provision apply to the current fact situation. This is the major and difficult part of legal reasoning. The main difficulty is that provisions, almost always, are composed of legal concepts the meaning of which is more or less unclear. This gives raise to semantic questions. We show in this chapter that the answers to these questions are determinative for when the higher-level rules come into operation.

The semantics of legal concepts is dependent upon the sentences of which they form parts, i.e., the provisions. To provide a setting and demarcation for the discussion let us look at a sample collection of provisions.

In older Swedish legislation provisions are often drafted as subjunctive sentences. For instance sec. 5 of the Sale of Goods Act is expressed as

\[(11) \text{"Is a sale of goods made but no price settled, [the consequence be that] the vendee pay what the vendor demands, where this cannot be deemed unreasonable."}\]

In contemporary legislation the subjunctive mood has been replaced by imperative statements. This is indicated either by the use of the imperative mood—exploited e.g., in penal law—or by deontic expressions such as in e.g., sec. 3 of the Consumer Sale of Goods Act

\[(12) \text{"Has not the vendor delivered the goods in due time and this is not due to the vendee or an event for which he runs the risk, the vendee may cancel the deal..."}\]

Provisions and the legal concepts they consist of give rise to difficult semantic problems. One reason is that provisions contain linguistic constructions the semantics of which is not fully understood. Take for example the mood of provision 11 and the modality used in provision 12. The semantics of sentences in the indicative mood is fairly well understood. The model theory of classical logic is a semantic system for indicative sentences. The semantics of subjunctive and imperative statements is less well understood however. These and other things provisions may contain are difficulties of considerable proportions within the philosophy of logics. Citing Davidson there remains "a staggering list of difficulties and conundrums" ([8], p. 321), to be solved before we know how to represent adequately subjunctive and imperative sentences and sentences containing probability and causal statements, adverbs, attributive adjectives, verbs of belief, perception, intention, action, etc. Provisions may contain everything in this list.

The Reformulation Problem. This study will not get involved in the philosophical problems of analyzing the structure of sentences containing linguistic forms such as those just listed because of two reasons. The first is as follows. The achieving of a one-to-one correspondence (a direct mapping) between the provisions and their representation is no end in itself. This study focuses on how lawyers reason about provisions. It would be a mistake to presuppose that a lawyer is bound to the literal form given a provision in the legislation. It is quite clear that the lawyer is free to interpret provisions differently and also that he does so. The reason why a provision is given a certain formulation is to be found in the issuer, i.e., the law-maker. It is natural from the law-maker’s point of view to express provisions as commands. At the time provision 11 was made law the common legal drafting technique

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2 In Swedish: “Är köpet slutet utan att priset blivit bestämt, erlägg en köparen vad säljaren fordrar, där det ej kan anses oskäligt.”

3 In Swedish: “Har säljaren ej avlämnat varan i rätt tid och beror det ej av köparen eller händelse för vilken denne står faran, får köparen häva köpet.”
for expressing commands was to use the subjunctive mood (coniunctivus hortativus), the contemporary technique is to use the imperative mood or deontic expressions. What is natural from the issuer’s point of view does not have to be natural also for the addressees. The group of addressees we are interested in here is lawyers carrying out the practical legal work of applying provisions to fact situations. It is not reasonable to assume that these reason about provisions such as rules 11 and 12 as subjunctive and imperative sentences, respectively. I argue that in his practical work the lawyer disregards the linguistic form given to the provision by the law-maker. The lawyer applies instead a counterpart to the provision reformulated to indicative sentences which could look something like:

(13) If a sale of goods is made and no price settled and the vendor demands a reasonable amount and the vendee pays that amount then the vendee fulfills his obligation.

and

(14) If a sale of goods is made and no price settled and the vendor demands a reasonable amount and the vendee does not pay that amount then the vendee fails to fulfill his obligation.

and

(15) If the vendee fails to fulfill his obligation then a lawsuit is successful.

etc.

These sentences contain the connective ‘if...then...’ If the sentences are to be viewed as purely indicative we must understand this connective as a material implication. Thus conceived, the sentences express legally acceptable states of affairs. The sentence 15, for instance, expresses that the following state of affairs is not legally acceptable: “the vendee fails to fulfill his obligation and a lawsuit will be unsuccessful.” The lawyer applies provisions to fact situations. A fact situation may be understood as a state of affairs. Interpreting the provisions as describing legally acceptable states of affairs is not indisputable but seems at least rather natural.

The Problem of Truth and Indicative Sentences. The position put forward above may be questioned by maintaining that in legal reasoning norms are treated, not as descriptions being indicative sentences, but as imperative sentences, cf. e.g., Ross [40,41]. Within philosophy and legal theory it has been put forward that legal norms (and their parts) are not capable of ‘truth’: ‘Many philosophers and logicians have thought that norms are essentially void of truth-value, “outside the realm of truth and falsehood,” belong to “practical” as distinct from “theoretical” discourse.’ [54]. These opinions cannot simply be ignored. It is necessary to give a further and more detailed motivation concerning why indicative sentences and truth are appropriate for depicting the kind of knowledge focused on in the study.

We argue that the question whether norms are capable or incapable of truth cannot be answered without taking into account the category of lawyers for which the question is put. Norms are conceptions and different groups of lawyers have disparate conceptions. The thought that norms cannot be comprehended as true or false is intuitively acceptable concerning some groups but counterintuitive as to other groups. The answer depends on the function carried out by the lawyers in the respective group.

A main function of a law-maker is to issue norms. How does a law-maker understand these norms? The law-maker gives a command when he issues a norm. In this role the norm does not convey information, neither to the law-maker nor the addressees, that something is or is not the case. Therefore, it is close at hand to assume that a lawyer who carries out the
legislative function does not comprehend norms as true or false. Norms are rather orders, permissions or authorizations to him, cf. e.g., Kelsen ([31], p. 73) “... the norms enacted by the legal authority, imposing obligations and conferring rights upon the legal subjects are neither true nor false, ...”

But again these are the norms as viewed from the law-maker and not from the group of addressees of interest here, i.e., lawyers who in practical legal work apply norms to fact situations.

A lawyer carrying out this function does not comprehend norms as commands, only as descriptions to be applied to fact situations. He uses his knowledge of norms in a way that exhibits no real difference from the way he uses his knowledge of the surrounding world. He applies in both cases description of norms respectively descriptions of objects to fact situations. If a description applies it is true relative to the fact situation, otherwise false. Here the thought that norms are incapable of truth is not only counterintuitive, but, I maintain, incorrect. The adequate representation of this norm knowledge is indicative sentences and therefore classical logic could be suitable.

This group of lawyers has a knowledge of norms corresponding closest to what Kelsen terms ‘rules of law.’ “The statements formulated by the science of law ... do not impose obligations nor confer rights upon anybody; they may be true or false” ([31], p. 73). A rule of law states the existence of a norm and describes it, and it is accordingly true or false. Raz gives the following more formal description of rules of law (which he terms normative statements): “One may say that a normative statement has the general form that p ought to be the case, and that it is true if, and only if, there is, in a certain normative system, a norm to the effect that p ought to be the case” ([39], p. 47). The ‘descriptive’ view of legal norms is rather accepted, for discussions cf. Hedenius [24,25], Strömholm ([44], p. 95). Kelsen for a long time defended this view, e.g., ‘But the “ought” of the legal rule does not have a prescriptive character, like the “ought” of the legal norm—its meaning is descriptive. This ambiguity of the word “ought” is overlooked when ought-statements are identified with imperative statements’ ([31], p. 75). Kelsen later abandoned this position ([30], p. 2). As explanation he gave the existence of contradictory norms. This seems rather to be a confusion between the object and the metalevel than an argument against classical logic however.

Studies of practical legal work strengthen the assumption that when lawyers apply norms they conceive these as indicative rather than imperative or subjunctive sentences. They describe their knowledge in terms of indicative sentences and seem to treat them as such in their reasoning [21]. This is not to say that the imperative approach ought be rejected however. Important groups of lawyers carry out other functions than the application of norms, e.g., law-makers. Judges have besides the function of applying norms an executive function as well. A norm issued by the law-maker conveys a command that the judge should (i) apply the description inherent in the norm to relevant fact situations, and, (ii) in his turn issue a command as to the execution of the consequences of the norm. Perhaps imperative sentences are necessary in a representation of this kind of legal knowledge and possibly also of the layman’s knowledge of law.

This was the first reason why a part of Davidson’s list [8] might be unnecessary in a representation of legal reasoning. The second is as follows. It cannot be denied that there may exist cases where the representation of the kind of knowledge we are interested in must include linguistic constructions such as those listed by Davidson. For instance, lawyers

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4 In legislative material the law-maker sometimes illustrates a norm’s applicability by giving model cases. When he assumes this function the law-maker treats the norm as true or false.

5 We have chosen in this study to use the term ‘legal norm’ for the conceptions and rule complexes behind a provision, cf. e.g., Strömholm ([44], p. 178). Kelsen uses the term in a different sense here: a ‘legal norm’ is the prescription a legal rule expresses to its addressees. Therefore, a legal norm is an imperative sentence, in contrast to the rule of law which describes a legal rule and is an indicative sentence.
perhaps conceive provisions as causal statements and consequently the interpretation of ‘if...then...’ as material implication is inadequate. However, that sentences may be causal statements or comprise other linguistic forms in Davidson’s list is a general problem in knowledge representation. Before a more common agreement as to their representation in general is settled it is premature to discuss whether some specific analysis is needed when sentences of this kind express legal arguments.

**Vague Legal Concepts.** This background renders it reasonable to put a demarcation to the study here. We will only discuss the semantics of provisions that lawyers reformulate and interpret as indicative sentences, and, moreover, only those of these that comprise only the classical truth-functional connectives.

With this demarcation the remaining source to semantic problems is that the legal concepts in the provisions often have an unclear meaning. The problem of subsumption—i.e., to assess whether the legal concepts in provisions apply to a certain fact situation—is an essential element of practical legal work. When vague concepts are involved this problem is particularly awkward.

Recall rule 2.

\[(2) \text{if fair ground for dismissal exists then the employee may be dismissed.}\]

This rule is applicable to fact situations involving an ‘employee,’ a ‘dismissal,’ and a ‘fair ground.’ Intuitively, it is close at hand to distinguish the first and second from the third. We are inclined to understand the two former as the names for physical or primitive ‘facts’ but the latter as something ‘non-physical’ and more diffuse, e.g., a name for a moral judgment. The reason why we deem ‘employee’ and ‘dismissal’ as ‘primitive’ is that we have a clear conception of what an ‘employee’ and a ‘dismissal’ are. They need no further definition. Concerning ‘fair ground’ we have no such clear conception. For want of a more appropriate name we will term concepts such as ‘employee,’ ‘dismissal,’ etc. ‘sharp’ legal concepts. Concepts such as ‘fair ground’ we term ‘vague’ legal concepts.

Legal concepts have what Hart terms an open texture ([22], p. 124), i.e., “whichever device, precedent or legislation, is chosen for the communication of standards of behaviour, these, however smoothly they work over the great mass of ordinary cases, will, at some point where their application is in question, prove indeterminate.” This means that which concepts are to be considered as sharp and vague, respectively, depends upon several factors. Of course, some concepts are intrinsically vague, e.g., ‘fair ground.’ Also, however concepts may be vague under some circumstances but sharp under other. The user-category exercises influence here. Users, who are unacquainted with the conditions under which someone is to be regarded as an employee, will consider that concept as vague. The legal matter in hand is also of importance. There exists no ultimate and universal level of detail that can be reached e.g., by defining ‘employee’ as a ‘person’ that has the properties so-and-so, cf. Gardner ([14], p. 95), Oliphant [37]. Most often some other legal matter exists where this definition does not suffice. Concerning ‘person’ we have for instance the law on abortion in which ‘person’ itself becomes a vague concept: whether a physician who illegally aborts a foetus be sued for man-slaughter depends upon whether the foetus is considered a person or not. The purpose of a knowledge system often is determinative for which concepts are to be considered as vague and sharp, respectively.

In a reasonably expressive formal language we have means for depicting entities being objects, relations, and functions. The set of entities supposed to be depicted by the symbols of a language may be termed the language’s conceptualization [15].

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6 There exist various reasons why legal concepts are vague. The concept ‘fair ground’ is an example where the legislator has intentionally omitted guidelines concerning how the concept should be understood. This is a law-making technique that may be adopted when the legislator considers it appropriate to let legal usage, doctrine, etc. gradually give a legal concept content. Another reason is that the use of sharp legal concepts would render the law unmanageably huge and hard to survey.
A lawyer has a depiction of the physical reality. When the lawyer uses sharp legal concepts in his reasoning he refers to this depiction. For the sake of argument, assume that ‘employee’ is a sharp legal concept. This would mean that there existed a rather precise idea of what the statement “John Doe is an employee” stands for—‘employee’ denotes a relation and ‘John Doe’ an object and both the relation and the object are ‘intelligible’—and therefore it would be reasonable to depict the concept as the sole primitive fact: ‘Employee(John_Doe).’

But how is the position concerning vague legal concepts?

Vague concepts, e.g., ‘fair ground’ in rule 2, give rise to difficult problems. Since we have no precise idea as to what a vague concept refers to, it is not reasonable to depict these only as sole primitive facts: e.g., ‘Fair_ground(John_Doe).’ A lawyer uses vague concepts extensively in his reasoning. It is interesting to examine to what extent such concepts can be represented; this question is crucial for the success of legal knowledge systems. Before we undertake the representation problem however, we must acquire a deeper understanding of vague legal concepts, in particular of the lawyer’s knowledge of these concepts. We will term this topic the epistemic question henceforth.

On one side we have legal rules and legal concepts that form part of these and on the other side a counterpart to these concepts—the entities in the conceptualization. Meaning is given to the legal concepts by their relation to the entities of the conceptualization. It is not obvious what semantic notion most adequately captures this meaning in a legal context. One competitor to truth values is, for instance, performance values, cf. von Wright [52]. In this study the position is taken up that truth is an appropriate notion at least for the kind of legal knowledge we focus on. This choice has been motivated above. But a conclusion that truth is an appropriate notion does not suffice. We need a deeper understanding of the truth notion applied to legal concepts. The reason is the existence of vague legal concepts such as ‘fair ground.’ That a concept is vague means that it is not clear what it refers to. A denotational semantics for such concepts raises therefore several questions. So, in specific for vague legal concepts, we must investigate what it means for these to be true. We will term this the truth theoretical question henceforth.

To summarize: The existence of vague legal concepts raises ‘epistemic’ and semantic questions. The semantic question has a truth theoretical nature. The ‘epistemic’ question is:

- What does the lawyer’s knowledge of vague legal concepts consist of?

and the ‘truth theoretical’ question is:

- What does it mean for a vague legal concept to be true?

2.2. The Epistemic Question

We begin by considering the epistemic question. Legal knowledge consists to some extent of ‘primitive facts’ which are reasonable to depict as objects, relations, and functions. Is it possible to describe the knowledge about a vague legal concept in terms of such facts? If so, to what extent? Are there any other intelligible entities besides these facts?

The point of departure below is the way expert lawyers prefer to describe their knowledge. The experiences accounted for are acquired during a project focusing on the treatment of vague legal concepts in practical legal work within the Swedish legal system [21].

Let us, as an example, consider the vague legal concept in rule 2 ‘fair ground’ for dismissal. If we asked an expert lawyer to share with us his knowledge of this concept, he would, in all probability, begin by enumerating a number of descriptions of fact situations in which he knows that fair ground has been deemed applicable and non-applicable, respectively [21]. These descriptions often contain ‘precise’ objects, relations and functions. A ‘positive’ fact situation—i.e., a situation in which ‘fair ground’ applies—is e.g.,
(16) The absence (a function) of the employee (an object) is $t$ and $t$ is greater than (a relation) one week and the position (a relation) of the employee is ordinary and no extenuating circumstances (an object) have been alleged (a relation).

and a negative fact situation is e.g.,

(17) The absence (a function) consists of shorter delays (a relation) and the employer (an object) has not taken (a relation) any measures in order to handle the situation (an object).

An expert’s knowledge of a vague concept comprises often a lot of descriptions of fact situations such as descriptions 16 and 17. These form two collections, a ‘positive’ and a ‘negative.’ The positive collection includes descriptions of fact situations relative to which the concept has been considered applicable, e.g., by a court. The negative collection includes ditto relative to which the concept has been considered non-applicable.

Although the extensions of the collections vary as legal usage evolves and the moral attitude of the society changes it is fair to say that they constitute the most firm content of a vague legal concept. A vague concept may lack collections however. An obvious presupposition is that the applicability of the concept relative to the fact situations has been examined e.g., by a court in an adjudication, or by the legislator as model situations described in legislative materials, or in legal writings by a legal theorist the authority of which, on this specific issue of law, is widely accepted in the judiciary (an authority of this kind is rare but exists nevertheless concerning some issues) or in some other way which is being recognized as authoritative.

Besides these more or less ‘reliable’ collections we will encounter other entities. When the expert is invited to give a more full account of the concept he will probably point out general criteria that always must be fulfilled for the concept to be applicable. Concerning ‘fair ground,’ one such general criterion is that

(18) the misbehaviour implies the unlikelihood of betterment.

Criteria such as criterion 18 are true and false, respectively, relative to two sets. These are subsets of the positive and negative collections of the vague concept to which the criterion belongs. The criterion ‘unlikelihood of betterment,’ for instance, is clearly the case relative to the description 16 and not the case relative to description 17. Were this its sole content, a general criterion would not add anything to our understanding of the vague concept. But something more than this seems to exist. In most cases, an expert lawyer has some ability to judge whether or not a new situation fulfills the criterion. In performing this judgment it seems clear that he makes use of external aspects, such as moral opinions. It is however reasonable to assume that the lawyer, rather than making a ‘free judgment,’ reasons in quite a structured way concerning these aspects (cf. e.g., Sundby’s theory of norms ([46], p. 214) and for discussions on this theory Bing [4] and Hasseltvedt, Herrestad [23]). There normally exists a genuine possibility of making a correct prognosis of what the conclusion will be in a legal case ([44], p. 181). That the lawyer reasons in a ‘structured way’ means that higher-level rules exercise control over his reasoning.

Approximations of Legal Knowledge. We are in this study concerned with certain fundamental aspects of legal knowledge. An abstraction from details of less importance is necessary. Therefore, let us from now on understand legal expert knowledge according to the summary given in statements 19 and 20.

(19) The knowledge concerning a sharp legal concept, such as ‘employee’ in rule 2, consists of one or several primitive facts. Concerning a vague legal concept, such as ‘fair ground,’ the knowledge consists partly of two collections of descriptions of fact situations relative to which the concept has been considered applicable or non-applicable, respectively.
(20) Besides the two collections the knowledge concerning a vague concept also includes
general criteria. It is reasonable to assume that the test whether or not a new
situation fulfills these criteria is controlled by higher-level rules. These would thus
control the classification of new situations to either of the two collections.

Of course, the statements 19 and 20 are only approximations and simplifications of
actual legal knowledge. But they are sufficiently adequate for discussing the problems taken
up in the remainder of this paper. Approximations 19 and 20 have a support in practical
evidences. They correspond to the way expert lawyers prefer to describe their knowledge
[21].

Approximation 20 is a resort when approximation 19 does not tell whether a legal
concept is applicable or non-applicable. This resort should be used with care and only when
necessary. It provides at the best a well-founded guess concerning the applicability of the
concept. How do approximations 19 and 20 complement each other? We need not resort
to the higher-level rules of approximation 20 in case the denotations of a concept suffice
to establish whether it is applicable or non-applicable. The higher-level rules discussed in
approximation 20 are applicable precisely when it is uncertain whether a vague legal concept
denotes a certain situation description. Approximation 20 is thus applicable precisely when
approximation 19 does not provide any further understanding as to the applicability of a
legal concept.

An adequate denotational semantics for legal concepts should coincide with approxima-
tion 19. Such a semantic system should capture the applicability and non-applicability of a
legal concept in all circumstances under which approximation 20 should not be resorted to.

The next section contains a discussion aiming to take some steps towards such a seman-
tic system. It leads up to the identification of circumstances under which it seems unfruitful
to discuss semantics further and where a resort to the higher-level rules of approximation
20 therefore seems motivated. In the subsequent chapters problems of understanding these
higher-level rules and the difficulties of capturing them in a representation are discussed.
That discussion is not dependent on whether the mentioned identification in next section is
correct.

The discussion in next section concerning the circumstances under which the higher-
level rules should be resorted to will, if it is correct, shed light also on the question whether
or not classical logic agrees with legal concepts.

2.3. The Truth Theoretical Question
One objective of this study is to acquire a deeper understanding of the higher-level rules of
approximation 20. But equally important is to establish when the higher-level rules at all
should be resorted to. That a sentence contains vague legal concepts does not in all cases
entail that higher-level rules must be used for assessing whether the sentence applies to a
fact situation. The cases where this can be done without the use of higher-level rules must
be identified. The connection to semantics here is that, ideally, we should choose a semantics
such that a truth value is assigned to the sentence in all cases where it is natural to do so
without the help of higher-level rules.

We call this a truth theoretical question simply because its answer depends on how the
truth notion is conditioned in the legal domain. We have no intention of classifying the legal
truth notion to truth theories such as correspondence, coherence theories, etc.

Truth Values of Composite Sentences. A vague legal concept is ‘unknown’ relative to
a situation description that deviates from its two collections. A concept being ‘unknown’ in
this sense may be a component of a sentence, e.g., a provision. Higher-level rules should be
used to handle the truth value ‘unknown’ but only if the sentence as a whole is ‘unknown’
relative to a fact situation. That is, higher-level rules should be used only if it cannot be
established otherwise that the truth value of the whole sentence is ‘true’ or ‘false.’
The trivial case where higher-level rules should not be used is of course when the sentence does not include any components the truth values of which are ‘unknown.’ But also when the sentence does, there exist cases where it would be natural for a lawyer to assign truth values ‘true’ or ‘false’ without the use of higher-level rules.

Take as an example the sentence that describes ‘theft’ in the Swedish legislation. A direct translation of the original provision in the Swedish penal law is: “Anyone who without permission and with the intention of misappropriation takes what belongs to another should be sentenced, if the misappropriation entails injury, to at most two years imprisonment for theft.” The sentence is composed of two vague legal concepts: ‘taking without permission’ and ‘intention of misappropriation.’ As described above, the provision can roughly be reformulated to the sentence:

\[(21)\] if ‘taking without permission’ and ‘intention of misappropriation’ then ‘theft.’

‘Robbery’ is also composed of several legal concepts. A translation of the original provision is: “Anyone who steals by the means of violence, or threat constituting, or with respect to the victim apparently constituting, a pressing danger or . . . should be sentenced to imprisonment for robbery.” Reformulated this provision becomes:

\[(22)\] if theft is carried out by: ‘the means of violence’ or ‘threat constituting a pressing danger’ or ‘threat apparently constituting a pressing danger from the victim’s point of view’ then ‘robbery.’

In the antecedent of sentence 21 the first legal concept is ‘taking without permission.’ Assume a judge had established that this concept was false relative to the situation description at hand. Quite indisputably he would in this case stop his investigation and conclude that the antecedent as a whole was false. And if he failed to establish the truth or falsity of ‘theft by the means of violence’ in the antecedent of the sentence 22 his conclusion would yet be that the antecedent as a whole is true if ‘threat constituting a pressing danger’ proved to be true.

It is reasonable to assume that a lawyer treats legal concepts that have the truth value ‘unknown’ as described above. That is, despite the presence of ‘unknown’ a disjunction is considered ‘true’ if one of its disjuncts is true and a conjunction ‘false’ if one of its conjuncts is false. The sentence will get the truth value ‘true’ or ‘false’ if it is ‘true’ or ‘false’ independently of the truth value the ‘unknown’ concept later proves to have. So the presence of a concept being ‘unknown’ does not necessarily entail that higher-level rules must be applied. This is only necessary if the sentence as a whole has the truth value ‘unknown.’ That is, when the truth values of its components do not suffice to establish the truth value of the whole sentence.

**Semantic Systems.** We have above described how the truth value ‘unknown’ functions in the legal domain. Is this behaviour reflected in any existing semantic system? To approach that question let us first consider what is the semantic status of the truth value ‘unknown’ compared to the classical truth values ‘true’ and ‘false.’

Is ‘unknown’ a third truth value on a par with the classical truth values ‘true’ and ‘false’? If so there could exist an incompatibility between vague legal concepts and classical logic since classical logic is bivalent. Does bivalence agree with vague legal concepts? In order to answer this question we must first clarify what is meant by ‘bivalence.’

Advocating that bivalence means that

\[(23)\] each wff is either true or false and that these truth values are always known or easily established

would be to rule out of court any truth values besides ‘true’ and ‘false’—our ‘unknown’ for example—and recognize as bivalent only formalisms such as propositional calculus and the like. This is not how bivalence is commonly understood.
The common understanding of bivalence is that

(24) each wff is either true or false

without any statement concerning how these truth values are established. Classical predicate calculus is bivalent in this sense and also e.g., Tarski’s truth theory [19,47].

Conception 24 does not forbid intermediate ‘truth values.’ A formula may be ‘unknown.’ ‘Unknown’ may however not be a truth value on a par with ‘true’ and ‘false.’ That is to say, the assignment of ‘unknown’ may not be understood as representing precisely the idea that the formula is neither true nor false. But it may be understood as an epistemic variant of a classical truth value i.e., that a classical truth value exists for the formula but in this case it is concealed from us.

Does ‘unknown’ have an equal semantic status as ‘true’ and ‘false’ or is it just to be considered as an epistemic variant of these? Let us again take a look at practical legal work. What happens when a court (or some other authoritative body) examines the truth value of a concept relative to an arbitrary fact situation? There are three possible results of such an investigation: the court may conclude

(25) that the fact situation shall be classified to the positive collection

or

(26) classified to the negative collection

or

(27) that the claim be rejected, since it is absurd.

The court is not free to conclude that the concept is ‘unknown’ relative to the fact situation. There exists a prohibition against such conclusions, the non liquet prohibition, the violation of which is considered a serious mal-practice termed déni de justice, cf. ch. 3. Result 27 does not express that the court considers the concept as ‘unknown’ relative to the fact situation. Rather it says that the concept cannot be meaningfully applied to the fact situation.

At any given point of time the knowledge of a vague concept consists of two collections of fact situations which have been previously adjudicated by the judiciary. What about other fact situations? Relative to these the truth value of the concept will be unknown to us as long as they have not been adjudicated by any authoritative body. So in practical usage ‘unknown’ seems to be an epistemic notion. ‘Unknown’ is not a definite truth value. As the investigation progresses it will be replaced by ‘true’ or ‘false.’

It seems thus that ‘unknown’ in a legal context is just an epistemic variant of the classical truth values and consequently vague legal concepts are bivalent in the sense of conception 24. So the compatibility between legal concepts and classical logic cannot be refuted by reference to the fact that vague legal concepts are ‘unknown’ relative to fact situations not yet tried.

Some doubts may still remain concerning how bivalence agrees with the court’s result 27. Take as an example the legal concept ‘heir’ and a fact situation involving Fido and his offspring Caro. It may be conceived as meaningless to apply ‘heir’ here. Dogs do not inherit each other. To say that “Caro is the heir of Fido” lacks a classical truth value relative to the fact situation may seem more adequate than to say that it is false.

This has no consequences for the applicability of laws of classical logic however. The conditions under which something is meaningful are ‘preparatory’ to the laws of logic. If the concept ‘heir’ is meaningful with respect to human beings only, there exists a logical type ‘human being’ that must be satisfied before it is at all meaningful to apply the laws of logic to the concept, cf. Russell [42], von Wright [53]. It is thus not meaningful at all to apply
the law of excluded middle (bivalence) to the concept `heir' before it has been affirmatively answered that the objects involved are `human beings.'

A system may thus be bivalent although it contains formulas that lack truth values if the lack is conceived only as an epistemic variant of the classical truth values `true' and `false.' Examples of such systems are Kleene's three valued logic ([32], p. 333) and Kripke's truth theory [33].

In the examples about `theft' and `robbery' above, we described how a lawyer in practical work probably treats a sentence in which there exist legal concepts with truth values `unknown.' This corresponds well to Kleene's and Kripke's logics. Roughly, the two systems have the following semantics: composite wffs, that have components the truth value of which is `unknown,' will only get the truth value `true' (`false') if they are `true' (`false') independently of the truth values these components later prove to have. Otherwise the composite wff is `unknown.' For instance, a disjunction is true if one of its disjuncts is true and a conjunction is false if one of its conjuncts is false.

The intention here is not to give a definition of the legal truth notion. Before that can be done a lot of difficult questions must be examined and answered. We will confine ourselves here to the tentative observation that the semantics of Kleene's and Kripke's logics seems to correspond rather well with an intuitively acceptable view upon sentences containing vague legal concepts. When a composite wff is used to represent a sentence containing vague legal concepts, there seems, for instance, to exist no reason to let a component, the truth value of which is `unknown,' dominate the whole compound as in Bochvar's logic [5].

Summary. We have in this section discussed the notion of truth in a legal context. We have concluded that vague legal concepts give rise to a truth value `unknown.' This truth value is only an epistemic variant of the classical truth values `true' and `false.' In practical legal work sentences containing concepts being `unknown' are assigned truth values in a way close to the semantics of Kleene's three-valued logic. The question remains how we shall treat the case when a sentence, composite or non-composite, has the truth value `unknown' according to this semantics.

That is, where the truth values of true or false components do not suffice to give the sentence of which they form part the truth value `true' or `false.' For instance, when one of the conjuncts in a conjunction is `true' but the other `unknown.'

A concept has the classical truth value `true' relative to a fact situation if it is known to denote something in the fact situation, `false' if it is known not to denote anything in the fact situation. Case 28 exists precisely when the legal concepts that have truth values in a sentence do not suffice to determine the truth value of the whole sentence. Therefore it does not seem possible to reach any further by discussing denotational semantics here. Thereby we have reached the goal of this section. The circumstances are identified under which the higher-level rules of approximation 20 should be resorted to. They coincide with case 28 if the discussion above is correct.

Here we must modify our previous statement that the semantic system should be such that it captures all cases where higher-level rules should not be resorted to. The semantic

2 "The thought that before the condition of truth for a sentence at all can be asked for one has to be acquainted with the conditions under which the sentence is meaningful, has with good reason been nominated as one of the greatest and most influential discoveries of modern logic. One may say, that it bridges logic and semantics" ([53], p. 74). Original text in Swedish: "Tanken att man först måste känna villkoren för att ett uttryck skall vara meningsfullt, innan man kan fråga efter villkoren för dess sanning, har med rätta kallats en av den moderna logikens största och följdikaste upptäckter. Man kan säga, att den slår en bro mellan logik och semantik."
system is in itself an interpretation principle, i.e., it is given by higher-level rules. These
higher-level rules should be such that they capture all situations where higher-level rules
that modify the knowledge of lower levels should not be resorted to, i.e., all cases except
case 28.

In next chapter we approach how higher-level rules should informally be understood. We show that the higher-level rules of approximation 20 are similar in function to another
category of higher-level rules, those for reasoning by analogy from provisions. The chapter
ends with an informal description of a schema for higher-level rules in that category.
3. VAGUE LEGAL CONCEPTS AND ANALOGICAL REASONING IN LAW

The characteristics of the legal truth notion collected in section 2.3 lead up to the case 28 in which we could no longer refer to semantics for assigning truth values to legal concepts. Establishing truth values is particularly awkward in this case. We lack a theoretical basis for truth value assignment but are at the same time forced to assign at least a tentative truth value. It is not necessary to motivate in detail why case 28 is important. It is quite obvious that if case 28 is not handled somehow the limitation is to an extent of rendering a potential knowledge system useless. We will in this section discuss some steps towards a treatment of case 28.

First however, for the sake of completeness, we must note that it is not necessarily so that all cases except case 28 are unproblematic. In these it must be established that a sentence containing a vague legal concept is true or false. The sentence could in principle have variable parts—i.e., be a propositional function—and have quantifiers. Such a sentence could yet be true or false, namely if it is a theorem or its contradiction is a theorem. Since it may be undecidable whether a sentence of this kind is a theorem [7,49] it could be impossible to establish its truth value. In practice however, we can probably disregard from this. The sentences we focus on are antecedents to legal rules and as such they could hardly be theorems. However, we have not analyzed this question at any depth and cannot assert anything with certainty here.

It is rarely the case that a vague legal concept or a sentence containing such a concept is true or false relative to a situation description. In practice, case 28 seems to be the most likely. The truth value of the vague legal concept or the sentence will be ‘unknown’ and the problem is to decide to which collection the fact situation should properly be classified. The higher-level rules of approximation 20 come here into operation and must be represented in order to handle case 28.

Before we can begin to discuss how to actually represent higher-level rules we need a deeper understanding of rules at these levels. We will try to acquire this by examining reasoning by analogy in law. Legal analogy has been analyzed by several writers in legal philosophy. The result of these analyses can be used as a source for finding higher-level rules for legal analogy and it is therefore an appropriate object of study here. In the next section a background concerning analogy is given and it is explained why legal analogy gives rise to similar difficulties as the higher-level rules of approximation 20.

3.1. Analogical Reasoning in Law

Inference by analogy is used frequently by human beings. The difference between analogy and deduction was observed early. Already Aristotle distinguished analogy from deduction and induction [27], p. 52 f.) and in the work ‘On methods of proof,’ last century B.C., Filodemos pointed out that analogical inferences do not follow from the premises by necessity as deductive do ([50], p. 142). In contrast to deduction we have rather elusive intuitions concerning inferences based on similarity, such as analogy. In legal reasoning inferences based on similarity are frequently used. Besides analogy we have e.g., the commonly used inference e contrario, i.e., a case not subsumed under a provision may be settled contrary to what the rule prescribes if the ‘silence of the law’ can be assumed intentional; the candidate

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provisions are selected due to their similarity to the case in issue. In this light pessimism concerning the possibility of representing legal knowledge may be close at hand. It cannot be denied that some affinity exists between general and legal analogy. The prospects of representing legal analogy are far better than for general analogy, however. The former is more restricted and well-understood than the latter. There exists a demand for a legal justification and within jurisprudence descriptions of the inference rule have evolved.

Analogy has during the last centuries become an inference of increasing importance to legal reasoning. The reason why may be summarized as follows.

In our modern society it is obviously impossible to provide a written rule for every conceivable situation of legal relevance. Yet, a court is always forced to come up with a conclusion for every situation brought before it according to the non liquet prohibition (except for situations that obviously lack every legal implication imaginable)\(^9\) ([44], p. 375, [45], pp. 146 ff.). The court may not reject a situation on the ground that it is not covered by any regulation. This would be to commit a serious malpractice termed denial de justice\(^10\).

Furthermore, a fundamental component in the Swedish and related legal systems is predictability. “This requirement follows from the very idea of a legal system. There must exist an identifiable set of norms . . . the ordinary member of society must have a possibility to recognize the existence of law regulating his conduct.” ([10], p. 125). The principle of predictability is part of the idea of legal security (the doctrine of ‘rule of law’) ([44], pp. 231, 390). (We avoid the more common term ‘principle of legality’ here since it is sometimes used to denote a variant of the principle of predictability demarcated to penal law.)

On one hand a court must thus adjudicate every fact situation brought before it and on the other hand maintain the principle of predictability. This gives rise to a conflict concerning fact situation not directly subsumed under existing provisions. The compromise solution is to preserve, while obeying the non liquet prohibition, as much predictability as possible. Sometimes this is achieved by applying an inference called analogia legis.

Analogia legis may be applied to fact situations that bear some resemblance to existing provisions. A presupposition is that the fact situation at hand obviously is outside the extension of existing provisions. The legal concepts in these may accordingly not be so vague that they allow the fact situation to be subsumed ([44], p. 408). The court strives to choose the provision being, in the relevant sense, most similar to the current fact situation. This provision is modified to a rule which subsumes the fact situation, a rule that becomes part of legal usage and may be applied in later cases. The court thus constructs new analogous rules. In order to maintain predictability and distinguish relevant from irrelevant similarity the court must use higher-level rules when it constructs these analogical rules.

Besides fact situations that resemble existing provisions we have those that have no resemblance whatsoever to provisions in existing regulations. For them the solution must be derived from general legal principles. The process is called analogia iuris but, despite its name, it is not an analogical inference according to legal doctrine ([44], p. 411). We will not further discuss this group here.

### 3.2. The Affinity between Vague Legal Concepts and Analogia Legis

**The Affinities.** There exists a functional affinity between on one hand analogical reasoning called analogia legis and on the other hand analogical reasoning in context of vague legal concepts. To illustrate this let us consider what the two inferences do. Analogia legis is to construct from a provision such as

\(^9\) cf. above the distinction between meaningless and meaningful.

\(^10\) The original formulation of this principle is “Le juge qui refusera de juger, sous prétexte du silence, de l’obscurité ou l’insuffisance de la loi, pourra être poursuivi comme coupable de déni de justice.” It was introduced in Code Civil art. 4 after the French revolution and acquired gradually acceptance in the Scandinavian legal system during the 19th century.
If a sale of goods has been made but no price settled then the vendee should pay what the vendor demands if reasonable.

An analogous rule that subsumes a new fact situation. For example,

If a hire of goods has been agreed but no price has been settled then the hirer should pay what the letter demands if reasonable.

A vague legal concept such as ‘fair ground’ has a positive and negative collection of fact situations e.g.,

The absence of the employee is t and t is greater than one week and the position of the employee is ordinary and no extenuating circumstances have been alleged.

and

The absence consists of shorter delays and the employer has not taken any measures in order to handle the situation.

Here analogical interpretation is used to determine whether a new fact situation is similar enough to the previously ruled fact situations and therefore should be included in either of the two collections. Analogical reasoning of this kind as well as the analogical reasoning termed analogia legis deal with new fact situations and do similar things. In both it is judged whether there exists enough similarity between a new fact situation and, in the case of analogia legis, an antecedent of a provision, or, in the case of analogical reasoning in context of vague legal concepts, a previously adjudicated fact situation.

**The Differences.** Between analogia legis and analogical reasoning in context of vague legal concepts there exist also differences. Not unexpectedly, these are caused by the fact that the two inferences deal with vague and sharp concepts, respectively.

Let us begin with analogia legis. Due to the principle of predictability the structural similarity between the fact situation at hand and the antecedent in the rule is important here. Assume that the legal concepts are all sharp in the antecedent of the rule Sale $\land A_2 \ldots A_n \rightarrow B$. We require that the fact situation at hand should be interpreted to Sale $\land A_2 \ldots A_n$ for the rule to apply directly and to Hire $\land A_2 \ldots A_n$ for the rule to apply to hire ex analogia, etc.

When, on the other hand, vague concepts are involved the structure of the fact situation is irrelevant for the outcome. Applying ‘fair ground’ to a fact situation involving a new form of misdemeanour entails that this fact situation is compared for similarity to fact situations in the concept’s two collections. But the similarity between fact situations is here a measure of spirit, not of structural affinities. Two fact situations may be deemed similar in the spirit of law and depending on how the similarity in each case looks the inference may be termed analogy, extensive interpretation or whatever. Analogy of this kind between a new fact situation and one previously subsumed under a vague concept does not result in the construction of a new rule but adds to one of the collections a new fact situation. It is not an instance of analogia legis.

Of course, this is not to say that similarity in ‘spirit’ is irrelevant for analogia legis. In the end the relevance of a rule constructed by analogia legis must be assessed in the light of the spirit of the law. In the search for candidate rules for analogia legis the demand for an isomorphism between fact situations and rule antecedents is helpful however. And it is obviously more difficult to understand and represent similarity being based upon spirit only than ditto being based also on structure.

Thus, the statement that the higher-level rules of approximation 20 give rise to similar technical difficulties as the higher-level rules for analogia legis does not extend to say that the higher-level rules are the same in both. The two categories of higher-level rules have
similar functions however. The problem of how they should be understood is similar in the sense that if we understand the higher-level rules for *analogia legis* we are at least closer to understand the higher-level rules of analogical interpretation between fact situations.

A presupposition for *analogia legis* is that the rules involved contain only concepts being more or less sharp, cf. above. Therefore, it is reasonable to assume, as we will do, that the extensions of the legal concepts are known and fixed in the rules used as examples below. This implies that we can elaborate our examples at a ‘rule level’ where the reasoning deals with the words of the rules and the possibility of replacing these with other words having distinct extensions. Also, we will assume that the extensions of the replacement words are known and fixed.

We describe *analogia legis* in more detail in next section.

3.3. The Secondary Rule for Analogia Legis

The inference rule for *analogia legis* has been described as follows in legal writing

(31) A certain rule may be applied to a case not subsumed, or at least not with certainty subsumed, under the rule’s linguistic wording, if the case is not the object of a particular explicit rule, if the case has a substantial similarity to those the rule is intended for, if interests of some importance, which the rule is intended to meet, support such an application, and if no opposite interests exist recommending the rejection of such an application. ([44], p. 71)\(^{11}\)

This is not a secondary rule for *analogia legis*. It is only a schema that, if it is correct, describes what is common for all secondary rules for *analogia legis*. *Analogia legis* is not a generally valid inference rule. Its content is so to say ‘domain dependent.’ That is, properties of the legal field may restrict or extend the applicability of the inference rule. The inference rule for analogy is, for example, much more restricted in labour law than in contract law. In penal law the restriction is to the extent that it is commonly argued (although perhaps not adequately cf. ([11], p. 125)) that analogy is forbidden within the field. It is important to realize the significance of this ‘field related’ knowledge.

The applicability of an actual inference rule for *analogia legis* is demarcated to a certain legal field. An inference rule of this kind is about the provisions of the legal field in question and therefore properly understood as a secondary rule. Secondary rules for *analogia legis* must all comply with the schema 31. In order to actually formulate a secondary rule the schema 31 must be specified by the use of knowledge specific to the current legal field. This knowledge and the schema 31 are thus at a metalevel with respect to real secondary rules [20]. Extending Hart’s terminology we may term the field specific knowledge tertiary rules and the schema 31 a tertiary schema. Here it is important to note that it is not possible to specify in advance field-specific secondary rules for *analogia legis*. This must be done dynamically in each case. We will explain why and how in the next chapter.

Summary. In this chapter we showed the affinity between the higher-level rules for handling vague legal concepts and those for *analogia legis*. We introduced the notion of a schema for higher-level rules and roughly began to identify the levels to which various fragments of legal knowledge belong.

In the next chapter we shall deepen our study of the levels of legal knowledge with an example from *analogia legis*.

\(^{11}\) Original text in Swedish: “en viss regel kan användas på ett fall, som icke eller åtminstone ej med säkerhet omfattas av regelns språkliga ordalydelse, om fallet ej är föremål för en egen uttrycklig regel, om fallet företrar betydande likhet med dem som regeln avser, om intressen av viss vikt, som regeln avses tillgodose, talar för en sådan användning och om inga motstående intressen talar däremot.”
4. LEVELS OF LEGAL KNOWLEDGE

In this chapter we take a closer look at higher-level rules. We will use \textit{analogia legis} as an example. The purpose is to acquire an understanding of how rules at different levels operate together. Let us start by looking at an ordinary provision in the Swedish Sale of Goods Act.

(29) If a sale of goods has been made but no price settled then the vendee should pay what the vendor demands if reasonable.

This provision is not only applicable to sale of goods. For example the provision 29 could be analogically applied to e.g., hire of goods (cf. above), or extensively interpreted, or interpreted \textit{e contrario}, etc. That is, the provision 29 embraces a lot of primary rules. One but only one of these is the rule given by a literal reading of the tokens physically expressing the provision. The provision 29 is a schema for all these rules and since this schema is about primary rules we call it a secondary schema. The relation between secondary schemata and primary rules is given by secondary rules. For example, the relation between the schema 29 and real primary rules is given by secondary rules that could look something like

(32) In commercial law a provision may be applied to a case not subsumed, or at least not with certainty subsumed, under the rule’s linguistic wording \textit{if} the case is not the object of a particular explicit rule in any act belonging to commercial law, \textit{if}, according to the present conception of justice in commercial law, the case has a substantial similarity to those the rule in question is intended for, \textit{if} such an application is not against the groups, interests, etc. protected in commercial law: e.g., consumers, free enterprise, etc.

which is just an example of how a secondary rule for \textit{analogia legis} in commercial law could possibly look. In the same way there exist tertiary schemata for secondary rules and tertiary rules that give the relation between these schemata and the secondary rules. The tertiary schema from which the secondary rule 32 originates is

(31) A certain rule may be applied to a case not subsumed, or at least not with certainty subsumed, under the rule’s linguistic wording, \textit{if} the case is not the object of a particular explicit rule, \textit{if} the case has a substantial similarity to those the rule is intended for, \textit{if} interests of some importance, which the rule is intended to meet, support such an application, and \textit{if} no opposite interests exist recommending the rejection of such an application. [44]

Information about the relation between this schema and secondary rules such as 32 is given by tertiary rules. For want of a more relevant tertiary rule, let us take an example from penal law.

(33) If the legal field is penal law then for \textit{analogia legis} only the following similarities are relevant: ‘man’ may be substituted for ‘woman,’ etc.

Figure 34 illustrates the levels of legal knowledge.

With the schemata and rules involved in \textit{analogia legis} the picture looks as in figure 35.
The notion of schemata for rules at different levels may seem abstract and hard to grasp. The schemata are however important. They are the sole help a lawyer has when examining a case. The lawyer never knows in advance what are the actual rules applicable to a certain case. The only knowledge a lawyer has concerning these rules are the schemata at different levels. The lawyer’s job is to extract the applicable rules from these schemata. Of course, since no real rules exist at any level the extraction process involves a moment of discretion, i.e., a point at which the lawyer just decide without further consideration how a schema should be transformed to a real rule.

Since there exist no real rules in advance the lawyer cannot just select a rule and generalize it to a schema. The lawyer is bound to carry out his reasoning in quite a top-down fashion from schemata to rules. At some level a schema is selected and by discretion transformed into a real rule which is used for transforming schemata to real rules at the level below. These rules in their turn are used in the same way for the next level below, etc. Of course, a lawyer may reconsider and move up again in the hierarchy but as an overall characteristic the top-down process seems important to a faithful description of legal reasoning.

In previous chapters we discussed the expert lawyer’s knowledge of vague legal concepts. This knowledge, we argued, consists partly of two collections of fact situations. In this chapter we propose that a lawyer has only a schematic knowledge concerning legal rules. This does not extend with the same force to the fact situations. Concerning these the lawyer has a more firm knowledge.

Among writers who have made similar observations concerning the schematic nature of legal norms we note Bing ([4] p. 12). In a comment on Sundby’s theory of norms Bing observes a category of so-called ‘non-deterministic’ rules, i.e., rules that cannot be predefined. According to Sundby [46] these rules are used to assess vague legal concepts. Sundby terms them ‘assessments rules’ and they correspond to our higher-level rules. Bing concludes that, since assessments rules cannot be predefined, Sundby’s theory, if it is correct, implies important limitations for the representation of legal norms in computerized systems. It will be shown below that the infeasibility of predetermining higher-level rules or any other legal rules for that matter raises no fundamental hindrances for their representation on condition that these rules at least can be schematically described.

(35)
In the preceding chapters we offered a rather detailed account concerning the rules at the lowest level, i.e., the provisions (‘primary rules’ or ‘secondary schemata’ as we from now on conceive them, cf. figure 35). We argued for the possibility of reformulating these rules to ‘if...then...’ sentences; we advanced that the lawyer conceives these sentences as indicative sentences; we discussed what semantic system would be appropriate for these sentences. With respect to higher-level rules we are not in a position where we can carry out a similar analysis. The reason is that we do not have deep enough knowledge about higher-level rules. Most often these are not available in the same form as lower level rules. In the legislation higher-level rules appear only occasionally as provisions. Some higher-level rules are accounted for in legal writing. But the large body of the higher-level rules exist only in the minds or subconscious of the lawyers. It is a very difficult problem to establish what are the structure and content of these rules. General conclusions as to the form and semantics of higher-level rules cannot be drawn at this stage. In this paper the separate examples we give of higher-level rules happen all to be ‘if...then...’ sentences and we will assume without further notice that the semantics of classical logic applies to them.

One thing deserves to be mentioned explicitly. A transformation from a secondary schema to a primary rule is structure-preserving. At least we can assume this with some certainty due to the principle of predictability. In general however structure preservation is no demand for transformations from schemata to higher-level rules. That the structure is more or less preserved in the examples given in figure 34 is a pure coincidence.

Summary. Legal knowledge does not consist of rules that can mechanically be applied. Legal reasoning is to establish the form and content of candidates for applicable rules. The knowledge used is schemata for rules. Rules at an even higher level give information about the relation between these schemata and real rules. The lawyer’s knowledge of these rules is only schematic as well.

We need some machinery before we approach the problem of how to actually capture the higher-level rules in a formalism. In next chapter we suggest a representation for the lowest level which includes fact situations and the result of transforming secondary schemata, i.e., the primary rules.
5. A LOW-LEVEL REPRESENTATION

In this section we give a basic representation of legal concepts and primary rules. The representation is no end in itself. It is put forward with a specific purpose. The purpose is not to discuss how to most adequately describe fact situations and primary rules. So no claim is made that the actual formulas given below illustrate the most adequate way of depicting knowledge at that level. We have above emphasized the important role in legal reasoning played by the higher-level rules described in approximation 20 and the higher-level rules for e.g., *analogia legis*. Whether these can be represented we believe is crucial for the success of legal knowledge system. The sole purpose of the low-level representation is therefore to provide a basis from which we can discuss the difficulties of representing higher-level rules. The low-level representation will thus only be used to illustrate the problems which the higher-level rules must handle, i.e., what kind of formulas these must operate on.

Proposing a representation for this level will necessarily comprise a choice of a specification of approximation 19. Let us here choose the following specification:

(36) We understand a sharp legal concept, e.g., ‘employee’ in the rule 2, to be one or several names corresponding to primitive facts. A vague legal concept, e.g., ‘fair ground,’ is partly understood as two collections of descriptions of fact situations relative to which the concept is true and false, respectively. The descriptions of fact situations are conjunctions of names corresponding to primitive facts.

On the basis of approximation 36 we now choose a more detailed representation of sharp and vague legal concepts and primary rules.

**Legal Concepts.** Sharp concepts that cannot be further decomposed, e.g., ‘employee,’ we choose to represent as ground atomic formulas. An atomic formula, e.g., ‘Employee(John Doe)’ is a name corresponding to a single primitive fact. Atomic formulas can be compounded by the connective ‘∧’ to form situation descriptions. In the simplest case a situation description is just a single atomic formula.

Compound legal concepts are of two kinds: sharp and vague. Sharp compound legal concepts stands in the logical relation ‘←’ to a collection of conjunctive formulas. For instance, the sharp compound legal concept ‘in service’ is implied by the situation descriptions ‘Employee(John Doe)’ and ‘Commission_agent(Dan Moe).’

E.g.,

\[
\begin{align*}
\text{In\_service}(John\_Doe) & \leftarrow \text{Employee}(John\_Doe) \\
\text{In\_service}(Dan\_Moe) & \leftarrow \text{Commission\_agent}(Dan\_Moe).
\end{align*}
\]

The reason why we do not represent this as

\[
\begin{align*}
\text{In\_service}(x) & \leftarrow \text{Employee}(x) \\
\text{In\_service}(x) & \leftarrow \text{Commission\_agent}(x)
\end{align*}
\]

is that the situation descriptions originate from legal cases and the like. In principle the extent according to which a legal case may be generalized is always a matter of discretion.

---

12 We here assume that these two concepts are atomic and distinct. This assumption is only for the sake of argument. As we noted above (p. 9), the nature of the case in issue determines whether concepts are to be considered as vague or sharp. In some cases there is every possibility of disputes as to whether a person fulfilling some service should properly be classified as an employee or a commission-agent.
A vague compound legal concept stands also in the logical relation ‘⇔’ to collections of conjunctive formulas. The difference between sharp and vague compound legal concepts is as follows. Sharp compound legal concepts are true with respect to one collection of situation descriptions and false with respect to everything outside this collection. Assume that formula 38 is a permitted generalization of formula 37. Since formula 38 represents a sharp legal concept it can be reformulated to an equivalence

\[
\text{In\_service}(x) \\
\iff \text{Employee}(x) \\
\lor \text{Commission\_agent}(x)
\]

the right hand side of which is a disjunction of situation descriptions. Vague compound legal concepts on the other hand are true with respect to one collection of situation descriptions, false with respect to another collection and unknown with respect to everything outside these two collections. For instance, the vague compound legal concept ‘fair ground’ is implied by situation descriptions such as

\[
\text{Fair\_ground}(\text{John\_Doe}) \\
\iff \text{Employee}(\text{John\_Doe}) \\
\land \text{Ordinary\_position}(\text{John\_Doe}) \\
\land \text{Greater\_than}(\text{Seven\_days}, \text{Absence}(\text{John\_Doe})) \\
\land \text{No\_excuses}(\text{John\_Doe})
\]

(39)

\[
\text{Fair\_ground}(\text{Mike\_Roe}) \\
\iff \text{Employee}(\text{Mike\_Roe}) \\
\land \text{Position\_of\_trust}(\text{Mike\_Roe}) \\
\land \text{Greater\_than}(\text{Three\_days}, \text{Absence}(\text{Mike\_Roe})) \\
\land \text{No\_excuses}(\text{Mike\_Roe})
\]

Etc.

‘Not fair ground’ is implied by situation descriptions such as

\[
\text{Not\_fair\_ground}(\text{Sue\_Poe}) \\
\iff \text{Employee}(\text{Sue\_Poe}) \\
\land \text{Ordinary\_position}(\text{Sue\_Poe}) \\
\land \text{Less\_than}(\text{Six\_months}, \text{Absence}(\text{Sue\_Poe})) \\
\land \text{Abuse\_of\_alcohol}(\text{Sue\_Poe})
\]

(40)

\[
\text{Not\_fair\_ground}(\text{Al\_Soe}) \\
\iff \text{Employee}(\text{Al\_Soe}) \\
\land \text{Ordinary\_position}(\text{Al\_Soe}) \\
\land \text{Less\_than}(\text{Twelve\_months}, \text{Absence}(\text{Al\_Soe})) \\
\land \text{Ill}(\text{Al\_Soe})
\]

Etc.

With respect to everything outside collections 39 and 40 of situation descriptions the concept ‘fair ground’ is ‘unknown.’ Reformulation to equivalences, such as formula 38’, is thus not possible.

**Analogia Legis.** The problems of representing the higher-level rules of approximation 20 are similar to the problems of representing the higher-level rules for *analogia legis.* The
higher-level rules of approximation 20 operate on structures such as the situation descriptions in collections 39 and 40 while the higher-level rules for *analogia legis* operate on structures such as

\[
\begin{align*}
\text{Pay}(x, y, u) \\
\leftarrow \text{Vendee}(x, z) \\
\land \text{Vendor}(y, z) \\
\land \text{Not}\_\text{settled}\_\text{price}(z) \\
\land \text{Demands}(y, x, z, u) \\
\land \text{Reasonable}(u, z).
\end{align*}
\]

(41)

The problem in both cases is how to manipulate the antecedent side of the structure to make it subsume or not subsume a certain description of a fact situation.

**Summary.** In this chapter we suggested a low-level representation for fact situations and primary rules. The purpose was to provide a basis from which we can discuss the problems of incorporating the higher-level rules in a representation. These problems are the topic of the next chapter in which we will elaborate a representation for *analogia legis.*
6. ASPECTS OF REPRESENTING HIGHER-LEVEL RULES

Above we have focused on how higher-level rules may informally be understood. In this chapter we show that there exist prospects for capturing the levels of legal knowledge in a runnable formalism [20]. A formal representation of higher-level rules raises numerous questions. We are not in a position where we can advance a particular approach as being the best. Any representation must however take into account certain fundamental aspects of the higher-level rules. These aspects entail that metaprogramming may be conceived as a natural approach. We elaborate a representation of higher-level rules along this approach. In addition, we identify some questions raised by the approach and point out guidelines for further work.

Basic Requirements. There exist some demands that must be met by any representation of higher-level rules. To identify these let us first put an operational view on the higher-level rules to see what these actually are supposed to do. Recall figure 34.

We see that higher-level rules are relations between schemata and real rules. For example, a tertiary rule is a relation between a tertiary schema and a secondary rule. The tertiary rule ‘takes’ a schema and ‘instantiates’ its variable parts. Let us understand a tertiary rule as a program and the input and output arguments of this program as data structures. A tertiary schema would be a data structure with variable parts, a secondary rule a data structure with these variable parts ‘instantiated.’ The program’s output is thus its input argument with the variable parts instantiated.

The output argument is a data structure at one level. At the same time it is a program at another level, a program corresponding to a secondary rule the input of which again is data structures with variable parts (secondary schemata) and output instantiated data structures (primary rules). The two different understandings of the output argument as data and program respectively must be taken into account. Several levels in the representation seem therefore necessary. Higher-level rules are about other rules and consequently a kind of metarules. Therefore, it is close at hand to conceive metaprogramming as a natural approach for representing higher-level rules.

Metalogic and Metaprogramming. A metalanguage is a language where all or some of the terms are names for syntactical entities of some language. This other language is usually referred to as the object language. This distinction between a language and its metalanguage has been known for some time in mathematics and logic. Hilbert utilized a similar distinction in an attempt aiming at an intuitively convincing proof of the consistency of classical mathematics; proofs in an axiomatic theory of classical mathematics were made the object of a mathematical investigation termed metamathematics or proof theory [26]. Gödel numbers, used to represent mathematical formulas, can also be deemed a metalanguage for mathematics [16]. The terms ‘object language’ and ‘metalanguage’ were introduced by Tarski in an attempt aiming at specifying adequacy criteria for definitions of truth for formal languages [47].
The idea of representing formulas and terms of an object language as terms in a meta-language has been applied also in computing science. Here the languages are usually programming languages and syntactic entities of a programming language are represented as data structures, operated upon by programs written in a metaprogramming language. This was first demonstrated in the universal Turing machine [49]. The idea is also embodied in the processing units of all computers based on the von Neumann model [36]; instructions are stored as bits and bytes in the memory chips of the computer, being interpreted by a microprogram. By changing the microprogram, the interpretation of the instructions is changed.

**Metaprogramming in Logic.** Bowen and Kowalski have proposed a general scheme for metaprogramming in logic by amalgamating an object level language $L$ with a metalevel language $M$ [6]. The amalgamation requires

I. a naming convention which associates every term or formula $E$ in $L$ with a ground term $E$ in $M$,

II. a representation of provability in $L$ by a program $Pr$ for the predicate $Demo$ in $M$, and

III. two linking rules stating that

$$\frac{Pr \vdash_M Demo(A, B)}{A \vdash_L B} \quad \text{and} \quad \frac{A \vdash_L B}{Pr \vdash_M Demo(A, B)}.$$

The linking rules allow an object level execution to be replaced by a metalevel execution and vice versa. Feferman [9], and later Weyhrauch [51], termed these rules ‘reflection principles.’

The representation in the metalanguage of an object language term or formula $E$ we call a *name* for $E$ and write it here as $\overline{E}$.

**Adequacy.** Does the amalgamation provide means for adequately reflecting the levels of legal knowledge? The amalgamation as well as classical studies of the object-meta language partition—cf. e.g., the truth theory of Tarski’s [48]—requires that every term or formula $E$ in the object language $L$ be associated with a ground term $\overline{E}$ in the metalanguage $M$. Moreover, $L$ forms part of $M$. This is repeated at the meta-metalevel and so forth. The metaprogramming hierarchy may be conceived as ‘straight’ since each level has the names of the entities at the level below, i.e.,

```
meta-metalevel
  |
  metalevel
  |
object level
  |
universe of discourse
```

Can the ‘straight’ hierarchy 42 be used to adequately depict the hierarchy in figure 34? Clearly, there exists something that looks like a hierarchy in figure 34 but at the first glance it does not seem to be a straight one. For example, tertiary rules set up relations between tertiary schemata and secondary rules and these are entities belonging to different levels. The hierarchy in figure 34 can nevertheless be directly depicted in hierarchy 42. An object language forms part of its metalanguage which in its turn forms part of the metametalanguage, etc., cf. Tarski [47]. In the hierarchy 42 rules at the meta-metalevel may accordingly give the relation between meta-metaentities and metaentities. This corresponds
to tertiary rules being relations between tertiary schemata and secondary rules. For our specific problem there exist however reasons for keeping the levels distinct so we choose a slightly different approach.

We rotate the hierarchy in figure 34 ninety degrees and as intermediate notions we introduce the ‘names’ for the rules at the level below. The result is the straight hierarchy in figure 43.

The only difference of importance between figures 34 and 43 is that the latter makes the partition between knowledge belonging to separate levels more clear. Below we illustrate how metaprogramming in logic may possibly be used to represent the hierarchy in figure 43. A notion termed partial name will be exploited for capturing schemata [20]. We begin by explaining this notion.

Partial Names. In the amalgamation a name at the metalevel is used to represent an object level term or formula. A possible naming convention is to let structure descriptions of terms and formulas serve as their names at the metalevel [2,48]. That is, a name for the object level formula $P(x,z) \leftarrow Q(x,y) \land R(y,z)$ is formed by using the formula’s structure. To clarify the distinction between a name and what it denotes the structure can e.g., be written within quotation marks or, as employed here, under a bar $P(x,z) \leftarrow Q(x,y) \land R(y,z)$. There exists a function from the name to the object and an algorithm can be defined that distinctly finds the object from the structural-descriptive name. Using a standard set of characters a representation of a structural-descriptive name for the atomic formula $P(x,Bill)$ could be e.g., $\text{atom(pred(1), var(1).constant(212).Nil)}$ [6]. In this rather awkward notation the name for $P(x,z) \leftarrow Q(x,y) \land R(y,z)$ would be

$$
\text{clause(\text{atom(pred(1), var(1).var(2).Nil)}, \\
\text{conj(\text{atom(pred(2), var(2).var(3).Nil)}, \text{atom(pred(3), var(3).var(4).Nil}))}).}
$$

We definitely think the bar is clearer from a presentation point of view but the verbose, more structured, notation can be more convenient for metaprogramming in logic.

The use of structural-descriptive names at the metalevel entails of course that the names will resemble each other if the object level terms and formulas have the same structure.
For example, the names $P(x, z) \leftarrow Q(x, y) \land R(y, z)$ and $P(x, z) \leftarrow T(x, y) \land R(y, z)$ have a similar structure. A partial structural-descriptive name describes what is common to these two names and leaves the rest unspecified as a variable. The partial name would in this case be $P(x, z) \leftarrow q \land R(y, z)$.

Given a representation of a language in another language, we can express all programs of the object language in the metalanguage. Moreover we can ‘simulate’ the computational mechanism of the object language with a program in the metalanguage. We can reason about well-formed syntactical object language entities in the metalanguage but also about ‘unsaturated’ linguistic entities [12] ([13] p. 290). This adds to the expressive power. In an object language about arithmetic say, expressions such as ‘5 + 7’ are meaningful but e.g., unsaturated expressions such as ‘5 + ()’ meaningless. ‘5 + ()’ can however form part of a meaningful expression in a metalanguage e.g., ‘‘5 + ()’’ is a function.’ The notion of a partial name used in this study should be understood as an unsaturated expression. As an expression of the object language $P(x, z) \leftarrow q \land R(y, z)$ would be ill-formed and meaningless. But in the metalanguage $P(x, z) \leftarrow q \land R(y, z)$ can form part of a meaningful statement about clauses in the object language.

Generation of Programs. Above we argued that levels of legal knowledge are ‘created’ in a top-down fashion. We attempt to reflect this in the representation by letting programs at a level $n-1$ be generated at level $n$ and, at this level, simulate the execution at level $n-1$. Bowen and Kowalski [6] represent object level provability at the metalevel by a program defining the predicate Demo/2. Our approach is slightly different since we do not have available in advance an object level but generate it dynamically for each case.

At level $n$ we generate the programs and simulate the execution at level $n-1$. A level $n-1$ has no ‘life’ independent from the level $n$; a level $n$ can create and reject candidate levels $n-1$ depending on the results these reflect up to level $n$. Therefore we want to keep

- the control at the topmost level, and
- the levels clearly distinct.

At level $n$ structural-descriptive names are used to represent the entities at level $n-1$. Level $n-1$ may in its turn be a metalevel for some level $n-2$. The entities at level $n-2$ are in this case represented by names at level $n-1$ and by names for names at level $n$.

We will attempt to represent the levels of legal knowledge in this framework. Our ambition is to illustrate that prospects actually exist for capturing the levels of legal knowledge in a runnable formalism. We do not assess this attempt’s possible merits compared to other conceivable approaches.

Simulation is inefficient. In the examples below we disregard from this because we want to be as faithful as possible to the levels of legal knowledge; efficiency is throughout sacrificed for clarity. The efficiency could however be improved by using the right linking rule in cases where it can be established at level $n$ that the programs at a level $n-1$ should be accepted.

We will write logic programs with a standard logic syntax. The programs are runnable only on condition that symbols such as e.g., ‘\!' are interpreted as computable counterparts, e.g., ‘negation as failure’ or Prolog’s ‘\'+.'

An Attempt. We now elaborate an attempt to represent higher-level rules exploiting metaprogramming in logic, cf. [20]. The example includes three levels.

We begin with the representation of schemata. A schema describes what is common for a class of rules at the level below. Similarly, a partial name describes what is common for several structural descriptive names. Partial names are used to represent schemata.

We regard provisions as secondary schemata, cf. figure 35. Secondary schemata correspond to partial names for logic programs. At the tertiary level a secondary schema is thus represented as a name for a partial name for a logic program. For example the provision
If a sale of goods has been made but no price settled then the vendee should pay what the vendor demands if reasonable.

could be represented as

\[
\begin{align*}
\text{Pay}(x, y, u) \\
\quad \leftarrow \text{vendee}(x, z) \\
\quad \land \text{vendor}(y, z) \\
& \land \neg \text{Settled\_price}(z) \\
& \land \text{Demands}(y, x, z, u) \\
& \land \text{Reasonable}(u, z).
\end{align*}
\]

We may recall that a very simplified secondary rule for *analogia legis* in commercial law could look something like

(32) In commercial law a provision may be applied to a case not subsumed, or at least not with certainty subsumed, under the rule’s linguistic wording if the case is not the object of a particular explicit rule in any act belonging to commercial law, if, according to the present conception of justice in commercial law, the case has a substantial similarity to those the rule in question is intended for, if such an application is not against the groups, interests, etc. protected in commercial law: e.g., consumers, free enterprise, etc.

We represent secondary rules as logic metaprograms. Consequently, at the third level a secondary rule corresponds to a name for a logic metaprogram. For example, the secondary rule 32 may be represented as the name 45 for a logic metaprogram where e.g., the question in the first conjunct of the secondary rule roughly is captured by use of the provability predicate *DemoI* (explained below).

\[
\begin{align*}
\text{Analogous\_rule\_construction\_in\_commercial\_law}(\text{case, ar}) \\
\quad \leftarrow \text{Acts\_of\_commercial\_law}(\text{acts}) \\
& \land \neg \exists \text{queryDemoI}(\text{acts, case} \rightarrow \text{query}) \\
& \land \text{pr} \in \text{acts} \\
& \land \text{Intended\_for}(\text{pr, cases}) \\
& \land \text{Similarity}(\text{case, cases, sim}) \\
& \land \text{Regarded\_as\_substantial\_in\_commercial\_law}(\text{sim}) \\
& \land \text{Constructed\_analogous\_rule}(\text{pr, sim, ar}) \\
& \land \text{Protects\_consumers}(\text{ar}) \\
& \land \text{Protects\_free\_enterprise}(\text{ar})
\end{align*}
\]

The secondary rule 32 is an instantiation of the tertiary schema 31 repeated here

(31) A certain rule may be applied to a case not subsumed, or at least not with certainty subsumed, under the rule’s linguistic wording, if the case is not the object of a particular explicit rule, if the case has a substantial similarity to those the rule is intended for, if interests of some importance, which the rule is intended to meet, support such an application, and if no opposite interests exist recommending the rejection of such an application.  [44]
The tertiary schema 31 may be represented by

\[
\text{Analogous\_rule\_construction\_in\_legal\_field}((\text{case, ar}) \leftarrow \text{Acts\_of\_legal\_field}(\text{acts})
\land \neg \exists \text{queryDemo1}(\text{acts, case} \rightarrow \text{query})
\land pr \in \text{acts}
\land \text{Intended\_for}(pr, \text{cases})
\land \text{Similarity}(\text{case, cases, sim})
\land \text{Regarded\_as\_substantial\_in\_legal\_field}(\text{sim})
\land \text{Constructed\_analogous\_rule}(pr, \text{sim}, ar)
\land \text{Protects\_interest\_1}(ar)
\ldots
\land \text{Protects\_interest\_n}(ar)
\]

which is a partial name for a logic metaprogram. The ellipsis means that the number of conjuncts about protected interests is arbitrary.

When a partial name is instantiated it becomes a structural-descriptive name that denotes a specific program. In this case the partial name 46 is instantiated to the name 45. This is intended to reflect how schemata are transformed to rules, exemplified here by the tertiary schema 31 and the secondary rule 32. The transformation is controlled by tertiary rules such as rule 33 repeated here

(33) If the legal field is penal law then for analogia legis only the following similarities are relevant: ‘man’ may be substituted for ‘woman,’ etc.

Let us now turn to the representation of higher-level rules. A tertiary rule is a relation between tertiary schemata and names for secondary rules. The representation we choose for schemata is partial names, for rules logic programming rules. We represent a tertiary rule as a logic programming rule $\text{TR}(x; y) \leftarrow \text{Body}$ where $x$ ranges over partial names corresponding to tertiary schemata and $y$ over names for logic programming rules corresponding to secondary rules. The contents of $\text{Body}$ is irrelevant for the discussion here.

Let us focus on the tertiary level and consider what would be programs and data at that level. $\text{TR}/2$ is a program and its arguments are data. Programs as $\text{TR}(x, y) \leftarrow \text{Body}$ are used to compute complete names such as 45 from partial names such as 46. For example the query

\[
\text{TR}(\text{Analogous\_rule\_construction\_in\_legal\_field}(\text{case, ar}) \leftarrow \text{Acts\_of\_legal\_field} \ldots, x)
\]

could give

\[
x = \text{Analogous\_rule\_construction\_in\_commercial\_law}(\text{case, ar}) \leftarrow \text{Acts\_of\_commercial} \ldots
\]

In chapter 4 we claimed that the figure 34 should be understood such that the lawyer’s knowledge consists only of the schemata. The rules do not exist beforehand, they are created from the schemata in a top-down fashion. We strive to reflect this process in the representation. We show below how names and programs at different levels interact. It is easiest to understand the levels if we start with the primary level (I).

III $\text{PTR} \vdash_{\text{III}} \text{Demo11}(\text{PSR, Demo1(PPR, facts} \rightarrow \text{query}))$

II $\text{PSR} \vdash_{\text{II}} \text{Demo1(PPR, facts} \rightarrow \text{query})$

I $\text{PPR} \vdash_{\text{I}} \text{facts} \rightarrow \text{query}$
At level I we have a program \( PPR \) which is a set of constructed clauses corresponding to primary rules. It is important to remember that these clauses do not exist in advance but are constructed from the level above, the secondary level (II). \( \text{facts} \) and \( \text{query} \) are the facts at hand and the query to be analyzed, respectively. We want to know whether e.g., an analogous legal source \( PPR \) may be constructed such that \( \text{facts} \rightarrow \text{query} \) follows from \( PPR \), i.e., \( PPR \models \text{facts} \rightarrow \text{query} \). Such a set \( PPR \) is ‘acceptable’ if and only if there exists, at level II, a program \( PSR \), corresponding to ‘acceptable’ secondary rules, that entails that \( PPR \) gives \( \text{facts} \rightarrow \text{query} \). That is, from \( PSR \) should follow that a predicate \( \text{Demo}_{II}/2 \), which represents provability at level I (i.e., \( \cdot_{II} \)), holds for \( PPR \) and \( \text{facts} \rightarrow \text{query} \), i.e., \( PSR \models \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query}) \). \( PSR \) defines the predicate \( \text{Demo}_{II}/2 \) and determines the content of \( PPR \). The same thing is repeated at the tertiary level (III). Program \( PSR \) is an ‘acceptable’ representation of secondary rules and of \( \cdot_{II} \) if and only if there exists at level III a program \( PTR \), corresponding to ‘acceptable’ tertiary rules, that entails that \( PSR \) entails that \( PPR \) gives \( \text{facts} \rightarrow \text{query} \).

Since we include three levels only in our example we assume that tertiary rules corresponding to \( PTR \) are accepted by the user, not by some program at a quaternary level. In chapter 4 we claimed that the lawyer must use discretion to specify schemata into real rules at some level. In a knowledge system this can be reflected by leaving the system’s topmost level ‘open.’ That is, to let the schemata at this level be specified into real rules by external means, e.g., through user communication supplemented by appropriate information on the matter. The tertiary level is the topmost in our example and we assume that tertiary rules have been made available in this way.

Look at level II say, \( PSR \) can make the relation \( \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query}) \) hold in two ways. First, \( PSR \) determines the content of \( PPR \) which affects whether \( \text{facts} \rightarrow \text{query} \) follows or not. Second, \( PSR \) defines the predicate \( \text{Demo}_{II}/2 \). The same program clauses may define both \( \text{Demo}_{II}/2 \) and \( \text{Demo}_{II}/2 \) however.

\( PTR \) and \( PSR \) are programs defining, respectively, \( \text{Demo}_{II}/2 \) and \( \text{Demo}_{II}/2 \). \( \text{Demo}_{II}/2 \) represents provability at level II (\( \cdot_{II} \)). At level III we have thus a program \( PPR \) for the relation \( \text{Demo}_{II}/2 \) such that \( PPR \models \text{Demo}_{II}(PSR, \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query})) \) holds iff at level II \( PSR \models \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query}) \) holds. \( PSR \models \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query}) \) holds in its turn iff at level I \( PPR \models \text{facts} \rightarrow \text{query} \). That is, sub-levels should be computed such that the result satisfies the following equivalences.

For the positive case

\[
PTR \models_{III} \text{Demo}_{II}(PSR, \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query}))
\]

\[
\iff
PSR \models_{II} \text{Demo}_{II}(PPR, \text{facts} \rightarrow \text{query})
\]

\[
\iff
PPR \models_{II} \text{facts} \rightarrow \text{query}
\]

and similarly for the negative case

\[
PTR \models_{III} \text{Demo}_{II}(PSR, \text{Demo}_{II}(PPR, \neg(\text{facts} \rightarrow \text{query})))
\]

\[
\iff
PSR \models_{II} \text{Demo}_{II}(PPR, \neg(\text{facts} \rightarrow \text{query}))
\]

\[
\iff
PPR \models_{II} \neg(\text{facts} \rightarrow \text{query})
\]

At level III the query ‘\( \text{Demo}_{II}(PSR, x) \)’ computes a structural-descriptive name for a predicate adequate for handling e.g., \( \text{analogia legis} \) in a particular legal field. It may give
\(x = \text{Demo}_1(\text{PPR}, \text{facts} \rightarrow \text{query})\)' where \(\text{Demo}_1/2\) is a field specific predicate for \textit{analogia legis} and \(\text{PPR}\) a theory of constructed analogous rules. We can specify for what object level query \(\text{facts} \rightarrow \text{query}\) we like the predicate to be constructed. Instead of the ‘\(x\)’ in query ‘\(\text{Demo}_{II}(\text{PSR}, x)\)’ we can use a partial name, i.e., ‘\(\text{Demo}_{II}(\text{PSR}, \text{demo}_I(\text{ppr}, \text{facts} \rightarrow \text{query}))\)’

Assume the object level query is ‘\(\text{Pay}(\text{Jones}, \text{Smith}, \$100)\)’ and the facts at hand are that Jones has rented a car from Smith at unspecified price i.e., \(\text{Hirer}(\text{Jones}, \text{Car}) \land \text{Letter}(\text{Smith}, \text{Car}) \land \neg \text{Settled\_price} (\text{Car}) \land \text{Demands}(\text{Smith}, \text{Jones}, \text{Car}, \$100) \land \text{Reasonable}(\$100, \text{Car})\). The answer to the object level query will be ‘\(\text{yes}\)’ if it can be computed that

\[
\text{Demo}_1(\text{PPR}, \text{Hirer}(\text{Jones}, \text{Car}) \land \ldots \land \text{Reasonable}(\$100, \text{Car}) \rightarrow \text{Pay}(\text{Jones}, \text{Smith}, \$100)).
\]

We have at level III names for partial names corresponding to secondary schemata e.g., name 44. The answer to the object level query would be ‘\(\text{yes}\)’ e.g., if, using the name for a partial name 44, \(\text{PPR}\) is computed to contain a name (47) for a name.

\[
\begin{align*}
\text{Pay}(x, y, u) & \leftarrow \text{Hirer}(x, z) \\
& \land \text{Letter}(y, z) \\
& \land \neg \text{Settled\_price}(z) \\
& \land \text{Demands}(y, x, z, u) \\
& \land \text{Reasonable}(u, z)
\end{align*}
\]

(47)

If on the other hand the system fails to compute a \(\text{PPR}\) that gives \(\text{facts} \rightarrow \text{query}\) the answer to the query is, in principle, ‘\(\text{unknown}\)’. A strategic step is then to attempt to compute a theory \(\text{PPR}\) that gives \(\neg (\text{facts} \rightarrow \text{query})\), i.e., see if the query at the tertiary level ‘\(\text{Demo}_{II}(\text{PSR}, x)\)’ comes up with ‘\(x = \text{Demo}_1(\text{PPR}, \neg (\text{facts} \rightarrow \text{query}))\)’. If this also fails we have to settle with the conclusion that the knowledge system simply does not contain knowledge sufficient for handling the legal matter at hand.

**Open Questions.** In knowledge representation we are at some stage forced to decide what formal structures to assume as counterparts for informal knowledge. It is an open question what approximation of legal knowledge is most appropriate as basis for a representation. Above, we have strived to reproduce legal knowledge in a form being both amenable to machine representation and faithful to the knowledge. Still this is of course only an approximation; a number of questions remain in need of further examination concerning how to increase its faithfulness.

Among the approximations made in this chapter is that we assumed the schemata to be available at various levels and possible to represent straightforwardly as partial names. In principle however, higher-level rules affect also the form of the schemata. We have only briefly touched upon this subject in chapter 4. We can adapt the representation to this by also allowing a partial name’s structure to be variable. A step in this direction was that we allowed an arbitrary number of conjuncts in the partial name 46.

As an additional approximation we assumed, without further notice, that classical logic is applicable to higher-level rules. Perhaps this is true. The semantics of higher-level rules ought to examined more closely, however.

In our striving to achieve a faithful representation the aspect of efficiency has been disregarded. Presumably, there is a tradeoff between efficiency and faithfulness. Consequently,
it might be better to settle with a reasonable approximation than trying to account for every possible aspect of legal knowledge. Perhaps, a good measure for what is a reasonable degree of accuracy could be given by empirical studies evaluating the merits of knowledge systems based on a particular approximation.

We exemplified above how a standard set of characters can be used for representing structural-descriptive names. For names already, this representation is awkward and hard to read. For names for names it is increasingly unmanageable. We need more convenient programming languages for metaprogramming. On contemporary computers the notation used in this study can be realized. However, although it is definitely clearer to use bars than standard characters only, this notation is still quite unkind to the reader and, in addition, metaprogramming in logic may require a more verbose and structured notation. There ought to exist better alternatives. It is an open question what these are and which would serve clarity best.

Finally, we have the open hierarchy of the system. At the top-level we assume that the user provides an interpretation of the schemata and specify these into real rules. The system being ‘open-ended’ is important since a legal knowledge system hardly could be a closed ‘oracle.’ We cannot leave the user without help at this level however. We need a system for user interaction support. Such a system could be based on a combination of computer reasoning and hypertext for legal text document management. Some work in this direction has already been carried out [3] but it is still an open question how such a system best should be designed.
7. CONCLUSIONS AND FURTHER WORK

Provisions depict legal norms and we have argued that a representation of legal knowledge must strive to reflect this. We have advanced that one step in this direction is to incorporate legal interpretation principles in the representation. Moreover we have maintained that the function carried out by the lawyer has consequences for knowledge representation. In particular, this function determines the mood according to which a provision should be interpreted. We have argued that, if the lawyer's function is norm application, it is correct to represent his knowledge of provisions as indicative sentences. Furthermore we have put forward that legal knowledge exhibits a multilevel structure of interpretation principles. The model theory of classical logic is a well developed semantics for indicative sentences. In addition, logic clauses may be data at one level and program at another which makes logic programming particularly adapted for metaprogramming. We have illustrated the possibilities of this framework by an attempt to represent *analogia legis*. A number of questions remain, however, for further analyses.

A primary objective for further work is to achieve a deeper understanding concerning the following issues:

- how the higher-level rules should informally be understood, what are their structure, semantics, etc.,
- the formal counterparts appropriate for representing higher-level rules,
- metaprogramming schemes adequate for the levels of legal knowledge,
- how an intuitively acceptable semantics may be defined for legal concepts,
- how such a semantics may be represented.
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The Multilevel Structure of Legal Knowledge and its Representation

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Abstract
We identify aspects of legal reasoning that a faithful representation must account for. In particular we argue that legal reasoning is, rather than rule application, a process of interpretation or even construction of rules from a set of schematic knowledge. Legal interpretation principles exercise control over this process. The import is stressed of incorporating these principles in the representation. We advance that legal interpretation principles may provide answers to the semantic questions raised by vague legal concepts. We show that legal knowledge exhibits a multilevel structure of interpretation principles and exemplify with legal analogy how these operate at various levels. We point out the affinity between levels of legal knowledge and levels of metaprogramming in logic and demonstrate the potentials of this framework for capturing legal interpretation principles in a runnable formalism.

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