Introductory psi-calculus exercises

Some solutions are wholly or partially hinted in the reading material. You might want to try to formulate answers before looking it up. If you want, work in small groups. These exercises are not part of the examination but will help you understand what a psi-calculus is.

1. Formulate a psi-calculus corresponding to the pi-calculus, and prove all requirements.

2. Extend it by letting the terms be tuples of names. Do you now get a psi-calculus corresponding to the polyadic pi-calculus? (Hint: substitution must be a total function, though of course it could yield an error symbol for insensible things - can it be defined to satisfy the requirements?)

3. Extend the psi-calculus from exercise 1 to contain integers and arithmetic operations on those. An idea is to let the evaluation for expressions be part of the substitution, e.g. $(x+3)[x := 5] = 8$. The psi-calculus probably contains a lot of junk: agents that do not make sense, e.g. using integers as channels. Can you formulate typing constraints on agents such that all well typed agents are sensible? Will this typing satisfy subject reduction (i.e. if P is well typed and $P \xrightarrow{\alpha} P'$ then P' is well typed)?

4. A constraint calculus contains a set of base facts. A constraint store is simply a subset of these facts. In a corresponding psi-calculus a **tell** action of some facts simply corresponds to asserting these facts. Formulate this as a psi-calculus and prove the requirements. Suppose that we want to model a construct such as **tell** f **then** P, where P can execute only after f has been asserted. Can this be represented in the psi-calculus?

5. Try to extend exercise 4 by adding retracts of facts. A main problem is that relative timing now is important: it matters which of **tell** f and **retract** f occurs first in **tell** f $\mid$ **retract** f .If you give up turn to the next page and ponder the solution. Explain it to someone else and discuss its merits.

A constraint calculus with retracts (solution by Johannes Åman Pohjola)

Assume a base logic with facts F (to be used in **tell** and **retract** constructs) ranged over by f and consequences CO (to be used in **ask** constructs) ranged over by c, with associated implication $\Rightarrow \subseteq P^{\text{fin}}(F) \times CO$. Here $P^{\text{fin}}(F)$ is is the subsets of F with finite support. Define a psi-calculus as follows:

$$
\begin{array}{ll}
\mathbf{A} & = P^{\text{fin}}(F) \\
\mathbf{C} & = CO \cup F \cup \{-f : f \in F\} \\
\Psi \otimes \Psi' & = (\Psi \cup \Psi') - (\Psi \cap \Psi') \\
\mathbf{1} & = \emptyset \\
\Psi \vdash c & \text{if } \Psi \Rightarrow c \\
\Psi \vdash f & \text{if } f \in \Psi \\
\Psi \vdash -f & \text{if } f \notin \Psi
\end{array}
$$

Guarded **tell** and **retract** can be formulated as

$$
\begin{array}{ll}
\alpha \,.\, \mathbf{tell}\, f & \texttt{if } -f \texttt{ then } \alpha \,.\, (\!|\{f\}|\!) \texttt{ else } \alpha \,.\, \mathbf{0} \\
\alpha \,.\, \mathbf{retract}\, f & \texttt{if } f \texttt{ then } \alpha \,.\, (\!|\{f\}|\!) \texttt{ else } \alpha \,.\, \mathbf{0} \\
\mathbf{ask}\; c \;\mathbf{then}\; P \;\mathbf{else}\; Q & \texttt{if } c \texttt{ then } P \texttt{ else } Q
\end{array}
$$