Advanced Process Calculi

Lecture 1: the pi-calculus

Copenhagen, August 2013

Joachim Parrow
Learning outcomes

After completing the course you will be able to:

• Use modern process calculi to make high-level models.

• Explain key issues involved in their construction, abilities, and limitations.

• Use a prototype tool to analyze your models.
Teachers

• Joachim Parrow, professor in Computing Science, Uppsala University

• Jesper Bengtson, professor in Computer Science, IT-university Copenhagen

• Ramunas Gutkovas, PhD student, Uppsala University
How it works

• 4 lectures of 2x45 mins each. Slides will be available after each lecture.

• 4 afternoons of tutored exercise and lab sessions.

• Examination: individual project. Choose an application and model it. (Examined in September by Jesper Bengtson.)
Material

http://www.it.uu.se/research/group/mobility/apc-course-copenhagen-2013
Advanced Process Calculi
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- A calculus: something in which we can calculate things.
Advanced Process Calculi

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- Calculation presupposes a rigorously defined semantics.
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- Calculation = logically obtained conclusion
Advanced Process Calculi

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- Calculation presupposes a rigorously defined semantics.
- Calculation = logically obtained conclusion
- Note the plural form. There will be more than one...
Advanced Process Calculi
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- The objects that we calculate with will be processes.
Advanced Process Calculi

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• A process is something that exhibits behaviour through interactions with the environment.
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- Defined in an abstract and high-level way. Could be implemented as software or hardware.
Advanced **Process** Calculi

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- A process is something that exhibits behaviour through interactions with the environment.
- Defined in an abstract and high-level way. Could be implemented as software or hardware.

*Eg: ``Send data value 5 along the output channel``*
Advanced Process Calculi
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- Suggests that there are also basic ones...
Advanced Process Calculi

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- No fret! We will start out by recapitulating the pi-calculus, a very basic one. (pi-calculus experts in the audience: see you tomorrow!)
Advanced Process Calculi

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• No fret! We will start out by recapitulating the pi-calculus, a very basic one. (pi-calculus experts in the audience: see you tomorrow!)

• Advanced = Complicated? Rich? Powerful? Recent?
The pi-calculus

- Developed in 1987-1992 by Robin Milner, Joachim Parrow and David Walker.
- Goal: give a minimalistic compositional computational model encompassing concurrency with mobility and scoping.
• **minimalistic:** only include stuff necessary to capture concurrency, mobility and scoping
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• **mobility**: connections between processes may change during execution
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• **concurrency**: asynchronous processes communicate in binary atomic actions

• **mobility**: connections between processes may change during execution

• **scoping**: these connections may be local
• **minimalistic**: only include stuff necessary to capture concurrency, mobility and scoping

• **concurrency**: asynchronous processes communicate in binary atomic actions

• **mobility**: connections between processes may change during execution

• **scoping**: these connections may be local

• Departure: CCS, a process calculus having all of the above except mobility (Milner, 1979 - )
Compositionality

"The behaviour of a system is given by the behaviour of its parts"
Compositionality

"The behaviour of a system is given by the behaviour of its parts"

behaves as means that they can replace each other
Compositionality

"The behaviour of a system is given by the behaviour of its parts"

behaves as means that they can replace each other
Formally:

If the behaviour $[[A]]$ of a system $A$ is defined as the **transitions** between its **states**
Formally:

If the behaviour $[A]$ of a system $A$ is defined as the transitions between its states then the states and transitions of a system should be determined by the states and transitions of its components.
Formally:

If the behaviour $[[A]]$ of a system $A$ is defined as the **transitions** between its **states**

then the states and transitions of a system should be determined by the states and transitions of its **components**.

$[[A \times B]] = [[A]] \times [[B]]$
Scoping

When a **name** is introduced, the valid places of its use, aka **scope**, is defined.

These **A** all refer to the name introduced at

The outer **A** do not refer to that name, even though it is called the same
Scoping

"Declaration of local resource"

Local variable

```c
int i = 10;
{
    while (i > 0)
    {
        i--; k = k + i;
    }
}
if (i = 2)
    .
    .
```

Scope boundary
Scoping

"Declaration of local resource"

Local variable

.  
.  
k=i+1;
{int i=10;
    while (i>0)
        {i--; k=k+i;};
};
if (i=2)
.  
.  

Local channel

not accessible from outside

Scope boundary

Local variable

.  
.  
k=i+1;
{int i=10;
    while (i>0)
        {i--; k=k+i;};
};
if (i=2)
.  
.  

Local channel
Scoping

”Declaration of local resource”

Local variable

```
    k = i + 1;
    { int i = 10;
        while (i > 0) {
            i--; k = k + i;
        }
    };
    if (i = 2)
    .
    .
```

Local channel

Scope boundary

tisdag 20 augusti 13
Universal law #1: Alpha-conversion

A scoped name can systematically be replaced by any other name not already occurring in its scope
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Scoping

Universal law #1: Alpha-conversion

A scoped name can systematically be replaced by any other name not already occurring in its scope

```c
k=i+1;
{
    int k=10;
    while (k>0)
    {
        k--; k=k+k;
    }
    if (i=2)
    .
    .
```
Scoping

Universal law #2: Scope extension

A scope can be extended (or retracted) as long as it does not include more (or fewer) occurrences of the scoped name.

```plaintext
while (i > 0)
  { i--; k=k+i; };

k=k*2;

k=i+1;

{ int i=10;
  while (i>0)
    { i--; k=k+i; };

k=k*2;
```

```plaintext

```
Scoping

Universal law #2: Scope extension

A scope can be extended (or retracted) as long as it does not include more (or fewer) occurrences of the scoped name.

```c
while (i > 0)
    { i--; k=k+i; }
k=k*2;
```

```c
int i=10;
{k=i+1;
{int i=10;
    while (i>0)
        { i--; k=k+i; }
k=k*2;
};
```
Scoping

Universal law #2: Scope extension

A scope can be extended (or retracted) as long as it does not include more (or fewer) occurrences of the scoped name.

```c
int i=10;
k+=i+1;
while (i>0)
{
i--; k=k+i;}
k=k*2;
```
Alpha-conversion
Alpha-conversion
Scope extension
Mobility

Data moves from caller to called

```c
int k=6;
k=fac(k)+3;

int fac(int i)
{
    if (i<2) return 1;
    else return i*fac(i-1);
}
```

Example: call by value
Mobility

Data moves from caller to called

```cpp
int k=6;
k=fac(k)+3;

int fac(int 6)
{
    if (6<2) return 1;
    else return 6*fac(6-1);
}
```

Example: call by value
Mobility

Data moves in a network

send \((R, 127)\);
...

receive \((P, x)\);
if \((x>0)\) ...
Mobility

Data moves in a network

send (R, 127);
...

receive (P, 127);
if (127>0) ...
Mobility of Scopes

What happens when a scoped thing moves out of its boundary?

Example: call by reference

```plaintext
{int k=3;
 foo(k); if k==4 ...
}

int foo(ref int i)
{if (i<2) i++;}
```
Mobility of Scopes

What happens when a scoped thing moves out of its boundary?

Example: call by reference

```c
{int k=3;
 foo(k); if k==4 ...}
```

```c
int foo(ref int k)
{if (k<2) k++;}
```

Scope of `k` is increased to old `i`!
Mobility of Scopes

Example: transfer access to local resource

The access to $R$ is in a scope only including Server
Mobility of Scopes

Example: transfer access to local resource

The access to $R$ is in a scope only including Server

The scope includes also Client!
Mobility of Scopes

Universal law #3: Scope Extrusion

When a scoped item is moved, the scope follows the item
Mobility of Scopes

Universal law #3: Scope Extrusion

When a scoped item is moved, the scope follows the item
Mobility of Scopes

What happens when a thing moves into a scope?

```c
int foo(ref int i)
{
    int k=0;
    if (i<k) i++;
}
foo(k); if k==4 ...
```
Mobility of Scopes

What happens when a thing moves into a scope?

```c
int foo(ref int k)
{
    int k=0;
    if (k<k) k++;
}
```

```c
foo(k); if k==4 ...
```
Mobility of Scopes

What happens when a thing moves into a scope?

```c
int foo(ref int k)
{
    int m=0;
    if (k<m) k++;
}
```

The scope is alpha-converted!
Mobility of Scopes

Universal law #4: Scope Intrusion

When an item is moved inside a scope, that scope is alpha-converted
Mobility of Scopes

Universal law #4: Scope Intrusion

When an item is moved inside a scope, that scope is alpha-converted
A minimal model

Assume a set of names $a, b, \ldots z$

The agents $P, Q, \ldots$ are of the following forms:

- $\overline{a} u.P$ Output, send $u$ along $a$
- $a(x).P$ Input for $x$ along $a$
- $P | Q$ $P$ and $Q$ in parallel
- $(\nu z) P$ Restriction: $z$ is local in $P$
Example

\[ \overline{az}.P \]

Output \( z \) along \( a \)

\[ a(x).\overline{bx}.Q \]

Input something on \( a \), then output it on \( b \)
Example

\[
\bar{a}z. P \quad \text{Output } z \text{ along } a
\]

\[
a(x). \bar{b}x. Q \quad \text{Input something on } a, \text{ then output it on } b
\]
Example

\[ \overline{a}z.P \] Output \( z \) along \( a \)

\[ a(x).\overline{b}x.Q \] Input something on \( a \), then output it on \( b \)

\[ P \mid \overline{b}z.Q \] "Does not occur in"

\[ \overline{a}z.P \mid a(x).\overline{b}x.Q \] Assuming \( x \neq Q \)
Example

\(\overline{a}z.P\)  
Output \(z\) along \(a\)

\(a(x).\overline{b}x.Q\)  
Input something on \(a\), then output it on \(b\)

\(P | \overline{b}z.Q\)  
Assuming \(x \neq Q\)

\(\overline{a}z.P | a(x).\overline{b}x.Q \rightarrow P | \overline{b}z.Q\)
Rules for Transitions

\[ P \rightarrow Q \]

Means that \( P \) can evolve into \( Q \) through an action that is internal to \( P \), possibly an interaction between components of \( P \).
Rules for Transitions

$P \rightarrow Q$

Means that $P$ can **evolve** into $Q$ through an action that is **internal** to $P$, possibly an interaction between components of $P$.

What about **compositionality**? How can we calculate the transitions of $T \mid U$ in terms of the transitions of $T$ and $U$?
Bad News:

This is clearly impossible:

$$\overline{au}.P \quad \overline{bu}.P$$

Have the same transitions (namely none)
Bad News:

This is clearly impossible:

\[
\overline{a}u. P \mid a(x). Q \quad \overline{b}u. P \mid a(x). Q
\]

\[
P \mid Q
\]

Have different transitions
Labelled Transitions

Solution: Introduce **labels** on transitions to signify **output** and **input** actions.

\[ P \xrightarrow{\alpha u} P' \]

Says \( P \) can **output** \( u \) along \( \alpha \) and move to state \( P' \)

\[ P \xrightarrow{\alpha u} P' \]

Says \( P \) can **input** \( u \) along \( \alpha \) and move to state \( P' \)
Some Rules

Substitution: Replace all $x$ by $u$, while alpha-converting any scopes of $u$ to take care of scope intrusion.
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Substitution: Replace all $x$ by $u$, while alpha-converting any scopes of $u$ to take care of scope intrusion.

Example:

$a(x).\overline{b}x.\overline{Q} \xrightarrow{au} \overline{b}u.\overline{Q}$

Assuming $x \# Q$
Communication rule

$$P \xrightarrow{\overline{a}u} P', \quad Q \xrightarrow{a} Q'$$
$$\frac{P | Q \xrightarrow{\tau} P' | Q'}{P | Q \xrightarrow{\tau} P' | Q'}$$

Example:

$$\overline{a} z. P | a(x). \overline{b} x. Q \xrightarrow{\tau} P | \overline{b} z. Q$$
Communication rule

\[
\begin{align*}
P & \xrightarrow{\overline{a}u} P', & Q & \xrightarrow{a u} Q' \\
\hline
P | Q & \xrightarrow{\tau} P' | Q'
\end{align*}
\]

Example:

\[
\overline{a}z. P | a(x). \overline{b}x. Q \xrightarrow{\tau} P | \overline{b}z. Q
\]

Now, what about scope extrusions?
\[
\bar{a}x. P \quad \bar{a}x. \ P \quad a(y). Q \quad a(y). \ Q
\]

\[
\bar{a}x. P \mid a(y). Q \quad \tau \quad P \mid Q\{x/y\}
\]
Value Passing

\[ \bar{a}x.P \xrightarrow{\bar{a}x} P \quad a(y).Q \xrightarrow{a(y)} Q \]

\[ \bar{a}x.P \mid a(y).Q \xrightarrow{\tau} P \mid Q\{x/y\} \]
\[ \overline{\alpha x}.P \overline{\alpha x} P \quad a(y).Q \overset{a(y)}{\rightarrow} Q \]

\[ \overline{\alpha x}.P \mid a(y).Q \overset{\tau}{\rightarrow} P \mid Q\{x/y\} \]

\[ (\overline{\alpha x}.P)\backslash x \]
\( \overline{\alpha x} \cdot P \xrightarrow{\alpha x} P \quad \alpha(y) \cdot Q \xrightarrow{\alpha(y)} Q \)

\( \overline{\alpha x} \cdot P \mid \alpha(y) \cdot Q \xrightarrow{\tau} P \mid Q\{x/y\} \)

\( (\overline{\alpha x} \cdot P) \backslash x \xrightarrow{\overline{\alpha(x)}} P \)
\[ \bar{a}x.P \xrightarrow{\bar{a}x} P \quad a(y).Q \xrightarrow{a(y)} Q \]

\[ \bar{a}x.P \mid a(y).Q \xrightarrow{\tau} P \mid Q\{x/y\} \]

\[ (\bar{a}x.P)\setminus x \xrightarrow{\bar{a}(x)} P \]

\[ (\bar{a}x.P)\setminus x \mid a(y).Q \]
Scope Extrusion!

\[ \overline{a}x.P \xrightarrow{\overline{a}x} P \]
\[ a(y).Q \xrightarrow{a(y)} Q \]

\[ \overline{a}x.P \mid a(y).Q \xrightarrow{\tau} P \mid Q\{x/y\} \]

\[ (\overline{a}x.P)\backslash x \xrightarrow{\overline{a}(x)} P \]

\[ (\overline{a}x.P)\backslash x \mid a(y).Q \xrightarrow{\tau} (P \mid Q\{x/y\})\backslash x \]
The very first written note by Robin on what was to become the pi-calculus.

What do you think Robin did in the very first sentence?

1) Explained the main idea
x) Explained the motivation
2) Gave most of the credit to someone else
A finitary language for label-passing communication

Outline

This is an attempt to simplify the presentation of the ideas of Nielsen and Forkjær, who made the technical break-through in showing that CCS can be extended to label-passing without losing any of the algebraic laws.

I have chosen the very simplest form that seems to work, with just one kind of variable — a label variable — and no constants. (These could be added, but we don’t seem to need them to get something sensible.) There is no recursion yet — when we add recursion, we probably have to add process variables too.

In the version I presented on 29/4/87 to the Concurrency Group, I used positive and negative labels, \( x \) for input and \( x^* \) for output. Joachim Parrow and I observed that — with a little
"This is an attempt to simplify the presentation of the ideas of Nielsen and Folkjaar [sic], who made the technical breakthrough in showing that CCS can be extended to label-passing without losing any of the algebraic laws."

In the version I presented on 29/4/87 to the concurrency group, I used positive and negative labels, \( \text{x} \) for input and \( \text{f} \) for output. Joachim Parrow and I observed that—with a little
Example

$$(\nu z) \overline{a}z. P$$  Output a local $z$ along $a$

$a(x). \overline{b}x. Q$  Input something on $a$, then output it on $b$
Example

\[(\nu z) \overline{a}z. P\]

Output a local \(z\) along \(a\)

\[a(x). \overline{b}x. Q\]

Input something on \(a\), then output it on \(b\)

\[(\nu z) \overline{a}z. P \mid a(x). \overline{b}x. Q\]
Example

\[(\nu z) \overline{a}z. P\]  
Output a local \(z\) along \(a\)

\[a(x) \overline{b}x. Q\]  
Input something on \(a\), then output it on \(b\)

\[(\nu z)\]

\[P \mid \overline{b}z. Q\]

\[(\nu z) \overline{a}z. P \mid a(x) \overline{b}x. Q\]
Example

\[(νz)(P | \overline{b}z. Q)\]

Output a local \(z\) along \(a\)

Input something on \(a\),
then output it on \(b\)

Because of Scope Extrusion!
Example

\((\nu z) \overline{a} z. P\)  
\(a(x). \overline{b} x. Q\)

Output a local \(z\) along \(a\)
Input something on \(a\), then output it on \(b\)

\((\nu z) (P | \overline{b} z. Q)\)

\((\nu z) \overline{a} z. P \mid a(x). \overline{b} x. Q \xrightarrow{\tau} (\nu z) (P | \overline{b} z. Q)\)
Scope Extrusion Rules

Scope Opening

\[
P \xrightarrow{\overline{a}z} P' \\
(\nu z)P \xrightarrow{\overline{a}(\nu z)z} P' \\
a \neq z
\]

Scope Closing

\[
P \xrightarrow{\overline{a}(\nu z)z} P', \quad Q \xrightarrow{az} Q' \\
\frac{P \mid Q \xrightarrow{\tau} (\nu z)(P' \mid Q')}{z \# Q}
\]
Scope Extrusion Rules

Scope Opening

A new kind of action signifying an output of a scoped $z$ along $a$

$$
\begin{align*}
P & \xrightarrow{\bar{a}z} P' \\
(\nu z)P & \xrightarrow{\bar{a}(\nu z)z} P' \\
\end{align*}
$$

Scope Closing

$$
\begin{align*}
P & \xrightarrow{\bar{a}(\nu z)z} P', \\
Q & \xrightarrow{az} Q' \\
P \mid Q & \xrightarrow{\tau} (\nu z)(P' \mid Q') \\
\end{align*}
$$

$z \# Q$
Scope Extrusion Rules

Scope Opening

A new kind of action signifying an output of a scoped $z$ along $a$

Note that the scope has disappeared from the agent. Instead it sits on the transition label.

Scope Closing

$P \xrightarrow{\bar{a}z} P'$

$(\nu z)P \xrightarrow{\bar{a}(\nu z)z} P'$

$P \mid Q \xrightarrow{\tau} (\nu z)(P' \mid Q')$

$z \# Q$

$tisdag 20 augusti 13$
Scope Extrusion Rules

Scope Opening

\[ P \xrightarrow{\alpha z} P' \]
\[ (\nu z) P \xrightarrow{\alpha(\nu z)z} P' \]
\[ a \neq z \]

Scope Closing

\[ P \xrightarrow{\alpha(\nu z)z} P', \quad Q \xrightarrow{az} Q' \]
\[ P \mathbin{|} Q \xrightarrow{\tau} (\nu z)(P' \mathbin{|} Q') \]
\[ z \# Q \]

The scope reappears in the agent.
Scope Extrusion Example

$$(\nu z)\bar{a}z.\ P \mid a(x).\bar{b}x.\ Q \rightarrow (\nu z)(P\mid\bar{b}z.\ Q)$$
Scope Extrusion Example

\[
\bar{a}z. P \xrightarrow{\bar{a}z} P
\]

\[
\bar{a}u. P \xrightarrow{\bar{a}u} P'
\]

\[
(\nu z)\bar{a}z. P \mid a(x). \bar{b}x. Q \rightarrow (\nu z)(P | \bar{b}z. Q)
\]
Scope Extrusion Example

\[
\begin{align*}
&\quad (\nu z)\overline{a}z.P \rightarrow (\nu z)(P|\overline{b}z.Q) \\
\end{align*}
\]
Scope Extrusion Example

\[ \overline{a}z.P \rightarrow P \]
\[ (\nu z)\overline{a}z.P \rightarrow \overline{a}(\nu z)z P \]
\[ a(x)\overline{b}x.Q \stackrel{a\overline{z}}{\rightarrow} b\overline{z}.Q \]

\[ a(x).P \xrightarrow{a_u} P'[x := u] \]

\[ (\nu z)(\overline{a}z.P) | a(x)\overline{b}x.Q \rightarrow (\nu z)(P | \overline{b}z.Q) \]
Scope Extrusion Example

\[
\begin{align*}
\overline{a}z. P & \rightarrow P \\
(\nu z)\overline{a}z. P & \rightarrow \overline{a}(\nu z)
\end{align*}
\]

\[
\begin{align*}
a(x). \overline{b}x. Q & \rightarrow b\overline{z}. Q \\
(\nu z)\overline{a}z. P | a(x). \overline{b}x. Q & \rightarrow (\nu z)(P | \overline{b}z. Q)
\end{align*}
\]
All the rules

\[
\begin{align*}
\bar{a}u \cdot P & \rightarrow \bar{a}u \rightarrow P \\
a(x) \cdot P & \rightarrow P[x := u] \\
\frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} & \quad \text{bn}(\alpha) \# Q \\
\frac{P \xrightarrow{\bar{a}u} P', Q \xrightarrow{au} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q'} & \quad P \mid Q \xrightarrow{\tau} (\nu z)(P' \mid Q') \\
\frac{P \xrightarrow{\alpha} P'}{(\nu u)P \xrightarrow{\alpha} (\nu u)P'} & \quad u \# \alpha \\
\frac{P \xrightarrow{\bar{a}z} P'}{(\nu z)P \xrightarrow{\bar{a}(\nu z)z} P'} & \quad a \neq z
\end{align*}
\]
The first pi-calculus semantics (May ’87!)

5. Rules of action (Rules marked ∗ have a symmetric form)

**FREE ACTION**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FACT</td>
<td>p \rightarrow p'</td>
</tr>
<tr>
<td>SILENT ACT</td>
<td>t, p \rightarrow p</td>
</tr>
</tbody>
</table>

**BOUND ACTION**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-FACT</td>
<td>x(y), p \rightarrow [y/\beta] p'</td>
</tr>
</tbody>
</table>

**SUM**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUM</td>
<td>p_i \rightarrow p_j'</td>
</tr>
<tr>
<td>\sum_i p_i \rightarrow p_j</td>
<td></td>
</tr>
</tbody>
</table>

**COMPOSITION**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC-COM</td>
<td>p, p' \rightarrow p_i'</td>
</tr>
<tr>
<td>TC-COM</td>
<td>p_i, p' \rightarrow p_i'</td>
</tr>
<tr>
<td>FC-COM</td>
<td>p_i, p' \rightarrow p_i'</td>
</tr>
<tr>
<td>RC-RES</td>
<td>p \rightarrow p'</td>
</tr>
</tbody>
</table>

**RESTRICTION**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>B-RES</td>
<td>p \rightarrow p</td>
</tr>
</tbody>
</table>
The first pi-calculus semantics (May ’87)!

5. Rules of action

Free action

\[ \text{FACT: } \alpha \Rightarrow P \Rightarrow P \]

Bound action

\[ \text{B-FACT: } \alpha(P) \Rightarrow P \]

No input / output

SUM

\[ \sum \frac{P_1 \Rightarrow P'_1}{\forall P_2 \Rightarrow P'_2 \Rightarrow P'_1 \mid P_2} \]

Composition

\[ \text{b-com: } \frac{P_1 \Rightarrow \alpha P'_1 \quad P_2 \Rightarrow \alpha P'_2}{P_1 \mid P_2 \Rightarrow \alpha (P'_1 \mid P'_2)} \]

Restriction

\[ \text{B-RES: } \frac{P \Rightarrow \alpha P'}{P \mid x \Rightarrow P' \mid x} \]
A Surprise

Up to now we have included the two kinds of variable binding, \( x(y).P \) and \( P \backslash y \). Can we do with just one kind? If so, the calculus gets cleaner and more “canonical”. Well, we can!

**Prop** If \( x \neq y \), then \( x(y).P \sim (xy.P) \backslash y \)
The first pi-calculus semantics (May ’87)!

5. Rules of action (Rules marked $\alpha$ have argument form)

FREE ACTION | BOUND ACTION

To be continued —

we need to prove $\sim$ a congruence,
then associativity of $\|$ etc.

Prop If $x \neq y$, then $x(y).P \sim (xy.P)\backslash y$
Turned out to have:

- Wrong basic constructors
- Wrong definition of bisimulation
- No sensible algebraic laws
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Revision
Turned out to have:

- Wrong basic constructors
- Wrong definition of bisimulation
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Revision

Establish properties
Turned out to have:

- Wrong basic constructors
- Wrong definition of bisimulation
- No sensible algebraic laws
From the pi-calculus proof archive (1987): first ever proof of scope extension law
Mobile processes (or the pi-calculus)

Robin Milner, Joachim Parrow, David Walker

Process calculi such as TCSP, ACP, CCS have not, on the whole, allowed for shifting contiguity among agents (though they allow them to bifurcate and to die). The purpose of this talk is to present a very basic calculus in which shifting contiguity, modelled by the use of names to communicate
I thought "process", or "pointer", or "parallel", but I also thought it a usable name -- if not too arrogant, and signifying that it aspires to primitivity like the lambda-calculus. You could also think of it as a near successor to the lambda calculus. Consider:
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mu-calculus ... this significantly exists
Robin’s reply to my question ”why pi”?

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- mu-calculus ... this significantly exists
- nu-calculus ... I thought we might have used this name, (nu standing for "name"), but mu and nu sound so alike.
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- omicron calculus ... who would want that?

Robin’s reply to my question ”why pi”?

tisdag 20 augusti 13
Robin’s reply to my question ”why pi”?

I thought "process", or "pointer", or "parallel", but I also thought it a usable name -- if not too arrogant, and signifying that it aspires to primitivity like the lambda-calculus. You could also think of it as a near successor to the lambda calculus. Consider:

mu-calculus ... this significantly exists
nu-calculus ... I thought we might have used this name,
(nu standing for "name"), but mu and nu sound so alike.
omicron calculus ... who would want that?
which leads to
PI-CALCULUS
... I put it in parentheses to try it out..
Exercise

• $R$ represents a resource that can be started by communicating along a trigger port $e$.

• $S$ represents a server controlling (ie handing out access to) the resource.

• $C$ represents a client requesting the resource. There may be many clients.

• How ensure that the only clients who can start $R$ are those who have been granted access by $S$?
Exercise

- $R$ represents a resource that can be started by communicating along a trigger port $e$. 
  
$$ R = e \cdot R' $$

- $S$ represents a server controlling (ie handing out access to) the resource. 
  
$$ S = (ve)(ae \ldots | R) $$

- $C$ represents a client requesting the resource. There may be many clients. 

- How ensure that the only clients who can start $R$ are those who have been granted access by $S$? 
  
$$ C = a(t) \ldots $$
Exercise

• $F$ represent an agent enacting a function. It receives some value along a certain link and produces (a link to) some result:

$$F = f(x) \ldots rv.0$$

• A caller $C$ calls $F$ and waits for the result. There may be several callers.

$$C = fu.r(z) \ldots$$

• How make sure that that only the $C$ who called $F$ will receive the result of its call?
Exercise

- $F$ represents an agent enacting a function. It receives some value along a certain link and produces (a link to) some result:

- A caller $C$ calls $F$ and waits for the result. There may be several callers.

- How make sure that only the $C$ who called $F$ will receive the result of its call?

\[ F = f(x) \ldots \bar{r}v.0 \]

\[ F = f(x) \cdot g(r) \ldots \bar{r}v \cdot 0 \]

\[ C = \bar{f}u \cdot r(z) \ldots \]

\[ C = \bar{f}u \cdot (\nu r)\bar{g}r \cdot r(z) \cdot 0 \]
Exercise

- $F$ represent an agent enacting a function. It receives some value along a certain link and produces (a link to) some result:

  $F = f(x) \ldots \bar{rv}.0$

  $F = f(x) \cdot g(r) \ldots \bar{rv} \cdot 0$

- A caller $C$ calls $F$ and waits for the result. There may be several callers.

  $C = \bar{fu} \cdot r(z) \ldots$

  $C = \bar{fu} \cdot (vr)\bar{gr} \cdot r(z) \cdot 0$

- How make sure that that only the $C$ who called $F$ will receive the result of its call?

  But do we really know that $f$ gets both $x$ and $r$ from the same $C$?
Exercise

• A world with one server $S$ and several clients $C$.

• $S$ sends two names to any client who will listen. But both names must end up with the same client!

$$S = \overline{an}_1 \cdot \overline{an}_2 \cdot 0$$
$$C = a(x) \cdot a(y) \cdot \ldots$$
$$S \mid C \mid C \cdots \mid C$$
Exercise

• A world with one server $S$ and several clients $C$.

• $S$ sends two names to any client who will listen. But both names must end up with the same client!

\[ S = (\nu p)(\overline{ap} \cdot \overline{pn}_1 \cdot \overline{pn}_2 \cdot 0) \]
\[ C = a(q) \cdot q(x) \cdot q(y) \cdot \ldots \]
\[ S \mid C \mid C \ldots \mid C \]
Additional operators

**Sum**: $P + Q$ means an agent behaving as either $P$ or $Q$, resolved at the first action.

$$P \xrightarrow{\alpha} P'$$

$$P + Q \xrightarrow{\alpha} P'$$

Same kind of choice as in a nondeterministic automaton: just says that both branches are possible and nothing about how the choice is resolved.
Additional operators

**match, mismatch:**

\[ [x = y]P \] means an agent behaving as \( P \) if \( x \) and \( y \) are the same name, ow do nothing.

\[ [x \neq y]P \] means an agent behaving as \( P \) if \( x \) and \( y \) are not the same name, ow do nothing.

\[ P \xrightarrow{\alpha} P' \]

\[ [x = x]P \xrightarrow{\alpha} P' \]

\[ P \xrightarrow{\alpha} P', \quad x \neq y \]

\[ [x \neq y]P \xrightarrow{\alpha} P' \]
Additional operators

match, mismatch:

\[ [x = y]P \] means an agent behaving as \( P \) if \( x \) and \( y \) are the same name, otherwise do nothing.

\[ [x \neq y]P \] means an agent behaving as \( P \) if \( x \) and \( y \) are not the same name, otherwise do nothing.

\[
\begin{align*}
P & \xrightarrow{\alpha} P' \\
[x = x]P & \xrightarrow{\alpha} P' \\
[x \neq y]P & \xrightarrow{\alpha} P'
\end{align*}
\]

\[ [\varphi]P \text{ means “if } \varphi \text{ then } P” \]
Exercise

An agent receives a name along $a$. If the received name is $u$ then it is forwarded along $b$. Otherwise it is forwarded along $c$. 
Exercise

An agent receives a name along \( a \). If the received name is \( u \) then it is forwarded along \( b \). Otherwise it is forwarded along \( c \).

\[
a(x) \cdot ([x = u] bx \cdot 0 + [x \neq u] cx \cdot 0)
\]
Additional operators

replication and recursion

\( !P \) means an agent behaving as an unlimited number of copies of \( P \).

\( A \leftarrow P \) means that the agent identifier \( A \) represents whatever \( P \) is. Note that \( A \) can occur in \( P \), leading to recursion.

\[
\begin{align*}
  P \mid !P & \xrightarrow{\alpha} P' \\
  !P & \xrightarrow{\alpha} P'
\end{align*}
\]

\[
\begin{align*}
  A \leftarrow P, \quad P & \xrightarrow{\alpha} P' \\
  A & \xrightarrow{\alpha} P'
\end{align*}
\]
Exercise

Do the following agents behave the same, or else how are they different?

\[ A \leftarrow a(x) \cdot \overline{b}x \cdot A \]

\[ !a(x) \cdot \overline{b}x \cdot 0 \]
Exercise

Do the following agents behave the same, or else how are they different?

\[ A \leftarrow a(x) \cdot \overline{bx} \cdot A \quad \quad A \xrightarrow{au} \overline{bu} \cdot A \]

\[ \neg a(x) \cdot \overline{bx} \cdot \mathbf{0} \]
Exercise

Do the following agents behave the same, or else how are they different?

\[ A \leftarrow a(x) \cdot \overline{b}x \cdot A \quad A \xrightarrow{au} \overline{b}u \cdot A \]

\[ !a(x) \cdot \overline{b}x \cdot 0 \]

\[ P \mid !P \xrightarrow{\alpha} P' \]

\[ !P \xrightarrow{\alpha} P' \]
Exercise

Do the following agents behave the same, or else how are they different?

\[ A \leftarrow a(x) \cdot \overline{b}x \cdot A \]

\[ A \xrightarrow{au} \overline{b}u \cdot A \]

\[ !a(x) \cdot \overline{b}x \cdot 0 \]

\[ a(x) \cdot \overline{b}x \cdot 0 \xrightarrow{au} \overline{b}u \cdot 0 \]

\[ a(x) \cdot \overline{b}x \cdot 0 \mid !a(x) \cdot \overline{b}x \cdot 0 \xrightarrow{au} \overline{b}u \cdot 0 \mid !a(x) \cdot \overline{b}x \cdot 0 \]
Exercise

Do the following agents behave the same, or else how are they different?

\[ A \leftrightarrow a(x) . \bar{b}x . A \]

\[ A \xrightarrow{au} \bar{b}u . A \]

\[ !a(x) . \bar{b}x . 0 \]

\[ P \mid !P \xrightarrow{\alpha} P' \]

\[ !P \xrightarrow{\alpha} P' \]

\[ a(x) . \bar{b}x . 0 \xrightarrow{au} \bar{b}u . 0 \]

\[ a(x) . \bar{b}x . 0 \mid !a(x) . \bar{b}x . 0 \xrightarrow{au} \bar{b}u . 0 \mid !a(x) . \bar{b}x . 0 \]

\[ !a(x) . \bar{b}x . 0 \xrightarrow{au} \bar{b}u . 0 \mid !a(x) . \bar{b}x . 0 \]
Exercise

- $F$ represent an agent enacting a function. It receives some value along a certain link and produces (a link to) some result:

  
  $$F = f(x) \cdot g(r) \ldots \cdot rv \cdot 0$$

- A caller $C$ calls $F$ and wait for the result. There may be several callers.

  $$C = fu \cdot (vr)gr \cdot r(z) \cdot 0$$

- How make sure that that only the $C$ who called $F$ will receive the result of its call?
Exercise

Can we make the function $F$ callable several times?

Can we make it reentrant?

\[ F = f(x) \cdot g(r) \cdot \ldots \cdot \bar{r} \nu \cdot 0 \]

\[ C = \bar{f} u \cdot (\nu r) \bar{g} r \cdot r(z) \cdot 0 \]
Exercise

Can we make the function $F$ callable several times?

Can we make it reentrant?

\[
F = f(x) \cdot g(r) \ldots \bar{r}v \cdot 0
\]

\[
F \leftarrow f(x) \cdot g(r) \ldots \bar{r}v \cdot F
\]

\[
C = \bar{f}u \cdot (\nu r)\bar{g}r \cdot r(z) \cdot 0
\]
Exercise

Can we make the function $F$ callable several times?

Can we make it reentrant?

$$F = f(x) \cdot g(r) \ldots \bar{rv} \cdot 0$$

$$F \leftarrow f(x) \cdot g(r) \ldots \bar{rv} \cdot F$$

$$C = \bar{f} u \cdot (\nu r) \bar{gr} \cdot r(z) \cdot 0$$

This disregards that $F$ might receive $x$ and $r$ from different $C$. 
Exercise

Can we make the function $F$ callable several times?

$F = f(x).g(r)\ldots\bar{rv}.0$

Can we make it reentrant?

$F \leftarrow f(x).g(r)\ldots\bar{rv}.F$

$F =! f(x).g(r)\ldots\bar{rv}.0$

$C = \bar{f} u. (\nu r)\bar{gr}.r(z).0$

This disregards that $F$ might receive $x$ and $r$ from different $C$. 
Bisimulation

When do two agents ”behave the same”?
Bisimulation

When do two agents "behave the same"?

\[ \alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \]
Bisimulation

When do two agents ”behave the same”?

\[
\begin{align*}
\alpha \cdot 0 & \quad \alpha \cdot 0 + \alpha \cdot 0 \\
P & \quad P + P
\end{align*}
\]
Bisimulation

When do two agents "behave the same"?

\[
\begin{align*}
\alpha \cdot 0 & \quad \alpha \cdot 0 + \alpha \cdot 0 \\
P & \quad P + P \\
\alpha \cdot \beta \cdot 0 + \beta \cdot \alpha \cdot 0 & \quad \alpha \cdot 0 | \beta \cdot 0
\end{align*}
\]
Bisimulation

When do two agents "behave the same"?

\[
\begin{align*}
\alpha \cdot 0 & \quad \alpha \cdot 0 + \alpha \cdot 0 \\
\quad P & \quad P + P \\
\alpha \cdot \beta \cdot 0 + \beta \cdot \alpha \cdot 0 & \quad \alpha \cdot 0 | \beta \cdot 0 \\
\quad !P & \quad !P | P
\end{align*}
\]
## Bisimulation

When do two agents "behave the same"?

<table>
<thead>
<tr>
<th>$\alpha \cdot 0$</th>
<th>$\alpha \cdot 0 + \alpha \cdot 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$P + P$</td>
</tr>
<tr>
<td>$\alpha \cdot \beta \cdot 0 + \beta \cdot \alpha \cdot 0$</td>
<td>$\alpha \cdot 0</td>
</tr>
<tr>
<td>$!P$</td>
<td>$!P</td>
</tr>
<tr>
<td>$(P</td>
<td>Q)$</td>
</tr>
</tbody>
</table>
Bisimulation

When do two agents ”behave the same”?

\[
\begin{align*}
\alpha . 0 & \quad \alpha . 0 + \alpha . 0 \\
P & \quad P + P \\
\alpha . \beta . 0 + \beta . \alpha . 0 & \quad \alpha . 0 \mid \beta . 0 \\
!P & \quad !P \mid P \\
!(P \mid Q) & \quad !(P \mid Q) \mid Q \\
(P \mid Q) \mid R & \quad P \mid (Q \mid R)
\end{align*}
\]
Bisimulation

We seek an equivalence relation on agents such that

- Equivalent agents cannot possibly be distinguished in any intuitive way by following transitions
- It is compositional
Bisimulation

**First idea:** $P$ and $Q$ are equivalent if they have the same actions.

\[
\alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0
\]
Bisimulation

**First idea:** $P$ and $Q$ are equivalent if they have the same actions.

\[ \alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \]

Not so good, consider

\[ \beta \cdot \alpha \cdot 0 \quad \beta \cdot (\alpha \cdot 0 + \alpha \cdot 0) \]
Bisimulation

\[ \alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \]

Not so good, consider

\[ \beta \cdot \alpha \cdot 0 \quad \beta \cdot (\alpha \cdot 0 + \alpha \cdot 0) \]
Bisimulation

**Refined idea:** $P$ and $Q$ are equivalent if they have the same actions, leading to equivalent agents.
Bisimulation

Refined idea: $P$ and $Q$ are equivalent if they have the same actions, leading to equivalent agents.

Not so good, this definition is circular!
Bisimulation

Not so good, this definition is circular!
Bisimulation

Not so good, this definition is circular!

Can we use induction?
Bisimulation

**Inductive** definition? Does this work?

$P$ is equivalent to $Q$ if for all $\alpha$ such that $P \xrightarrow{\alpha} P'$
it holds that $Q \xrightarrow{\alpha} Q'$ and $P'$ is equivalent to $Q'$
and vice versa
Bisimulation

**Inductive** definition? Does this work?

$P$ is equivalent to $Q$ if for all $\alpha$ such that $P \xrightarrow{\alpha} P'$ it holds that $Q \xrightarrow{\alpha} Q'$ and $P'$ is equivalent to $Q'$ and vice versa

**We also need** $\text{bn}(\alpha) \# Q$
Bisimulation

**Inductive** definition? Does this work?

*P* is equivalent to *Q* if for all *α* such that *P* $\xrightarrow{\alpha} P'$ it holds that *Q* $\xrightarrow{\alpha} Q'$ and *P'* is equivalent to *Q'* and vice versa

We also need $\text{bn}(\alpha) \# Q$

But $\xrightarrow{\alpha}$ is not well founded!
Bisimulation

**Inductive** definition? Does this work?

We also need $bn(\alpha) \# Q$

But $\xrightarrow{\alpha}$ is not well founded!
Bisimulation

Correct definition is **coinductive** (due to Park 1981)

We seek the largest equivalence relation satisfying

\[ P \text{ is equivalent to } Q \text{ implies that } \forall \alpha. \text{bn}(\alpha) \# Q \text{ and } P \xrightarrow{\alpha} P' \]

it holds that \( Q \xrightarrow{\alpha} Q' \) and \( P' \) is equivalent to \( Q' \)

and vice versa

A relation \( \sim \) satisfying this property is called a **bisimulation**
Bisimulation

A relation satisfying this property is not necessarily an equivalence (for example the empty relation is a bisimulation) so a better formulation is

The binary relation $R$ is a bisimulation if, for all $P, Q$:

$P R Q$ implies that for all $\alpha$ such that $bn(\alpha) \# Q$ and $P \xrightarrow{\alpha} P'$

it holds that $Q \xrightarrow{\alpha} Q'$ and $P' R Q'$

and vice versa
The binary relation $R$ is a bisimulation if, for all $P, Q$: $P R Q$ implies that for all $\alpha$ such that $\text{bn}(\alpha) \neq Q$ and $P \xrightarrow{\alpha} P'$ it holds that $Q \xrightarrow{\alpha} Q'$ and $P' R Q'$ and vice versa.

Three equivalent definitions:

$P \sim Q$ if there exists a bisimulation relating $P$ and $Q$

$\sim$ is the union of all bisimulations

$\sim$ is the largest bisimulation

$\sim$ is called **bisimilarity**
Examples

\[ \alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \]

\[ P \quad P + P \]

\[ (P \mid Q) \mid R \quad P \mid (Q \mid R) \]
Examples

\[\alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \quad \{ (\alpha \cdot 0, \alpha \cdot 0 + \alpha \cdot 0), (0, 0) \} \]

\[P \quad P + P\]

\[(P \mid Q) \mid R \quad P \mid (Q \mid R)\]
Examples

\[ \alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \quad \{ (\alpha \cdot 0, \alpha \cdot 0 + \alpha \cdot 0), (0, 0) \} \]

\[ P \quad P + P \quad \{ (P, P + P) \} \cup \text{Id} \]

\[ (P | Q) \mid R \quad P \mid (Q \mid R) \]
Examples

\[ \alpha \cdot 0 \quad \alpha \cdot 0 + \alpha \cdot 0 \quad \{(\alpha \cdot 0, \alpha \cdot 0 + \alpha \cdot 0), (0, 0)\} \]

\[ P \quad P + P \quad \{(P, P + P)\} \cup \text{Id} \]

\[ (P \mid Q) \mid R \quad P \mid (Q \mid R) \quad \text{Surprisingly complicated:} \]

\[ \{((\nu a_1) \cdots (\nu a_n)((P \mid Q) \mid R), ((\nu a_1) \cdots (\nu a_n)(P \mid (Q \mid R)) \cdots} \]
Compositionality

If $P \sim Q$ then

- $P \mid R \sim Q \mid R$
- $P + R \sim Q + R$
- $!P \sim !Q$
- $\overline{a}u \cdot P \sim \overline{a}u \cdot P$
- $[x = y]P \sim [x = y]Q$
- $[x \neq y]P \sim [x \neq y]Q$
Compositionality

Does $P \sim Q$ imply $a(x) \cdot P \sim a(x) \cdot Q$?
Compositionality

Does \( P \sim Q \) imply \( a(x) \cdot P \sim a(x) \cdot Q \)?

No!
Compositionality

Does $P \sim Q$ imply $a(x).P \sim a(x).Q$?
No!

$P = 0, \quad Q = [x = y] \alpha \cdot 0$
Compositionality

Does $P \sim Q$ imply $a(x) \cdot P \sim a(x) \cdot Q$?

No!

$P = 0, \quad Q = [x = y] \alpha \cdot 0$

Neither has any transition so they are bisimilar.

But

$a(x) \cdot Q \xrightarrow{\alpha y} [y = y] \alpha \cdot 0 \xrightarrow{\alpha} 0$

$a(x) \cdot P \xrightarrow{\alpha y} 0 \xrightarrow{\alpha} 0$
Compositionality

The input prefix $a(x)$ means that $x$ can be substituted by something received

So, for compositionality we also require that agents are bisimilar under substitutions.

$P \sim Q$ if for all $\tilde{x}, \tilde{y}$ it holds that $P[\tilde{x} := \tilde{y}] \sim Q[\tilde{x} := \tilde{y}]$
Compositionality

\sim is a congruence and satisfies several useful laws

There are several versions of bisimilarity. The one presented here is called **strong early bisimilarity**. There are also several alternative ways to present the semantics.
The End for now

- Read in "An introduction to the pi-calculus"
- Work on the review questions. Tackle those you think you can manage and interest you.
- See you after lunch!