Advanced Process Calculi

Lecture 3: bisimulation in psi-calculi

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Psi-calculi

T	(Data) Terms	M, N
A	Assertions	Ψ, Ψ'
C	Conditions	φ, φ'

Channel Object Pattern Test (aka guard)

$\overline{M}\, N.P$	Output
$\underline{M}(\lambda\widetilde{x})N.P$	Input
$\mathbf{case}\ \varphi_1 : P_1 \,[\,]\, \cdots \,[\,]\, \varphi_n : P_n$	Case
$(\nu a)P$	Restriction
$P \mid Q$	Parallel
$!P$	Replication
$(\!\vert \Psi \vert\!)$	Assertion

To mean tell Ψ

Instance parameters

$$\boxed{X[\widetilde{x} := \widetilde{T}]}$$

Equivariance: $p \cdot (X[\tilde{x} := \tilde{T}]) = (p \cdot X)\big[(p \cdot \tilde{x}) := (p \cdot \tilde{T})\big]$

Freshness: if $\tilde{x} \subseteq \mathrm{n}(X)$ and $a \# X[\tilde{x} := \tilde{T}]$ then $a \# \tilde{T}$

Alpha-equivalence: if $p \subseteq \tilde{x} \times (p \cdot \tilde{x})$ and $(p \cdot \tilde{x}) \# X$ then
$X[\tilde{x} := \tilde{T}] = (p \cdot X)[(p \cdot \tilde{x}) := \tilde{T}]$

$$\begin{array}{l} \dot{\leftrightarrow} : \mathbf{T} \times \mathbf{T} \to \mathbf{C} \\ \otimes : \mathbf{A} \times \mathbf{A} \to \mathbf{A} \\ \mathbf{1} : \mathbf{A} \\ \vdash \subseteq \mathbf{A} \times \mathbf{C} \end{array}$$

Channel Symmetry: $\Psi \vdash M \dot{\leftrightarrow} N \implies \Psi \vdash N \dot{\leftrightarrow} M$

Channel Transitivity: $\Psi \vdash M \dot{\leftrightarrow} N \ \wedge \ \Psi \vdash N \dot{\leftrightarrow} L$
$\implies \Psi \vdash M \dot{\leftrightarrow} L$

Composition: $\Psi \simeq \Psi' \implies \Psi \otimes \Psi'' \simeq \Psi' \otimes \Psi''$

Identity: $\Psi \otimes \mathbf{1} \simeq \Psi$

Associativity: $(\Psi \otimes \Psi') \otimes \Psi'' \simeq \Psi \otimes (\Psi' \otimes \Psi'')$

Commutativity: $\Psi \otimes \Psi' \simeq \Psi' \otimes \Psi$

All the rules

$$\textsc{In}\ \frac{\Psi \vdash M \dot{\leftrightarrow} K}{\Psi \rhd \underline{M}(\lambda\widetilde{y})N.P \xrightarrow{\underline{K}\,N[\widetilde{y}:=\widetilde{L}]} P[\widetilde{y}:=\widetilde{L}]} \qquad \textsc{Out}\ \frac{\Psi \vdash M \dot{\leftrightarrow} K}{\Psi \rhd \overline{M}\,N.P \xrightarrow{\overline{K}\,N} P} \qquad \textsc{Case}\ \frac{\Psi \rhd P_i \xrightarrow{\alpha} P' \qquad \Psi \vdash \varphi_i}{\Psi \rhd \mathbf{case}\ \widetilde{\varphi} : \widetilde{P} \xrightarrow{\alpha} P'}$$

$$\textsc{Com}\ \frac{\Psi_Q \otimes \Psi \rhd P \xrightarrow{\overline{M}\,(\nu\widetilde{a})N} P' \qquad \Psi_P \otimes \Psi \rhd Q \xrightarrow{\underline{K}\,N} Q' \qquad \Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \dot{\leftrightarrow} K}{\Psi \rhd P \mid Q \xrightarrow{\tau} (\nu\widetilde{a})(P' \mid Q')}\ \widetilde{a}\#Q$$

$$\textsc{Par}\ \frac{\Psi_Q \otimes \Psi \rhd P \xrightarrow{\alpha} P'}{\Psi \rhd P|Q \xrightarrow{\alpha} P'|Q}\ \mathrm{bn}(\alpha)\#Q \qquad \textsc{Scope}\ \frac{\Psi \rhd P \xrightarrow{\alpha} P'}{\Psi \rhd (\nu b)P \xrightarrow{\alpha} (\nu b)P'}\ b\#\alpha,\Psi$$

$$\textsc{Open}\ \frac{\Psi \rhd P \xrightarrow{\overline{M}\,(\nu\widetilde{a})N} P'}{\Psi \rhd (\nu b)P \xrightarrow{\overline{M}\,(\nu\widetilde{a}\cup\{b\})N} P'}\ \begin{array}{c} b\#\widetilde{a},\Psi,M \\ b \in \mathrm{n}(N) \end{array} \qquad \textsc{Rep}\ \frac{\Psi \rhd P \mid !P \xrightarrow{\alpha} P'}{\Psi \rhd !P \xrightarrow{\alpha} P'}$$

+ freshness conditions in Par and Com

+ symmetric variants

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How can we prove **beyond doubt** that we have made no mistake similar to applied pi and CC-pi?

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...and that it satisfies the scoping laws, eg scope extension, that $(\nu a)P \mid Q$ and $(\nu a)(P|Q)$ have the same transitions if $a\#Q$

Strategy

Define an intuitive equivalence from the semantics

Prove that it is a congruence

Prove that it is satisfies universal laws like scope extension

ie exactly the same strategy as in the pi-calculus

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The environment used to satisfy φ
Will it always do this?
Can it change to make φ false?

The environment is any agent, and this can evolve!

Eg: $Q = (\!|\Psi|\!) \mid \tau \,.\, (\!|\Psi'|\!)$

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If so the agents behave the same

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The crucial question is if the psi-calculus satisfies

$$\Psi \vdash \varphi \ \Rightarrow \Psi \otimes \Psi' \vdash \varphi$$ **monotonicity**

The only thing we know for sure about any psi-calculus:

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So there are psi-calculi where this does not hold

Good: this means we can have psi-calculi describing concurrent constraints with retracts!

Bad: this means that some natural looking laws are not valid in these calculi

$$\mathbf{if}\ \varphi\ \mathbf{then}\ \tau.\,P \quad \overset{?}{\sim} \quad \mathbf{if}\ \varphi\ \mathbf{then}\ \tau.\,\mathbf{if}\ \varphi\ \mathbf{then}\ P$$

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This law is only valid if assertion composition is monotonic

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In fact this is expected: if the environment can evolve to retract facts the law should not be valid!

Universal laws

Universal laws are those valid in **all** psi-calculi

We should at least expect something like

$$
\begin{array}{rcll}
P & \sim & P \mid \mathbf{0} & \\
P \mid (Q \mid R) & \sim & (P \mid Q) \mid R & \\
P \mid Q & \sim & Q \mid P & \\
(\nu a)\mathbf{0} & \sim & \mathbf{0} & \\
P \mid (\nu a)Q & \sim & (\nu a)(P \mid Q) & \text{if } a \# P \\
\overline{M}\ N.(\nu a)P & \sim & (\nu a)\overline{M}\ N.P & \text{if } a \# M, N \\
\underline{M}(\lambda \widetilde{x})N.(\nu a)P & \sim & (\nu a)\underline{M}(\lambda \widetilde{x})(N).P & \text{if } a \# M, N \\
\textbf{case}\ \widetilde{\varphi} : \widetilde{(\nu a)P} & \sim & (\nu a)\textbf{case}\ \widetilde{\varphi} : \widetilde{P} & \text{if } a \# \widetilde{\varphi} \\
(\nu a)(\nu b)P & \sim & (\nu b)(\nu a)P & \\
!P & \sim & P \mid !P &
\end{array}
$$

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If any of thee fail something is wrong with our definitions!

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Universal laws

We also want compositionality

Ie the equivalence is a congruence

Bisimulation

First attempt, can we as in the pi-calculus define bisimulation only in terms of transitions?

The binary relation R is a *bisimulation* if

1. R is symmetric

2. $P\,R\,Q$ implies that $\forall\alpha.\mathrm{bn}(\alpha)\#Q$.
   $P \xrightarrow{\alpha} P'$ implies $Q \xrightarrow{\alpha} Q'$ and $P'\,R\,Q'$

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"Can be alpha-converted to each other"

$\boxed{F \vdash \varphi}$ means $F =_\alpha (\nu\widetilde{b})\Psi$

$\widetilde{b}\#\varphi$

$\Psi \vdash \varphi$

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This will not hold in any psi-calculus
Not even in monotonic ones!

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$P \stackrel{\bullet}{\sim} Q$ if for some bisimulation R it holds $\forall \Psi . R(\Psi, P, Q)$

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We are almost there, but not quite!

This definition will not satisfy compositionality!

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Proves $P \sim Q$

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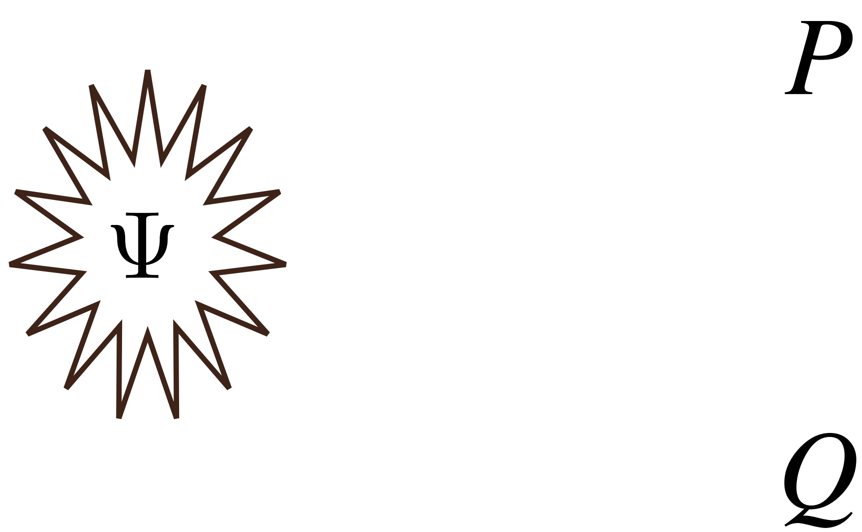
Ergo, $P \dot{\sim} Q$ but not $P \mid T \dot{\sim} Q \mid T$

Bisimulation graphically

In the environment Ψ, P and Q behave similarly.

If P' changes to P' then Q can mimic this to Q'



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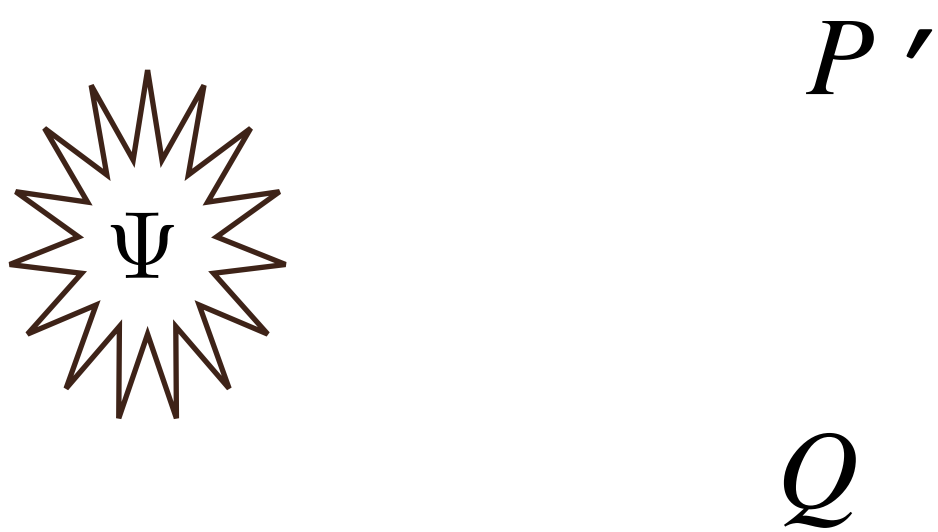

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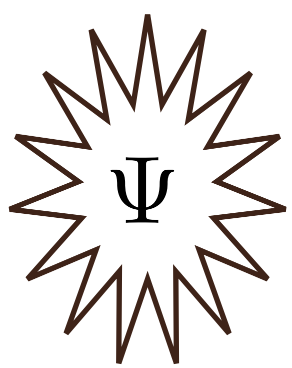

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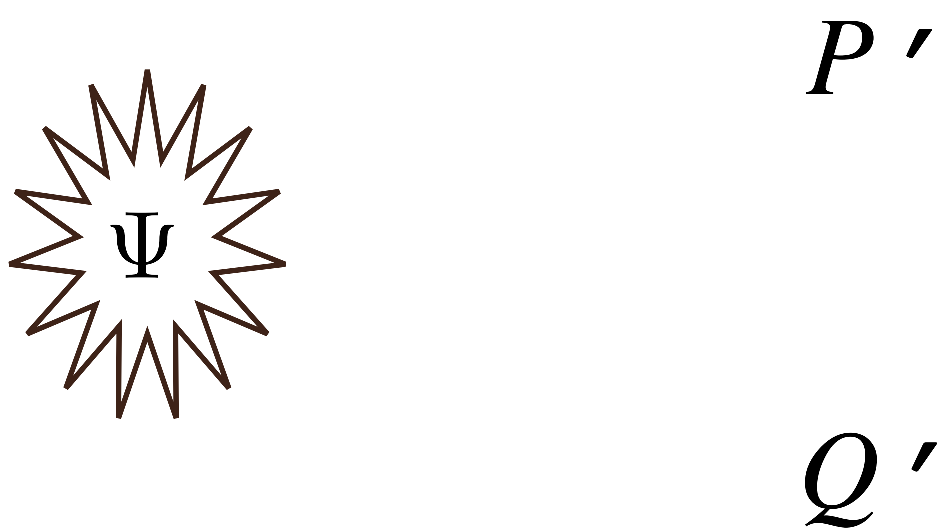

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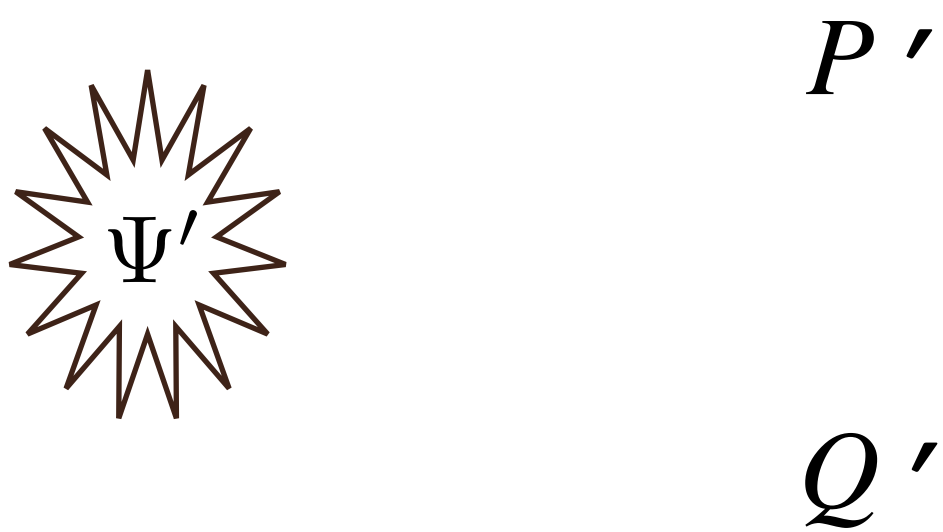

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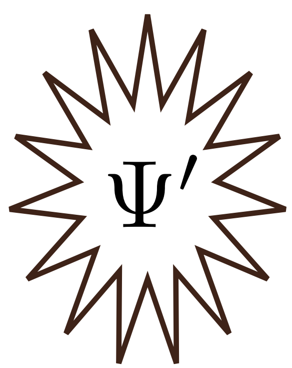

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The only way an agent in the environmnent can change the environmental assertion is to add more assertions to it!

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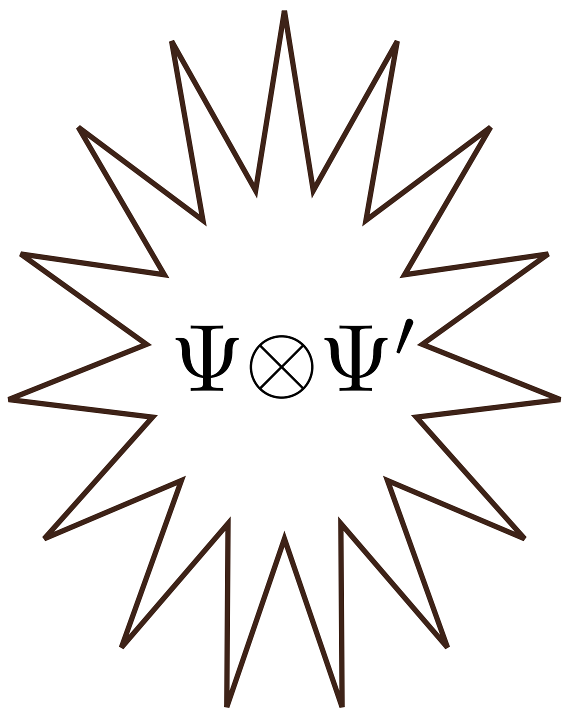

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R is a *bisimulation* if $R(\Psi, P, Q)$ implies

1. $R(\Psi, Q, P)$

2. $\forall \alpha . \mathrm{bn}(\alpha) \# Q, \Psi.$

   $\Psi \rhd P \xrightarrow{\alpha} P'$ implies $\Psi \rhd Q \xrightarrow{\alpha} Q'$ and $R(\Psi, P', Q')$

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$\dot{\sim}$ is the largest bisimulation.

$P \sim Q$ If for all sequences of substitutions σ, $P\sigma \mathrel{\dot{\sim}} Q\sigma$

Alternative definition

A binary relation R is a *context bisimulation* if $R(P, Q)$ implies

1. $R(Q, P)$
2. $\forall \alpha . \mathrm{bn}(\alpha) \# Q$
   $\mathbf{1} \rhd P \xrightarrow{\alpha} P'$ implies $\mathbf{1} \rhd Q \xrightarrow{\alpha} Q'$ and $R(P', Q')$
3. $\mathcal{F}(P) \simeq \mathcal{F}(Q)$
4. $\forall \Psi .\ R((\!|\Psi|\!) \mid P, \ \ (\!|\Psi|\!) \mid Q)$

Comparison

$\mathcal{R}(P, Q)$

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$$P \xrightarrow{\alpha} P' \implies \begin{array}{l} Q \xrightarrow{\alpha} Q' \\ \wedge\, \mathcal{R}(P', Q') \end{array}$$

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Manual proof: 1-2 hours, 70 lines
Isabelle proof: 1 day, 450 lines

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Results

- Standard Semantics
- Symbolic Semantics
- Compositionality
- Strong and Weak Bisimulation
- Barbed Congruence
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Definition of behaviour

Results

- Standard Semantics ← Definition of behaviour
- Symbolic Semantics ← More ”computable”
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If P and Q behave the same, then $P|R$ and $Q|R$ behave the same

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Efficient proof method

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Machine checked once and for all

Intuitive equivalence

Correctness: the holy grail

Theory Development in a Theorem Prover

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Benefit 4: **Generality** (easy to keep track of assumptions)

Flexibility

- **Theory development is like programming**: It almost never starts from scratch. You continually add, improve, amend, adjust...

Flexibility

- **Theory development is like programming**: It almost never starts from ... nually ... end, adjust...

Please change this one

- **Programming**: Every amendment needs a program recompilation.

- **Theory development**: Every amendment needs a re-check of all proofs. *A huge error source*.

- **Mechanised proofs** means we have a proof repository and can quickly assess ramifications of changes.

Generality

”hmm, some need grease,
some need oil,
I better give both to all...

The lemmas in a theory are like cogwheels, each with a little twisty set of assumptions

...otherwise I would have to remember who needs what.”

Generality

”hmm, some need ~~grease~~, Weakening
some need ~~oil~~, Idempotence
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The lemmas in a theory are like cogwheels, each with a little twisty set of assumptions

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Example from Psi

A binary relation R on agents is an MJbisim if R(P,Q) implies

1. F(P)=F(Q) (static equiv)
2. R(Q,P)
3. Forall Psi. R({Psi}|P, {Psi}|Q)
4. Forall a s.t. bn(a)#Q. P -a-> P' => Q-a->Q' and R(P',Q')

(here transitions without assertion means bottom assertion)

Conjecture 1.
a) Psi |> P -a-> P' implies {Psi}|P -a-> {Psi}|P'.
b) {Psi}|P -a-> T implies exists P'. T = {Psi}|P' and Psi |> P -a-> P'
Proof: For a: by the PAR rule and F({Psi})=Psi. For b: case analysis on deirivation of {Psi}|P-a->T, and here only PAR can be used. Details are left as an exercise for the reader :)

Conjecture 2.
{Psi}|{Psi'} ~ {Psi+Psi'}
Proof: Directly from definitions. Obvious :)

Conjecture 3. If R is an MJbisim up to ~ and R(P,Q) then there is an MJbisim R' such that R'(P,Q)
Proof: By intimidation :)

Lemma 1
If R is an MJbisim then R* =def {(Psi, P,Q): R({Psi}|P, {Psi}|Q)} is a bisimulation up to ~
Proof. We need to check 4 conditions. Assume R*(Psi,P.Q). Then R({Psi}|P, {Psi}|Q).
1. Psi + F(P) = Psi + F(Q). Follows from F({Psi}|P) = F({Psi}|Q).
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3. All Psi' . R*(Psi+Psi', P, Q). Follows from All Psi' . R({Psi'}|{Psi}|P, {Psi'}|{Psi}|Q), and Conjecture 2. Note that here we probably need associativity.
4. Psi |> P-a-> P' implies exists Q' . Psi |> Q-a-> Q' and R(Psi,P',Q'). So assume Psi |> P-a-> P'. Then by Conjecture 1a {Psi}|P -a-> {Psi}|P'. By Condition 4 on MJbisim and R({Psi}|P, {
{Psi}|Q -a-> T with R({Psi}|P',T). Conjecture 1b then gives that there exists a Q' such that T = {Psi}|Q' and {Psi} |> Q -a-> Q'. Also R({Psi}|P',{Psi}|Q') by definition implies R*(Psi,P',Q'), a
QED

Lemma 2.
If R* is a bisimulation then R = def {({Psi}|P, {Psi}|Q): R*(Psi,P,Q)} is an MJbisim up to ~.
Proof. We need to check 4 conditions. Assume R(T,U). By definition there are Psi,P,Q s.t. T={Psi}|P, U={Psi}|Q, R*(Psi,P,Q).
1. F(T)=F(U). Follows from R*(Psi,P,Q) and thus Psi+F(P) = Psi+F(Q).
2. R(U,T). Follows from R*(Psi,Q,P) and definitions.
3. Forall Psi' . R({Psi'}|T, {Psi'}|U). Follows from Forall Psi' . R*(Psi'+Psi,P,Q), Definitions and Conjecture 2.
4. T -a-> T implies exists U' . U -a-> U' and R(T',U'): So assume T -a-> T'. Then by T={Psi}|P and Conjecture 1b we get P' such that Psi |> P -a-> P'. By R*(Psi,P,Q) we get Psi |> Q -a-> Q
R*(Psi,P',Q'). By conjecture 1a we get {Psi}|Q -a-> {Psi}|Q'. So choose U' = {Psi}|Q'. We thus have U -a-> U', and by R*(Psi,P',Q') and definition also R(T',U').
QED

Corollary
P ~ Q iff there exists an MJbisim R such that R(P,Q)
Proof.
=>: Suppose P ~ Q. Then there is a bisimulation R* such that R*(bot,P,Q). Define R as in Lemma 2, using this R*. It follows that R is an MJbisim and R(0|P, 0|Q), and therefore R U {(P
MJ-bisimulation up to ~. By Conjecture 3 there is than an MJbisim as required.
<=: Suppose R is an MJ-bisimulation up to ~ and R(P,Q). Then R(0|P, 0|Q). By Conjecture 3 there is an MJbisim R' such that R'(0|P,0|Q). So by Lemma 1 there is a bisimulation (up to ~
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{Psi}|Q -a-> T with R({Psi}|P',T). Conjecture 1b then gives that there exists a Q' such that T = {Psi}|Q' and {Psi} |> Q -a-> Q'. Also R({Psi}|P', {Psi}|Q') by definition implies R*(Psi,P',Q'), a
QED

Lemma 2.
If R* is a bisimulation then R = def {({Psi}|P, {Psi}|Q): R*(Psi,P,Q)} is an MJbisim up to ~.
Proof. We need to check 4 conditions. Assume R(T,U). By definition there are Psi,P,Q s.t. T={Psi}|P, U={Psi}|Q, R*(Psi,P,Q).
1. F(T)=F(U). Follows from R*(Psi,P,Q) and thus Psi+F(P) = Psi+F(Q).
2. R(U,T). Follows from R*(Psi,Q,P) and definitions.
3. Forall Psi' . R({Psi'}|T, {Psi'}|U). Follows from Forall Psi' . R*(Psi'+Psi,P,Q), Definitions and Conjecture 2.
4. T -a> T implies exists U' . U -a-> U' and R(T',U'): So assume T -a-> T'. Then by T={Psi}|P and Conjecture 1b we get P' such that Psi |> P -a-> P'. By R*(Psi,P,Q) we get Psi |> Q -a-> (
R*(Psi,P',Q'). By conjecture 1a we get {Psi}|Q -a-> {Psi}|Q'. So choose U' = {Psi}|Q'. We thus have U -a-> U', and by R*(Psi,P',Q') and definition also R(T',U').
QED

Corollary
P ~ Q iff there exists an MJbisim R such that R(P,Q)
Proof.
=>: Suppose P ~ Q. Then there is a bisimulation R* such that R*(bot,P,Q). Define R as in Lemma 2, using this R*. It follows that R is an MJbisim and R(0|P, 0|Q), and therefore R U {(P
MJ-bisimulation up to ~. By Conjecture 3 there is than an MJbisim as required.
<=: Suppose R is an MJ-bisimulation up to ~ and R(P,Q). Then R(0|P, 0|Q). By Conjecture 3 there is an MJbisim R' such that R'(0|P,0|Q). So by Lemma 1 there is a bisimulation (up to ~
R*(bot,P,Q), which implies P~Q.

Entire manual proof
from email archive
70 lines text
2h work

A binary relation R on agents is an MJbisim if R(P,Q) implies

1. F(P)=F(Q) (static equiv)
2. R(Q,P)
3. Forall Psi. R({Psi}|P, {Psi}|Q)
4. Forall a s.t. bn(a)#Q. P -a-> P' => Q-a->Q' and R(P',Q')
(here transitions without assertion means bottom assertion)

Conjecture 1.
a) Psi |> P -a-> P' implies {Psi}|P -a-> {Psi}|P'.
b) {Psi}|P -a-> T implies exists P'. T = {Psi}|P' and Psi |> P -a-> P'
Proof: For a: by the PAR rule and F({Psi})=Psi. For b: case analysis on deirivation of {Psi}|P

Conjecture 2.
{Psi}|{Psi'} ~ {Psi+Psi'}
Proof: Directly from definitions. Obvious :)

Conjecture 3. If R is an MJbisim up to ~ and R(P,Q) then there is an MJbisim R' such that R
Proof: By intimidation :)

Lemma 1
If R is an MJbisim then R* =def {(Psi, P,Q): R({Psi}|P, {Psi}|Q)} is a bisimulation up to ~
Proof. We need to check 4 conditions. Assume R*(Psi,P.Q). Then R({Psi}|P, {Psi}|Q).
1. Psi + F(P) = Psi + F(Q). Follows from F({Psi}|P) = F({Psi}|Q).

```
lemma bisimContextBisimPar:
  fixes Ψ :: 'b
  and   P  :: "('a, 'b, 'c) psi"
  and   Q  :: "('a, 'b, 'c) psi"

  assumes "Ψ ⊳ P ∼ Q"

  shows "⦃Ψ⦄ ∥ P ∼c ⦃Ψ⦄ ∥ Q"
proof -
  let ?X = "{(⦃Ψ⦄ ∥ P, ⦃Ψ⦄ ∥ Q) | Ψ P Q. Ψ ⊳ P ∼ Q}"
  from assms have "(⦃Ψ⦄ ∥ P, ⦃Ψ⦄ ∥ Q) ∈ ?X" by blast
  thus ?thesis
  proof(coinduct rule: contextBisimWeakCoinduct)
    case(cStatEq P Q)
    thus ?case by(auto dest: bisimE)
  next
    case(cSim P Q)
    have "eqvt ?X" by(force dest: bisimClosed simp add: eqvt_def)
    hence "eqvt({(𝟭, P, Q) | P Q. (P, Q) ∈ ?X})"
      by(auto simp add: eqvt_def permBottom)
    thus ?case using cSim by(blast dest: bisimE intro: contextSimAssertionId)
  next
    case(cExt Ψ PsiP PsiQ)
    from `(PsiP, PsiQ) ∈ ?X` obtain Ψ' P Q where "Ψ' ⊳ P ∼ Q" and A: "PsiP = ⦃Ψ'⦄ ∥ P"
                                                            and B: "PsiQ = ⦃Ψ'⦄ ∥ Q" by auto
    from `Ψ' ⊳ P ∼ Q` have "Ψ' ⊗ Ψ ⊳ P ∼ Q" by(rule bisimE)
    hence "Ψ ⊗ Ψ' ⊳ P ∼ Q" by(metis statEqBisim Commutativity)
    hence "Ψ ⊳ ⦃Ψ'⦄ ∥ P ∼ ⦃Ψ'⦄ ∥ Q" by(rule_tac bisimParPresAuxSym) auto
    with A B show ?case by blast
  next
    case(cSym P Q)
    thus ?case by(blast dest: bisimE)
  qed
qed
```

Corresponding Isabelle proof
475 lines text
8h work

```
lemma bisimContextBisimPar:
  fixes Ψ :: 'b
  and   P  :: "('a, 'b, 'c) psi"
  and   Q  :: "('a, 'b, 'c) psi"

  assumes "Ψ ⊳ P ∼ Q"

  shows "⦃Ψ⦄ ∥ P ∼c ⦃Ψ⦄ ∥ Q"
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  let ?X = "{(⦃Ψ⦄ ∥ P, ⦃Ψ⦄ ∥ Q) | Ψ P Q. Ψ ⊳ P ∼ Q}"
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  thus ?thesis
  proof(coinduct rule: contextBisimWeakCoinduct)
    case(cStatEq P Q)
    thus ?case by(auto dest: bisimE)
  next
    case(cSim P Q)
    have "eqvt ?X" by(force dest: bisimClosed simp add: eqvt_def)
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                                                    and B: "PsiQ = ⦃Ψ'⦄ ∥ Q" by auto
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    hence "Ψ ⊗ Ψ' ⊳ P ∼ Q" by(metis statEqBisim Commutativity)
    hence "Ψ ⊳ ⦃Ψ'⦄ ∥ P ∼ ⦃Ψ'⦄ ∥ Q" by(rule_tac bisimParPresAuxSym) auto
    with A B show ?case by blast
  next
    case(cSym P Q)
    thus ?case by(blast dest: bisimE)
  qed
qed
```

Different syntax
Same structure

Lemma 2.
If R* is a bisimulation then R = def {({Psi}IP, {Psi}IQ): R*(Psi,P,Q)} is an MJbisim up to ~.
Proof. We need to check 4 conditions. Assume R(T,U). By definition there are Psi,P,Q s.t. T={Psi}IP, U={Psi}IQ, R*(Psi,P,Q).
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2. R(U,T). Follows from R*(Psi,Q,P) and definitions.
3. Forall Psi' . R({Psi'}IT, {Psi'}IU). Follows from Forall Psi' . R*(Psi'+Psi,P,Q), Definitions and Conjecture 2.
4. T -a> T implies exists U' . U -a-> U' and R(T',U'): So assume T -a-> T'. Then by T={Psi}IP and Conjecture 1b we get P' such that Psi I> P -a-> P'. By R*(Psi,P,Q) we get Psi I> Q -a-> Q' and R*(Psi,P',Q'). By conjecture 1a we get {Psi}IQ -a-> {Psi}IQ'. So choose U' = {Psi}IQ'. We thus have U -a-> U', and by R*(Psi,P',Q') and definition also R(T',U').
QED

```
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proof -
  let ?X = "{(⦃Ψ⦄ || P, ⦃Ψ⦄ || Q) | Ψ P Q. Ψ ⊳ P ∼ Q}"
  from assms have "(⦃Ψ⦄ || P, ⦃Ψ⦄ || Q) ∈ ?X" by blast
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    case(cExt Ψ PsiP PsiQ)
    from `(PsiP, PsiQ) ∈ ?X` obtain Ψ' P Q where "Ψ' ⊳ P ∼ Q" and A: "PsiP = ⦃Ψ'⦄ || P"
                                               and B: "PsiQ = ⦃Ψ'⦄ || Q" by auto
    from `Ψ' ⊳ P ∼ Q` have "Ψ' ⊗ Ψ ⊳ P ∼ Q" by(rule bisimE)
    hence "Ψ ⊗ Ψ' ⊳ P ∼ Q" by(metis statEqBisim Commutativity)
    hence "Ψ ⊳ ⦃Ψ'⦄ || P ∼ ⦃Ψ'⦄ || Q" by(rule_tac bisimParPresAuxSym) auto
    with A B show ?case by blast
  next
    case(cSym P Q)
    thus ?case by(blast dest: bisimE)
  qed
qed
```

The cost?

One measure of effort: **”manhours”**

This particular proof:

Isabelle effort is four times the manual proof

In general

This factor varies wildly

The cost?

One measure of effort: **”manhours”**

Theory development is not exclusively
- not even mainly -
about writing down proofs.

So the factor is not so important.

The cost?

Study of time spent by 4 persons over 25 months on developing the Psi framework

The cost?

Study of time spent by 4 persons over 25 months on developing the Psi framework

1/3 of the effort went into Isabelle formalisation

2/3 of the total effort has been fully formalised