Divide and Conquer: Towards Faster Pseudo-Boolean Solving

Jan Elffers

KTH Royal Institute of Technology

NordConsNet Workshop 2018
Gothenburg, Sweden
May 29, 2018

Joint work with Jakob Nordström

To appear at IJCAI-ECAI 2018
The Boolean satisfiability (SAT) problem:

Given Boolean variables $x_1, \ldots, x_n$ and set of clauses $C_1, \ldots, C_m$, is there assignment to the variables satisfying all clauses?

Example:

$$(x_1 \lor x_2) \land (x_1 \lor x_3) \land (x_2 \lor x_3)$$

Clauses are disjunctions of literals $x$, $\overline{x}$. 
Introduction

Encoding as SAT used to solve various problems:

- Planning and scheduling problems.
- Hardware verification problems.
- Problems in combinatorics.

Much progress on so-called *SAT solvers* in past decades [BS97, MS99, MMZ⁺01].

Main algorithm: CDCL (Conflict Driven Clause Learning)
The pseudo-Boolean SAT problem

Limitation of propositional SAT:
Clauses are fairly bad at encoding real-world constraints.

We consider the generalization of SAT to linear inequalities.
\((x_1 \lor x_2) \land (x_1 \lor x_3) \land (x_2 \lor x_3)\) is equivalent to \(x_1 + x_2 + x_3 \geq 2\).
The pseudo-Boolean SAT problem

We represent linear inequalities over \{0, 1\} in normalized form:

- All inequalities are of type $\geq$.
- Negative coefficients replaced by negative literals.

$x_1 + x_2 + x_3 \leq 1$ becomes $\overline{x}_1 + \overline{x}_2 + \overline{x}_3 \geq 2$.

We call the right hand side the *degree*.

We use $c$ for coefficients and $\ell$ for literals.
Cutting planes proof system

Given a set of linear inequalities including \( x_i \geq 0, \overline{x}_i \geq 0 \ \forall i \).

Rules:

- **Addition:**
  \[
  \sum c_i \ell_i \geq w \quad \sum c'_i \ell'_i \geq w' \\
  \sum c_i \ell_i + \sum c'_i \ell'_i \geq w + w'
  \]

- **Multiplication:** for all positive integers \( d \),
  \[
  \sum c_i \ell_i \geq w \quad \sum d \cdot c_i \ell_i \geq d \cdot w
  \]

- **Division:** for all positive integers \( d \),
  \[
  \sum c_i \ell_i \geq w \quad \sum \lceil c_i/d \rceil \ell_i \geq \lceil w/d \rceil
  \]

Exponentially stronger than proof system underlying CDCL.
Earlier pseudo-Boolean SAT solvers

Conversion to clauses ("resolution-based"):  
- MiniSat+ [ES06]  
- Sat4j [LP10]  
- OpenWBO [MML14]  
- NaPS [SN15]

Reasoning with linear inequalities ("cutting planes-based"):  
- Galena [CK05]  
- Pueblo [SS06]  
- Sat4j [LP10]
Our pseudo-Boolean SAT solver

We present a new pseudo-Boolean SAT solver, *RoundingSat*. Strengths:

- Reasons with linear inequalities, so more formulas solvable.
- Highly optimized, written in C++.
The CDCL algorithm

Backtracking search, enhanced with
  ▶ Unit propagation.
  ▶ Clause learning.
If all but one literals in a clause falsified:

\[ x_1 \lor x_2 \lor x_3 \lor x_4 \]

then last literal must be satisfied:

\[ x_1 \lor x_2 \lor x_3 \lor x_4 \]

Unit propagation uses this rule to find implications. If \( C \) propagates \( \ell \), then \( C \) is the reason of \( \ell \).
The CDCL algorithm: clause learning

If unit propagation falsifies a clause, derive a *learnt clause*. Learnt clause directs search away from the conflicting state.
PB extension of CDCL

Early developments: [DG02, CK05].
  ▶ Extend unit propagation.
  ▶ Extend clause learning to pseudo-Boolean learning.
PB extension of CDCL: unit propagation

One uses slack function:
for $C = \sum c_i \ell_i \geq w$, $\rho$ partial assignment,

$$slack(C, \rho) = \sum_{\ell_i \text{ not falsified by } \rho} c_i - w$$

Lower slack $\Rightarrow$ closer to propagating.
PB extension of CDCL: learning

We use generalized resolution to combine linear inequalities.

Takes linear combination such that some variable occurring with opposite signs cancels.

\[
\text{Res}(2x + y \geq 1, \overline{x} + \overline{z} \geq 1, x)
\]

\[
= \text{Res}(2x + y \geq 1, 2\overline{x} + 2\overline{z} \geq 2, x)
\]

\[
= 2x + y + 2\overline{x} + 2\overline{z} \geq 1 + 2
\]

\[
= y + 2\overline{z} \geq 1
\]
Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
- $C' : 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3.$
PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.
- $C' : 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.

We set $x_1 = 0$. 
PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
- $C' : 2\bar{x}_1 + 2\bar{x}_2 + 2\bar{x}_3 + 2\bar{x}_4 \geq 3.$

We set $x_1 = 0$.

$C$ propagates $x_2$, $x_3$ and $x_4$. 
Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
- $C' : 2\bar{x}_1 + 2\bar{x}_2 + 2\bar{x}_3 + 2\bar{x}_4 \geq 3.$

We set $x_1 = 0$.

$C$ propagates $x_2, x_3$ and $x_4$.

Now $C'$ is falsified, so we start conflict analysis.
PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
- $C' : 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3.$

We set $x_1 = 0$.

$C$ propagates $x_2, x_3$ and $x_4$.

Now $C'$ is falsified, so we start conflict analysis.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = C'$.
- $\text{reason}(x_2, \rho) = \text{reason}(x_3, \rho) = \text{reason}(x_4, \rho) = C.$
PB extension of CDCL: learning

while termination criterion does not hold do
    \( \ell \leftarrow \) literal assigned last on the trail \( \rho \);
    if \( \bar{\ell} \) occurs in \( C_{\text{confl}} \) then
        \( C_{\text{reason}} \leftarrow \text{reason}(\ell, \rho) \);
        \( C_{\text{reason}} \leftarrow \text{reduceReason}(C_{\text{reason}}, C_{\text{confl}}, \ell, \rho) \);
        \( C_{\text{confl}} \leftarrow \text{Res}(C_{\text{confl}}, C_{\text{reason}}, \bar{\ell}) \);
    end
    \( \rho \leftarrow \text{removeLast}(\rho) \);
end
return \( C_{\text{confl}} \);

(Green: new compared to CDCL)
PB extension of CDCL: reason reduction

We discuss the method of [CK05] and the one of RoundingSat.

Operations used:

- **Weakening**: if $x_1 + x_2 + x_3 \geq 2$, then $x_1 + x_2 \geq 1$.
- **Saturation**: if $x + 3y \geq 2$, then $x + 2y \geq 2$.
- **Division**: as defined before,

$$\frac{\sum c_i \ell_i \geq w}{\sum \lceil c_i / d \rceil \ell_i \geq \lceil w / d \rceil}$$
Reason reduction example.

- \( \rho = (\overline{x}_1, x_2, x_3, x_4) \).
- \( C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3. \)
- \( \ell = x_4, \text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6. \)

\[ C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \]
Reason reduction example.

- $\rho = (\overline{x_1}, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x_1} + 2\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

\[ C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \]

1. Try generalized resolution.

2.
Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$

\[
\text{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : x_5 \geq 1
\]

1. Try generalized resolution.
2.
Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{confl} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $reason(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$C_{reason} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$

$Res(C_{confl}, C_{reason}, \overline{x}_4) : x_5 \geq 1$

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

1. \( \rho = (\overline{x}_1, x_2, x_3, x_4) \).
2. \( C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3 \).
3. \( \ell = x_4, \ text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \).

\[ C_{\text{reason}} : 2x_1 + 2x_3 + 2x_4 + x_5 \geq 4 \]

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

- \( \rho = (\overline{x_1}, x_2, x_3, x_4) \).
- \( C_{\text{confl}} = 2\overline{x_1} + 2\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} \geq 3 \).
- \( \ell = x_4, \ r(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \).

\[ C_{\text{reason}} : 2x_1 + 2x_3 + 2x_4 + x_5 \geq 4 \]

1. **Try generalized resolution.**
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $reason(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}}: 2x_1 + 2x_3 + 2x_4 + x_5 \geq 4$$

$$Res(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4): 2\overline{x}_2 + x_5 \geq 1$$

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

► $\rho = (x_1, x_2, x_3, x_4)$.
► $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
► $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_3 + 2x_4 + x_5 \geq 4$$

$$\text{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : 2\overline{x}_2 + x_5 \geq 1$$

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_4 + x_5 \geq 2$$

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction example.

- \( \rho = (\overline{x}_1, x_2, x_3, x_4) \).
- \( C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3 \).
- \( \ell = x_4, \overline{\text{reason}}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \).

\[
C_{\text{reason}} : 2x_1 + 2x_4 + x_5 \geq 2
\]

1. **Try generalized resolution.**
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_4 + x_5 \geq 2$$

$$\text{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : 2\overline{x}_2 + 2\overline{x}_3 + x_5 \geq 1$$

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction example.

- $\rho = (\overline{x_1}, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x_1} + 2\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

\[C_{\text{reason}} : 2x_1 + 2x_4 + x_5 \geq 2\]

\[\text{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x_4}) : 2\overline{x_2} + 2\overline{x_3} + x_5 \geq 1\]

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

\[ C_{\text{reason}} : 2x_1 + 2x_4 \geq 1 \]

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{conf}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : \quad x_1 + x_4 \geq 1$$

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

\[C_{\text{reason}}: \ x_1 + x_4 \geq 1\]

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x_1}, x_2, x_3, x_4)$.
- $C_{confl} = 2\overline{x_1} + 2\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} \geq 3$.
- $\ell = x_4$, $reason(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

\[ C_{reason} : x_1 + x_4 \geq 1 \]

\[ Res(C_{confl}, C_{reason}, \overline{x_4}) : 2\overline{x_2} + 2\overline{x_3} \geq 1 \]

1. Try generalized resolution.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction example.

- \( \rho = (\overline{x}_1, x_2, x_3, x_4) \).
- \( C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3 \).
- \( \ell = x_4, \text{ reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \).

\[
C_{\text{reason}} : \quad x_1 \quad + \quad x_4 \quad \geq \quad 1
\]

\[
\text{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : \quad 2\overline{x}_2 + 2\overline{x}_3 \geq 1
\]

1. **Try generalized resolution.** Works, so terminate.
2. If not falsified, weaken non-falsified literal and saturate.
Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$
Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x_1}, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x_1} + 2\overline{x_2} + 2\overline{x_3} + 2\overline{x_4} \geq 3$.
- $\ell = x_4$, $\text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

1. Weaken non-falsified literals in $C_{\text{reason}}$ with coefficient not divisible by coefficient of $x_4$. 
Reason reduction of RoundingSat

Same example:

- \( \rho = (\overline{x}_1, x_2, x_3, x_4) \).
- \( C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3 \).
- \( \ell = x_4, \text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \).

\[
C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 \geq 5
\]

1. Weaken non-falsified literals in \( C_{\text{reason}} \) with coefficient not divisible by coefficient of \( x_4 \).
Reason reduction of RoundingSat

Same example:

- \( \rho = (\overline{x}_1, x_2, x_3, x_4) \).
- \( C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3 \).
- \( \ell = x_4, \text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6 \).

\[ C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 \geq 5 \]

1. Weaken non-falsified literals in \( C_{\text{reason}} \) with coefficient not divisible by coefficient of \( x_4 \).
2. Divide by coefficient of \( x_4 \).
Reason reduction of RoundingSat

Same example:

\[ \rho = (\overline{x}_1, x_2, x_3, x_4). \]
\[ C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3. \]
\[ \ell = x_4, \ \text{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6. \]

\[ C_{\text{reason}} : \ x_1 + x_2 + x_3 + x_4 \geq 3 \]

1. Weaken non-falsified literals in \( C_{\text{reason}} \) with coefficient not divisible by coefficient of \( x_4 \).
2. Divide by coefficient of \( x_4 \).
Experimental results: PB16 decision track, small integers

Entries: number of solved instances (satisfiable + unsatisfiable)

**Bold**: solver is (one of) the best in category

<table>
<thead>
<tr>
<th></th>
<th>RoundingSat</th>
<th>Sat4j Res+CP</th>
<th>Sat4j Res</th>
<th>Open-WBO</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB05 aloul</td>
<td>36 + 21</td>
<td>36 + 21</td>
<td>36 + 3</td>
<td>36 + 6</td>
</tr>
<tr>
<td>PB06 manquiho</td>
<td>14 + 0</td>
<td>14 + 0</td>
<td>14 + 0</td>
<td>3 + 0</td>
</tr>
<tr>
<td>PB06 ppp-problems</td>
<td>4 + 0</td>
<td>4 + 0</td>
<td>4 + 0</td>
<td>3 + 0</td>
</tr>
<tr>
<td>PB06 uclid</td>
<td>1 + 47</td>
<td>1 + 47</td>
<td>1 + 47</td>
<td>1 + 49</td>
</tr>
<tr>
<td>PB06 liu</td>
<td>16 + 0</td>
<td>16 + 0</td>
<td>16 + 0</td>
<td>17 + 0</td>
</tr>
<tr>
<td>PB06 namasivayam</td>
<td>72 + 128</td>
<td>72 + 128</td>
<td>72 + 128</td>
<td>72 + 128</td>
</tr>
<tr>
<td>PB06 prestwich</td>
<td>10 + 0</td>
<td>11 + 0</td>
<td>9 + 0</td>
<td>14 + 0</td>
</tr>
<tr>
<td>PB06 roussel</td>
<td>0 + 22</td>
<td>0 + 22</td>
<td>0 + 4</td>
<td>0 + 4</td>
</tr>
<tr>
<td>PB10 oliveras</td>
<td>34 + 32</td>
<td>34 + 32</td>
<td>34 + 33</td>
<td>34 + 33</td>
</tr>
<tr>
<td>PB11 heinz</td>
<td>2 + 0</td>
<td>2 + 0</td>
<td>2 + 0</td>
<td>2 + 0</td>
</tr>
<tr>
<td>PB11 lopes</td>
<td>42 + 26</td>
<td>37 + 25</td>
<td>37 + 25</td>
<td>33 + 28</td>
</tr>
<tr>
<td>PB12 sroussel</td>
<td>31 + 0</td>
<td>21 + 0</td>
<td>23 + 0</td>
<td>29 + 1</td>
</tr>
<tr>
<td>PB16 elffers</td>
<td>0 + 287</td>
<td>0 + 229</td>
<td>0 + 142</td>
<td>0 + 213</td>
</tr>
<tr>
<td>PB16 nossum</td>
<td>68 + 0</td>
<td>39 + 0</td>
<td>39 + 0</td>
<td>55 + 0</td>
</tr>
<tr>
<td>PB16 quimper</td>
<td>43 + 214</td>
<td>43 + 213</td>
<td>43 + 213</td>
<td>46 + 241</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>373 + 777</td>
<td>330 + 717</td>
<td>330 + 595</td>
<td>345 + 703</td>
</tr>
</tbody>
</table>
Experimental results

- RoundingSat dominates Sat4j (both versions).
- RoundingSat and Sat4j Res+CP better than resolution-based solvers on 3 categories.
- OpenWBO sometimes better than RoundingSat, sometimes worse.
Conclusion

RoundingSat shows that reasoning with linear inequalities can be competitive on many different domains. And sometimes, it is crucial for performance.

Future work:
- Extend to optimization track in non-trivial way.
Conclusion

RoundingSat shows that reasoning with linear inequalities can be competitive on many different domains. And sometimes, it is crucial for performance.

Future work:
- Extend to optimization track in non-trivial way.

Thank you!
References I

Roberto J. Bayardo Jr. and Robert Schrag.
Using CSP look-back techniques to solve real-world SAT instances.

Donald Chai and Andreas Kuehlmann.
A fast pseudo-Boolean constraint solver.

Heidi E. Dixon and Matthew L. Ginsberg.
Inference methods for a pseudo-Boolean satisfiability solver.


Matthew W. Moskewicz, Conor F. Madigan, Ying Zhao, Lintao Zhang, and Sharad Malik.
Chaff: Engineering an efficient SAT solver.

João P. Marques-Silva and Karem A. Sakallah.
GRASP: A search algorithm for propositional satisfiability.
Preliminary version in *ICCAD ’96*.

Masahiko Sakai and Hidetomo Nabeshima.
Construction of an ROBDD for a PB-constraint in band form
and related techniques for PB-solvers.
Hossein M. Sheini and Karem A. Sakallah.
Pueblo: A hybrid pseudo-Boolean SAT solver.
Journal on Satisfiability, Boolean Modeling and Computation, 
Preliminary version in DATE ’05.